ALGEBRAICAL EXERCISES

WITH

SOLUTIONS

IN TWO PARTS.

COMPILED FROM
DR. WOOD'S ALGEBRA, CAMBRIDGE, DUBLIN, CALCUTTA,
MADRAS, BOMBAY AND OTHER SCHOOL AND COLLEGE
EXAMINATION PAPERS, REMODELLED, SIMPLI-
FIED, ARRANGED IN PROGRESSIVE ORDER,
AND ADAPTED TO GENERAL USE
IN SCHOOLS.

BY

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> "As tennis is a game of no use in itself, but of great use in respect it maketh a quick eye, and a body ready to put itself into all postures, so in the Mathematics, that use which is collateral and intervenient is no less worthy than that which is principal and intended."
>
> *Bacon, Advancement of Learning.*

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ALGEBRAICAL

EXERCISES.

Exercise 1.

NOTATION.

If $a=1$, $b=2$, $c=3$, $d=4$, $e=5$, $f=6$, find the numerical values of the following expressions : -

1. $6a+5b+3c+d$ Ans 29
2. $4c+2d+f-2a$ Ans 24
3. $4ac+5bd+9d-f$ Ans 82
4. $4abc-bcd+cdf-abd$ Ans 64
5. $abcd+bcde+abde$ Ans 184
6. $14a+13b+12c+11d+10e+9f$ Ans 224
7. $\frac{4a}{b}+\frac{8c}{3}+\frac{8d}{e}-\frac{a}{f}$ Ans $16\frac{7}{30}$
8. $\frac{abc}{d}+\frac{bcd}{a}-\frac{2bdf}{ae}$ Ans $6\frac{3}{10}$
9. $8a+bcd-\frac{2ab}{4}$ Ans 31
10. $\frac{2a+5b}{c}+\frac{3b+2c}{d}-\frac{a+b+c+d}{2e}$ Ans 6
11. $\frac{b+c+3e}{e+c-d}$ Ans 5
12. $\frac{2a+b}{2a-b}+\frac{c+d}{c+2d}+\frac{e+f}{f-a}$ Ans $6\frac{46}{55}$

If $a=1$, $b=2$, $c=3$, $d=4$, $e=0$, find the values of the followin expressions: –

13. $7abc-5bcd+4cde+3acd-10ace$ Ans -42

14. $a-2b+3c+4d-5e$ Ans 22

15. $\frac{2d}{\text{[illegible]}}-\frac{be}{\text{[illegible]}}-\frac{15a}{\text{[illegible]}}+\frac{4b}{\text{[illegible]}}$ Ans $8\frac{3}{4}$

If $a=8$, $b=6$, $c=4$, find the value of :

16. $\frac{5a}{b+c}-\frac{b}{a+c}+\frac{a}{b-c}-\frac{b}{a-c}+\frac{c}{b+2c}$ Ans 6

If $a=1$, $b=2$, $c=3$, $d=4$, $e=5$, $f=0$, find the values of: –

17. $a^2+b^2+(2c)^2+d^2$ Ans 57

18. $e^3+d^3-c^3-b^3$ Ans 154

19. $a^2b^2c^2+b^2c^2d^2-f^2$ Ans 612

20. $a^3+b^3+c^3-20$ Ans 16

21. $a^4+3a^3b+3ab^3+b^4$ Ans 47

22. $d^4+4b^2d^2+b^4-2a^2b-3ab^2$ Ans 512

23. $\frac{b^2c^2}{2a}+\frac{8ab}{d^2}-\frac{16}{a^2}$ Ans 3

24. $\frac{2f+c}{a-f}+\frac{a+9}{3c-2}$ Ans $6\frac{3}{7}$

25. $\frac{a^2+b^2}{c}+\frac{c^2+e^2}{b}+\frac{e^2-d^2}{c}+\frac{a}{b}$ Ans $21\frac{1}{2}$

26. $\frac{8a^2+3b^2}{a^2+b^2}+\frac{4c^2+6b^2}{c^2-b^2}+\frac{c^2+d^2}{c^2}$ Ans 17

27. $\frac{4}{a+b^2+c}+\frac{6}{a^2+b+c}+\frac{12}{d^3+c^3-a^3}$ Ans $1\frac{19}{30}$

28. $\frac{a^3+3a^2b+3ab^2+b^3}{a^2+2ab+b^2}$ Ans 3

29. $\frac{d^{t}}{\text{[illegible]}d}$ Ans $\frac{16}{81}$

30. $\dfrac{b^{2c} - c^{2c}}{b^c - c^c}$ Ans 35

31. $\dfrac{2 + b^c + d^c}{b^2 + d^2 - ad}$ Ans $4\frac{5}{8}$

32. $\dfrac{e^c - d^c}{e^2 + ed + d^2}$ Ans 1

If $a=1$, $b=3$, $c=5$, $d=0$, find the values of: –

33. $\dfrac{a^2b^2+1}{a^2+b^2} - \dfrac{1+a^2c^2}{a^2+c^2} + \dfrac{4a+b^2+b^2c^2}{b^2+c^2} - \dfrac{a^2+2ab+b^2}{b^2-2bc+c^2}$ Ans 3

34. $a^4 - 7a^3b + 6a^2b^2 - 4ab^3 + b^4$ Ans [illegible]

35. $3a^2 + \dfrac{2ab^2}{c} - \dfrac{c^3}{b^2}$ Ans $-7\frac{13}{45}$

36. $\dfrac{a^2 + b^2 - c^2 + 2ab}{a^2 - b^2 - c^2 + 2bc}$ Ans 3

If $x = 2$, find the values of: –

37. $x^2 + (x^2 - 42x + 89)^{\frac{1}{2}}$ Ans 7

38. $(4x^4 - 12x^3 + 12x^2 - 4)^{\frac{1}{3}}$ Ans $\sqrt[3]{12}$

If $a = \frac{1}{4}$, $b = \frac{1}{5}$, find the value of: –

39. $\dfrac{a}{b} - \left(\dfrac{1+a}{1-b}\right)^{\frac{1}{2}}$ Ans 0

If $a=1$, $b=2$, $c=3$, $d=5$, $e=8$, find the values of the following expressions: –

40. $b(c+d)$ Ans 16

41. $d(a+d)$ Ans 30

42. $b^3(a^2+d^2-c^2)$ Ans 68

43. $c^2(e^2+a^2-b^2)$ Ans 549

44. $\dfrac{a^2+b^2+c^2}{a^2+b^2}$ Ans $2\frac{4}{5}$

45. $\sqrt{2b+4d+5e}$ Ans 8

46. $(a+b+2c)(5e+2d-c)$ Ans 423

47. $(a^2+b^2+c^2+d^2)(d^2+e^2-a^2)$ Ans ~~2370~~ [illegible]32

48. $(4c-2a)^2$ Ans 100

49. $c\sqrt{d^2-3c}+d\sqrt{d^2+3c}$ Ans 67

50. $e+(\sqrt{e+1}+1)+(e-\sqrt[3]{e})\sqrt{e-4}$ Ans 27

51 $\sqrt{(a^2+2ab+b^2)}+\sqrt[3]{(a^3+3a^2b+3a\overset{c}{b}^2+b^3)}$ Ans 6

If $a=25$, $b=9$, $c=4$, $d=1$, find the values of –

52 $\sqrt{a}+2\sqrt{b}+3\sqrt{c}+4\sqrt{d}$ Ans 21

53. $\sqrt{(4a)}+\sqrt{(9b)}+\sqrt{(16c)}-\sqrt{(25d)}$ Ans 22

54. $3\sqrt{a}+2\sqrt{(4b)}-4\sqrt{(9c)}+\sqrt{(16d)}$ Ans 7

55. $\sqrt[3]{(5a)}+2\sqrt[3]{(3b)}-\sqrt[3]{(2c)}+4\sqrt[3]{d}$ Ans 13

56. $\sqrt{a^2}-2\sqrt[3]{b^3}+3\sqrt[4]{c^4}-4\sqrt[3]{d}$ Ans 15

57. $\sqrt{(bc)}+3\sqrt{(acd)}-4\sqrt{(b^2d)}+\sqrt{(c^2d^2)}$ Ans 4

If $a=0$, $b=2$, $c=4$, $d=6$, find the values of –

58. $3\sqrt[3]{(2b^2-a)}+2\sqrt[3]{(b^2+c^2+7)}-\sqrt[3]{\{2(b+c)^2-(b+d)^2\}}$

Ans [illegible]

59. $\{a+(b+c)^2-d\}\{(a+b)^2+(d-c)^2\}\{(a+b+c)^2-d\}$

Ans 7200

If $a=1$, $b=2$, $c=3$, [illegible]$=4$, $e=5$, find the numerical values of the following expressions :–

60. $a+4d-e$ Ans 12

61. $(a+2d)(c+4a)-(a+c)(b+d)+(c+d)(d+c)$ Ans 102

62. $\dfrac{2a+b}{c}-\dfrac{4c-2a}{2}+a+d$ Ans $1\frac{1}{3}$

63. $(3a-2c+d)^2-(d+e)^2+(2d+b)^2$ Ans 20

64. $\dfrac{a^3+b^3+c^3-3abc}{a+b+c}$ Ans 3

65. $\dfrac{a^2+2ab+b^2}{a+b}+\dfrac{b^2+2bc+c^2}{b+c}+\left(\dfrac{a}{c}\right)^2$ Ans $8\frac{1}{4}$

66 $\sqrt{}(3c^2 + 4d^2 + 2e - 1)$ Ans 10

67 $\sqrt{}(e^2 + 2d^2 + 3c^2 - a^2 +$ [illegible]$)$ Ans 12

68 $\sqrt[3]{}(4d^2 + 7b + c^2e + 2)$ Ans 5

69 $\sqrt[4]{}(4b^2 + 3c^2 + abcd + 14)$ Ans 3

70 $4bc^2 - \{d^2 - (b^2 + c^2)\}^2$ Ans 135

71 $\{d - (a + b + c)\}\{(d + c) - (a + b)\}$ Ans -8

72. $\{(b + c) - (d - a)\}^2 + \{(c + d) - (b - a)\}^2 + \{(b + d) - (c - a)\}^2$
$+ (b + c + d - a)^2$ Ans 120

If $a = -x = \frac{1}{2}$, $b = 0$, find the value of

73 $x^4 - (a - b)x^3 + (a - b)b^2x - b^4$ Ans $\frac{1}{8}$

If $a - b = x = 3$, and $a + b + x = 2$, find the value of –

74 $(a - b)\{x^2 - 2ax^2 + a^2x - (a + b)b^2\}$ Ans -6

If $a = 5$, $b = 3$, $c = 1$, find the values of –

75 $$\frac{a^3 - b^3}{a^2 + ab + b^2} + \frac{2a + 5b}{2b - c} + \frac{a^2 - c^2}{a + 2b + c}$$ Ans 9

76 $\sqrt{}(5ab^2) + \sqrt[3]{}(9bc) - 2\sqrt[4]{}(3a + b - 2c)$ Ans 14

Exercise 2.

ADDITION

Add together,

1 $4a - 6b$, $2a + 3b$, $a - 2b$, $a + b$ Ans $8a - 4b$

2 $5x^2 + 4y^2$, $4x^2 - 8y^2$, $-2x^2 + y^2$, $4x^2 - y^2$ Ans $11x^2 - 4y^2$

3 $4a + 5b + 6c$, $2a + 3b + c$, $a + b + 3c$ Ans $7a + 9b + 10c$

4 $4x - 2y + z$, $2x - y - z$, $-4x + 4y + 5z$ Ans $2x + y + 5z$

5 $6a + 5b - 4x$, $2a + b + x$, $-7a - b - 2x$ Ans $a + 5b - 5x$

6. $x - 4y + z,\ 4x + 4y - 3z,\ x + y + z$ Ans $6x + y - z$

7. $a + b + c,\ a + b - c,\ a - b + 2c$ Ans $3a + b + 2c$

8. $a - 2b + 3c + 4d,\ 4a + 4b - 4c - d,\ 6a + 2b - c - d$
Ans $11a + 4b - 2c + 2d$

9. $4x^3 - 2x^2 + 6x + 5,\ 2x^3 + 4x^2 - 8x - 6,\ 3x^3 - 8x^2 - 7x + 12$
Ans $9x^3 - 6x^2 - 9x + 11$

10. $x^4 - 2x^3 + 4x^2 - 6x + 2,\ x^4 - 4x^3 + 6x^2,\ 4x^4 + 5x^3 + 6x^2 + 7x + 8,$
$10x^4 - x^2 - 8$ Ans $16x^4 - x^3 + 15x^2 + x + 2$

11. $2x^3 - 4ax^2 + 6ax - 8\ \ 8x^3 + 6ax^2 - 12ax + 2,\ 3x^3 - ax^2 - ax + 2$
Ans $13x^3 + ax^2 - 7ax - 4$

12. $2ab - ax^2 + by^2 + cx,\ 4ab + 4ax^2 - 2by^2 - 4cx,\ -4ab + 2ax^2$
$- 4by^2 - cx$ Ans $2ab + 5ax^2 - 5by^2 - 4cx$

13. $2x^3 + y^4 + 2z^3,\ -4x^3 - 5z^3,\ 4x^3 - 3y^4 + 7z^3$
Ans $2x^3 - 2y^4 + 4z^3$

14. $4x^2 - 4xy + 2y^2 + 2x - 2y + 8,\ 6x^2 - 8xy + 2y^2 - 2x + 15y - 16,$
$-9x^2 + 10xy - 3y^2 - 4x + y + 1$ Ans $x^2 - 2xy + y^2 - 4x + 14y - 7$

15 $4x^4 - 3x^3y + 2x^2y^2 - 4xy^3 + y^4,\ 7x^4 - 2x^3y - x^2y^2 - 8xy^3 - y^4,$
$2x^4 + 6x^3y + y^4,\ x^4 + y^4$ Ans $14x^4 + x^3y + x^2y^2 - 12xy^3 + 2y^4$

16. $x^3 + xy^2 + xz^2 - x^2y - xyz + 4x^2z,\ 4x^3 - 2xy^2 - 4xz^2 - x^2y$
$+ 6xyz - 6x^2z,\ -2x^3 + 4xy^2 + 2xz^2 - 3x^2y - 2xyz + 2x^2z$
Ans $3x^3 + 3xy^2 - xz^2 - 5x^2y + 3xyz$

17. $2\sqrt{(a + x)} - 4\sqrt{(a - x)} + 6\sqrt{(a^2 - x^2)},\ -3\sqrt{(a + x)} + 2\sqrt{(a^2 - x^2)}$
$- 6\sqrt{(a - x)},\ 5\sqrt{(a^2 - x^2)} + 6\sqrt{(a - x)} - 4\sqrt{(a + x)}$
Ans $13\sqrt{(a^2 - x^2)} - 5\sqrt{(a + x)} - 4\sqrt{(a - x)}$

18. $ax^4+bx^3+cx^2+dx+1$, $bx^4+cx^3+dx^2+ex+1$, $cx^4+dx^3+ex^2+x+1$, $-2bx^4-4dx^2+1$

Ans $(a-b+c)x^4+(b+c+d)x^3+(c-3d+e)x^2+(d+e+1)x+4$

19. $mx^4-2x^3+ax^2-bx+y$, $nx^4+mx^3+bx^2+2bx+2y$, $px^4+qx+ny$

Ans $(m+n+p)x^4+(m-2)x^3+(a+b)x^2+bx+(n+3)y$

20. ax^2+bx+c, px^2+qx-c, ax^2-px+q

Ans $(2a+p)x^2+(b+q-p)x+q$

21. $px^m+rx^n+sx^r$, $nx^m-2px^n+mx^r$ and $sx^m-nx^n+4x^r$

Ans $(n+p+s)x^m-(2p-r+n)x^n+(m+s+4)x^r$

Collect the coefficients of x^4 in the following expressions:—

22. $4x^5+6x^4+2x^3+x^2-1$, $-x^4+2x^3+11$, $3x^6+11x^4-6$, and $5x^5+2x^4-12$ Ans 18

23. Find the sum of $2x^6+4x^5-6x^2-x+1$, $8x^5-2x^3+12$, $16x^4+18x^3+4x^2-11$, x^6+1, $7x^5-x^4-x^3+4x^2-2x+1$, and x^4+2x^2+4

Ans $3x^6+19x^5+16x^4+15x^3+4x^2-3x+8$

24. $9x^4+12x^3+6x-11$, $4x-8x^2+6x^3-11$, $8x^2-4x$, $13x^3+6x^2-x+18$ and $11x^5+2x^2+14$

Ans $11x^5+9x^4+31x^3+5x+10$

25. Find the aggregate of $9+x^{-2}+6x^{-4}+8x^{-3}$, $6+x^{-1}+x^{-2}-4x^{-3}$, $8+x^{-3}+4x^{-4}$ and $8x^{-3}-6x^{-1}+3$

Ans $26-5x^{-1}+2x^{-2}+13x^{-3}+10x^{-4}$

26. The addands are $\frac{1}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{7}x^2 + x + \frac{1}{4}$, $\frac{3}{4}x^4 + \frac{1}{4}x^3 + \frac{3}{14}x^2 + \frac{1}{2}x + \frac{2}{7}$ and $2x^4 + \frac{4}{5}x^3 + \frac{3}{11}x^2 + \frac{1}{6}x + \frac{1}{6}$, what is the sum ?

Ans $\frac{13}{4}x^4 + \frac{9}{5}x^3 + \frac{97}{154}x^2 + \frac{5}{3}x + \frac{101}{84}$.

27. The summands are $\frac{2}{7}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \frac{1}{2}$, $\frac{2}{7}x^4 + \frac{4}{5}x^3 - \frac{1}{8}x^2 + \frac{1}{7}x + \frac{1}{4}$ and $\frac{3}{7}x^4 - \frac{2}{3}x^3 - \frac{1}{4}x^2 - \frac{2}{5}x - \frac{1}{6}$; what is the total ?

Ans $x^4 - \frac{1}{5}x^3 + \frac{1}{8}x^3 - \frac{51}{35}x + \frac{1}{12}$.

28. Find the sum of $(2a + b + c)x^3 + (3a + 2b + c)x^2 + (2b - a - 3c)x + a$, $(3a - 3b + c)x^3 + (4a - 5b - 2c)x^2 + (8b - 4a + 6c)x + b$, and $(2b - 5a - 2c)x^3 + (3b - 7a + c)x^2 + (5a - 10b - 3c)x + c$

Ans $a + b + c$.

29. Find the sum of $(a + b)(x + y)^2 + (c + d)(x + y) + cx$, $(a - b)(x + y)^2 + (c - d)(x + y) + cy$ and $-2a(x + y)^2$

Ans $3c(x + y)$.

30. Find the sum of $4x^{\frac{1}{3}} + 5x^{\frac{1}{4}} + 6x^{\frac{1}{5}} + 8$, $8x^{\frac{1}{3}} + 8x^{\frac{1}{4}} + 9x^{\frac{1}{5}} + 11$, and $11x^{\frac{1}{3}} - 11x^{\frac{1}{4}} - 15x^{\frac{1}{5}} - 18$ Ans $23x^{\frac{1}{3}} + 2x^{\frac{1}{4}} + 1$

Exercise 3.

SUBTRACTION.

1. From $8a + 16b$ take $3a + 10b$ Ans $5a + 6b$
2. From $8a - 3b + c$ take $4a - 2b - c$ Ans $4a - b + 2c$
3. From $4a + 3b - 3c + 4d$ take $-4a + 5b - c$ Ans $8a - 2b - 2c + 4d$
4. From $6x^2 - 3x + 1$ take $2x^2 + 5x + 9$ Ans $4x^2 - 8x - 8$
5. From $4x^4 + 2x^3 + 6x^2 - 9x - 2$ take $-14x^4 + 4x^3 - 7x^2 + 2x - 3$ Ans $4x^4 - 2x^3 + 27x^2 - 11x + 1$

6. From $4x^2 - 3ax + 2a^2$ take $2x^2 - 4ax + 3a^2$

Ans $2x^2 + ax - a^2$

7. From $x^4 + 6x^2 + 7x + 8$ take $2x^4 + 2x^3 + 5x^2 - 3x - 2$

Ans $-x^4 - 2x^3 + x^2 + 10x + 10$

8. From $a^4 + 4a^3 + 6a^2 + 4a + 1$ take $-8a^4 + 4a^3 + 6a^2 - 4a - 1$

Ans $9a^4 + 8a + 2$

9. From $ax^2 + bx + c$ take $bx^2 + cx + d$

Ans $(a - b)x^2 + (b - c)x + c - d$

10. From $mx^4 + nx^3 + px^2 + qx + r$ take $2x^4 + 3nx^3 - qx^2 + rx + s$

Ans $(m - 2)x^4 - 2nx^3 + (p + q)x^2 + (q - r)x + r - s$

11. From $4\sqrt[3]{a^2 - y} - 3\sqrt{y - a^2}$ take $2\sqrt{y - a^2} + 3\sqrt[3]{a^2 - y}$

Ans $\sqrt[3]{a^2 - y} - 5\sqrt{y - a^2}$

12. From the sum of the first two of the following expressions $a + b + c + d$, $a - b + c - d$, $a + b - c + d$, take the sum of the last two.

Ans $2c$

13. From $4x^4 - 3x^3 + 2x^2 + 1$ take $2x^4 - 6x^3 + 4x^2 - 1$

Ans $2x^4 + 3x^3 - 2x^2 + 2$

14. From $4a^{\frac{2}{3}} - 6a^{\frac{1}{2}} + 7a^{-3}$ take $5a^{\frac{2}{3}} + 8a^{\frac{1}{2}} - 10a^{-3}$

Ans $-a^{\frac{2}{3}} - 14a^{\frac{1}{2}} + 17a^{-3}$

15. From $x^{-2} + 4x^{-1}y^{-1} + 2y^{-1}$ take $4x^{-2} - 4x^{-1}y^{-1} + 3y^{-1}$

Ans $-3x^{-2} + 8x^{-1}y^{-1} - y^{-1}$

16. The sum of two quantities is $4x^4 + 6x^3 + 3x - 1$, and one of them is $2x^4 - x^3 + 4x + 2$; find the other.

Ans $2x^4 + 7x^3 - x - 3$

17. The subtrahend is $x^p + qx^m + r$ and the remainder is $4x^p + 2qx^m + 3r$; find the minuend.

Ans $5x^p + 3qx^m + 4r$

18. From $(2a + b - c)x^2 + (a + b + c)x + a + b + c$ take $(a + b - c)x^2 + (a + c)x + a + b$

Ans $ax^2 + bx + c$

19. From $12x^{\frac{1}{3}} - 6x^{\frac{1}{4}} + 8x^{\frac{1}{5}}$ take $8x^{\frac{1}{3}} - 2x^{\frac{1}{4}} - 3x^{\frac{1}{5}} + 2$
Ans $-2 + 4x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 11x^{\frac{1}{5}}$

20. From $\frac{5}{7}x^4 - \frac{1}{8}x^2 - 1$ take $\frac{2}{5}x^4 + 8x^3 + \frac{1}{16}x^2 - 2$
Ans $\frac{11}{35}x^4 - 8x^3 - \frac{3}{16}x^2 + 1$

Exercise 4.

BRACKETS.

Simplify the following expressions by removing the brackets.

1. $3a - 2b + (2a - b)$ Ans $5a - 3b$
2 $a + b - c - (a - b - c)$ Ans $2b$
3. $1 - (a - 1) + (a^2 - 1) - (a^2 + a - 1)$ Ans $2 - 2a$
4. $a + 2b + (7a - 4b) - (2a - b) - (4a + 3b)$ Ans $2a - 4b$
5. $a + b - c + (a - b - c) - (a - b + c)$ Ans $a + b - 3c$
6. $2x + 3y + 4z - (x + y + z) - (-x - y - z)$ Ans $2x +$ [illegible] $+ 4z$
7. $x - \{b - c - (d - x)\}$ Ans $c + d - b$
8. $4a - (5b - c) - \{(a - b) - (2b - 2c)\}$ Ans $3a - 2b - c$
9. $3a - \{4b - (2c + a - b)\}$ Ans $4a - 5b + 2c$
10. $4a - \{b - (a - c)\}$ Ans $5a - b - c$
11. $10a - \{4a - (5 - 6a) - 2a\}$ Ans $2a + 5$
12 $2 - \{1 - (1 - a)\} - 4$ Ans $-a - 2$
13. $x - \{-1 - a - (1 - a)\}$ Ans $x + 2$
14. $5a - [2a - \{1 - (2a - 1)\}]$ Ans $a + 2$
15. $4a - (4c - a) - [5c - \{(a - c) - \overline{c - a}\}]$ Ans $7a - 11c$

16. $a-[2b+\{3c-3a-(a+b)\}+\{2a-(b+c)\}]$ Ans $3a-2c$

17. $14-\{4-2x-[1-(3-x)]\}$ Ans $3x+8$

18. $16-x-[7x-\{8x-\overline{2+x}\}]$ Ans x [illegible] 14

19. $x^4-[4x^3-\{6x^2-(4x-1)\}]-(x^4+4x^3+6x^2+4x+1)$
$+8x^3+8x$ Ans 0

20. $-[-1-\{1-(a-\overline{1-a})-a\}]$ Ans $3-3a$

21. $-[-\{-(-\overline{a-1})\}]$ Ans $a-1$

22. $2a+(b-\overline{c+a})-(c-\overline{a+b})$ Ans $2a+2b-2c$

23. $a-[-a-1+\{-1-\overline{a-1}-2\}-1]$ putting Asn $3a+4$

24. $-[-1-\{-\overline{1-2}-1\}]$ Ans 1

25. $x^3-\{x^2+2x+1-(2x^2+3)\}-8$ Ans $2x^2-2x-6$

26. Put the 2nd, 4th and the 6th terms of the following expression within brackets, beginning with the 4th, and the rest within another, beginning with the 3rd. $2a+b-4c-d+e+f$ Ans $-(d-b-f)-(4c-2a-e)$

Exercise 5.

MULTIPLICATION.

Multiply –

1. $2x^4$ by $8x^3$ Ans $16x^7$
2. $3a^2$ by $4a^3$ Ans $12a^5$
3. $2a^2x$ by $3a^4x^2$ Ans $6a^6x^3$
4. $8x^2y^2z^2$ by $5x^5yz^3$ Ans $40x^7y^3z^5$
5. $5a^2x^2$ by $6y^2z^2$ Ans $30a^2x^2y^2z^2$

6. $a^m x^n$ by $a^n x^m$ Ans $a^{m+n}x^{m+n}$

7. $a^{m+2}x^{2n}$ by $a^{2-m}x^{-n+2}$ Ans a^4x^{2n+2}

8. $5x^{\frac{2}{3}}$ by $15x^{\frac{4}{3}}$ Ans $75x^2$

9. $6x^{\frac{2}{5}}$ by $ax^{\frac{2}{3}}$ Ans $6ax^{\frac{16}{15}}$

10. $4a^2 - 4ab$ by $2ab$ Ans $8a^3b - 8a^2b^2$

11. $4a^2 - 7b^2$ by $4a^2b^2$ Ans $16a^4b^2 - 28a^2b^4$

12. $3x^2 + 4y^2 + 5z^2$ by $2x^2y^2z^2$ Ans $6x^4y^2z^2 + 8x^2y^4z^2 + 10x^2y^2z^4$

13. $4x^2y^2 + 5y^2z^2 - 6yz$ by $2xy^2z$ Ans $8x^3y^4z + 10xy^4z^3 - 12xy^3z$

14. $2x + y$ by $4x - 2y$ Ans $8x^2$ [illegible] $- 2y^2$

15. $2a^mb^{n+2}$ by $-6a^{m+1}b^{-n+2}$ Ans $-12a^{2m+1}b^4$

16. a^5x^m by $a^{-3}x^{a-2m}$ Ans a^2x^{a-m}

17. $a^4b^4x^p$ by $a^pb^2x^4$ Ans $a^{p+4}b^6x^{p+4}$

18. $a + b$ by $a + b$ Ans $a^2 + 2ab + b^2$

19. $a + b$ by $a - b$ Ans $a^2 - b^2$

20. $a^2 + ab + b^2$ by $a - b$ Ans $a^3 - b^3$

21. $a^2 - ab + b^2$ by $a + b$ Ans $a^3 + b^3$

22. $9x^2 - 6x + 4$ by $3x + 2$ Ans $27x^3 + 8$

23. $4x^3 + 2x^2 + 3x - 6$ by $2x - 10$

Ans $8x^4 - 36x^3 - 14x^2 - 42x + 60$

24. $2 + 3x - 10x^2$ by $1 - 6x + 3x^2$

Ans $2 - 9x - 22x^2 + 69x^3 - 30x^4$

25. $x^3 - 4x^2 + 11x - 24$ by $x^2 + 5x - 3$

Ans $x^5 + x^4 - 12x^3 + 43x^2 - 153x + 72$

26. $x^3 + 2x^2 + 6x + 3$ by $x^3 - x + 4$

Ans $x^6 + 2x^5 + 5x^4 + 5x^3 + 2x^2 + 21x + 12$

27. $2x+3$ by $2x+3$ Ans $4x^2+12x+9$

28. $x^3+4x^2-6x+12$ by $3x^2+6x+12$
Ans $3x^5+18x^4+18x^3+48x^2+144$

29. x^2-ax+a^2 by x^2+ax+a^2 Ans $x^4+a^2x^2+a^4$

30. $x^2+2ax-a^2$ by $x^4+a^2x^2+a^2$
Ans $2ax^5+2a^3x^3-(a^2-1)a^2x^2+2a^3x-a^4$

31. $x^4+x^3y+x^2y^2+xy^3+y^4$ by $x-y$ Ans x^5-y^5

32. $(4x+1)^2$ by $4x+1$ Ans $64x^3+48x^2+12x+1$

33. $x^2+y^2-xy+x+y-1$ by $x+y-1$ Ans $x^3+y^3+3xy-1$

34. $81x^4+27x^3y+9x^2y^2+3xy^3+y^4$ by $3x-y$ Ans $243x^5-y^5$

35. $x^2+y^2+z^2+xy-xz+yz$ by $x-y+z$ Ans $x^3-y^3+z^3+3xyz$

36. $3x^3-2x^2-5x-3$ by $7x^2+4x-9$
Ans $21x^5-2x^4-70x^3-23x^2+33x+27$

37. $x^3+4x^2+5x-24$ by $x^2-4x+11$ Ans $x^5+151x-264$

38. $x^2+(a+b)x+ab$ by $x+c$
Ans $x^3+(a+b+c)x^2+(ab+ac+bc)x+abc$

39. a^3+2a^2+2a+1 by a^3-2a^2+2a-1 Ans a^6-1

40. $a^4-2a^3b+3a^2b^2-2ab^3+b^4$ by $a^2+2ab+b^2$
Ans $a^6+2a^3b^3+b^6$

41. x^2-ax+b by $x-c$ Ans $x^3-(a+c)x^2+(ac+b)x-bc$

42. $a^4-a^2b^2+b^4$ by a^2+b^2 Ans a^6+b^6

43. $4x^5-6x^3-7x+8$ by $3x^4+2x^2-5$
Ans $12x^9+8x^7-18x^6-41x^5+12x^4-14x^3+46x^2+35x-$ 40

44. $(a+b)x^2+(c-d)x+m$ by $(a-b)x^2+(c+d)x+p$
Ans $(a^2-b^2)x^4+2(ac+bd)x^3+\{(m+p)a-(m-p)b+c^2-d^2\}x^2$
$+\{m(c+d)+p(c-d)\}x+mp$

45. $2x^2-xy-2y^2$ by $x^3-xy^2+2y^3$ Ans $2x^5-x^4y-4x^3y^2$
$+5x^2y^3-4y^5$

Find the continued product of :—

46. $a - b$, $a + b$, and $a^2 + b^2$ Ans $a^4 - b^4$

47. $a^2 - ab + b^2$, $a^2 + ab + b^2$, and $a^4 - a^2b^2 + b^4$
Ans $a^8 + a^4b^4 + b^8$

48. $x^2 + 5$, $x - 1$, and $x + 1$, $x^2 - 5$ Ans $x^6 - x^4 - 25x^2 + 25$

49. $x - 1$, $x + 1$, $x^2 + 1$, $x^4 + 1$, and $x^8 + 1$ Ans $x^{16} - 1$

50. $2x + 3$, $3x + 2$, $4x + 1$, and $x - 4$
Ans $24x^4 - 30x^3 - 225x^2 - 150x - 24$

51. $x - 3$, $x + 4$, $x - 5$ and $x + 6$ Ans $x^4 + 2x^3 - 41x^2 - 42x + 360$

52. $x + 3$, $x - 7$, $x + 4$, and $x - 10$
Ans $x^4 - 10x^3 - 37x^2 + 286x + 840$

53. $3x - 4y$, $x - 2y$, $x + 2y$ and $3x + 4y$
Ans $9x^4 - 52x^2y^2 + 64y^4$

Find the product of :—

54. $x^{\frac{1}{2}} + y^{\frac{1}{2}}$ and $x^{\frac{1}{2}} - y^{\frac{1}{2}}$ Ans $x - y$

55. $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$ and $a^{\frac{1}{3}} + b^{\frac{1}{3}}$ Ans $a + b$

56. $x^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{2}}$ and $x^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{2}} + a^{\frac{1}{2}}b^{\frac{1}{2}}$
Ans $x + 2xy^{\frac{1}{2}} + xy - ab$

57. $x^{\frac{1}{2}} + x^{\frac{1}{3}}y^{\frac{1}{6}} + x^{\frac{1}{6}}y^{\frac{1}{3}} \overset{+y^{\frac{1}{2}}}{\wedge}$ and $x^{\frac{1}{6}} - y^{\frac{1}{6}}$ Ans $x^{\frac{2}{3}} - y^{\frac{2}{3}}$

58. $x + y$ and $x^{\frac{1}{2}} - y^{\frac{1}{2}}$ Ans $x^{\frac{3}{2}} - xy^{\frac{1}{2}} + x^{\frac{1}{2}}y - y^{\frac{3}{2}}$

59. $a^{\frac{1}{3}} - a^{\frac{1}{6}}b^{\frac{1}{6}} + b^{\frac{1}{3}}$ and $a^{\frac{1}{3}} + a^{\frac{1}{6}}b^{\frac{1}{6}} + b^{\frac{1}{3}}$ Ans $a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$

60. $a^{\frac{1}{3}} + b^{\frac{1}{3}} - c^{\frac{1}{3}}$ and $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}$ Ans $a^{\frac{2}{3}} + b^{\frac{2}{3}} - c^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}}$

61. $x^{-2} - y^{-2}$ and $x^{-2} + y^{-2}$ Ans $x^{-4} - y^{-4}$

62. $x^{-\frac{1}{3}} - y^{-\frac{1}{3}}$ and $x^{-\frac{1}{3}} + y^{-\frac{1}{3}}$ Ans $x^{-\frac{2}{3}} - y^{-\frac{2}{3}}$

63. $x^3 - mx^2 + 1$ and $x^3 + mx^2 + 1$ Ans $x^6 - m^2x^4 + 2x^3 + 1$

64. $x^{m(n-1)} + y^{n(m-1)}$ by $x^{-mn} - y^{-(n-1)}$
Ans $x^{-m} + x^{-mn}y^{n(m-1)} - x^{m(n-1)}y^{-(n-1)} - y^{mn-2n+1}$

65. $x^{\frac{p}{p+q}} + x^{\frac{2p}{p+q}}$ by $x^{\frac{q}{p+q}} - x^{\frac{2q}{p+q}}$
Ans $x + x^{\frac{2p+q}{p+q}} - x^{\frac{p+2q}{p+q}} - x^2$

66. $\frac{2}{3}x^2 - \frac{4}{5}x + \frac{1}{4}$ by $\frac{3}{4}x - \frac{1}{2}$ Ans $\frac{1}{2}x^3 - \frac{14}{15}x^2 + \frac{47}{80}x - \frac{1}{8}$

67. $\frac{2}{7}x^2 - \frac{1}{2}x + \frac{2}{3}$ by $\frac{7}{4}x^2 + \frac{2}{3}x - \frac{4}{3}$
Ans $\frac{1}{2}x^4 - \frac{115}{168}x^3 + \frac{57}{126}x^2 + \frac{10}{9}x - \frac{8}{9}$

68. $x^{2^n} + y^{2^n}$ by $x^{2^n} - y^{2^n}$ Ans $x^{2^{n+1}} - y^{2^{n+1}}$

69. Find the coefficient of x^3 in the result of :—
$(4x^2 - 1)(2x + 3)(4x + 7)$ Ans 76

70. In the product of $1 + 2x + 3x^2 + 4x^3 + 5x^4$ and $1 + 3x + 5x^2 + 7x^3 + 9x^4$, find the coefficient of x^5 Ans 74

DIVISION.

Divide.

1. $45x^4y^5$ by $5x^2y^3$ Ans $9x^2y^2$
2. $48x^6$ by $-12x^3$ Ans $-4x^3$
3. $18x^2y^2$ by $6x^2$ Ans $3y^2$
4. $20a^5b^5c^6$ by $-4a^3b^2c^4$ Ans $-5a^2b^3c^2$

5.	$a^4x^4y^5$ by a^2xy^2	Ans $a^2x^3y^3$
6	$x^{2m}y^{5n}z^2$ by $x^my^{2n}z^2$	Ans x^my^{3n}
7.	$a^{\frac{4}{3}}b^{\frac{1}{3}}c$ by a^2bc^2	Ans $a^{-\frac{2}{3}}b^{-\frac{2}{3}}c^{-1}$
8.	$4x^3-8x^2+48x$ by $4x$	Ans $x^2-2x+12$
9.	$3a^4-12a^3+6a^2$ by $-2a^2$	Ans $-\frac{3}{2}a^2+6a-3$
10.	$x^3y-3x^2y^3+9y^2$ by $3xy$	Ans $\frac{x^2}{3}-xy^2+\frac{3y}{x}$
11	$4x^5+6x^4-8x^3+5x^2+6x-1$ by $2x^2$	Ans $2x^3+3x^2-4x+\frac{5}{2}+\frac{3}{x}-\frac{1}{2x^2}$
12.	$3x^m+6x^{3m}+5x^{2m}+6$ by $3x$	Ans $1+2x^{2m}+\frac{5}{3}x^m+\frac{2}{x^m}$
13.	$5x^2-6x^{\frac{3}{2}}+2x-4x^{\frac{1}{2}}-1$ by $4x^2$	Ans $\frac{5}{4}-\frac{3}{2}x^{-\frac{1}{2}}+\frac{x^{-1}}{2}-x^{-\frac{3}{2}}-\frac{1}{4}x^{-}$
14.	x^2-2x+1 by $x-1$	Ans $x-1$
15.	x^2-5x+6 by $x-3$	Ans $x-2$
16.	x^2+x-56 by $x-7$	Ans $x+8$
17.	$4x^2-6x+3$ by $x+7$	Ans $4x-34$, rem. 241
18.	$2x^3+32x^2-3x-48$ by $x+16$	Ans $2x^2-3$
19.	$6x^3+14x^2-4x+24$ by $2x+6$	Ans $3x^2-2x+4$
20.	$8x^5-12x^4-4x^2+8x-3$ by $2x-3$	Ans $4x^4-2x+1$
21.	$7x^3-24x^2+58x-21$ by $7x-3$	Ans x^2-3x+7
22.	a^3-x^3 by $a-x$	Ans a^2+ax+x^2
23.	a^6-a^6 by a^2-x^2	Ans $a^4+a^2x^2+x^4$
24.	x^6-1 by x^3+1	Ans x^3-1

25. a^3+x^3 by $a+x$ Ans a^2-ax+x^2.

26. a^5+x^5 by $a+x$ Ans $a^4-a^3x+a^2x^2-ax^3+x^4$

27. a^6+x^6 by $a+x$ Ans $a^5-a^4x+a^3x^2-a^2x^3+ax^4-x^5$;

28. a^6+x^6 by a^2+x^2 Ans $a^4-a^2x^2+x^4$

29. $9x^2-64$ by $3x-8$ Ans $3x+8$

30. x^4-81y^4 by $x-3y$ Ans $x^3+3x^2y+9xy^2+27y^3$

31. a^6-x^6 by $a+x$ Ans $a^5-a^4x+a^3x^2-a^2x^3+ax^4-x^5$

32. Find the middle term and the last three terms of the quotient of $a^{81}-x^{81}$ divided by $a-x$

Ans $a^{40}x^{40}$; $+a^2x^{78}+ax^{79}+x^{80}$

33. Find the 49th term of the quotient of $a^{91}+b^{91}$ divided by $a+b$ Ans $a^{42}b^{48}$

34 Find the 21st term of the quotient of $a^{40}-b^{40}$ divided by $a+b$ Ans $a^{19}b^{20}$

35. Write down the $(m-5$ th term of the quotient of a^m-b^m divided by $a-b$ Ans a^5b^{m-6}

Divide.

36 $2a^4+27ab^3-81b^4$ by $a+3b$

Ans $2a^3-6a^2b+18ab^2-27b^3$

37. $x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5$ by x^3+y^3

Ans x^2+xy+y^2

38. $2x^4-7x^3-10x^2+x+2$ by x^2-5x+2 Ans $2x^2+3x+1$

39. $x^4+x^3-9x^2-16x-4$ by x^2+5x+4

Ans x^2-4x+7, rem $-35x-32$

40. $21x^5-2x^4-70x^3-23x^2+33x+27$ by $7x^2+4x-9$

Ans $3x^3-2x^2-5x-3$

41. $6x^5-24x^4y+47x^3y^2-49x^2y^3+62xy^4-45y^5$ by $2x^2-7xy+9y^2$ Ans $3x^3-2x^2y+3xy^2-5y^3$

42 $x^3 + 64$ by $x^2 + 4x + 8$ Ans $x - 4$, rem. $8x + 96$

43. $3x^4 - 10x^3y + 22x^2y^2 - 22xy^3 + 15y^4$ by $x^2 - 2ax + 3y^2$
Ans $3x^2 - 4ax + 5y^2$

44. $x^7 - 5x^5 + 7x^3 + 2x^2 - 6x - 2$ by $1 + 2x - 3x^2 + x^4$
Ans $x^3 - 2x - 2$

45. $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$
Ans $a^2 + b^2 + c^2 - ab - ac - bc$

46. $6x^7 - 3x^6 + 9x^5 + 4x^3 - 4x^2 + 7x - 3$ by $2x^2 - x + 3$
Ans $3x^5 + 2x - 1$

47 $x^{12} + x^6 - 2$ by $x^4 + x^2 + 1$ Ans $x^8 - x^6 + 2x^2 - 2$

48. $27x^8 - 18x^6 - 27x^5 + 9x^3 + 6x^2 + 2x + 1$ by $9x^5 - 6x^2 - 1$
Ans $3x^3 - 2x - 1$

49. Find the remainder when $x^4 + 2$ is divided by $x - 2$ Ans 18

50. The dividend is $x^7 - 5x^5 + 7x^3 + 2x^2 - 8x + 2$, the remainder is $-2x + 4$ and the quotient is $x^3 - 2x - 2$; find the divisor
Ans $x^4 - 3x^2 + 2x + 1$

51. The divisor is $x^2 - 2x + 6$, the quotient is $4x^3 - 2x + 6$ and the remainder is $4x - 3$; find the dividend
Ans $4x^5 - 8x^4 + 22x^3 + 10x^2 - 20x + 33$

Divide.

52. $a^4 - 2a^2b^2 + b^4 + 4abc^2 - c^4$ by $a^2 + 2ab + b^2 - c^2$
Ans $a^2 - 2ab + b^2 + c^2$

53. $a^3 + b^3 - c^3 + 2ac^2 - 2a^2c + 2bc^2 - 2b^2c - abc$ by $a + b - c$
Ans $a^2 - ab + b^2 - bc + c^2 - ac$

54. $x^5 - px^4 + qx^3 - qx^2 + px - 1$ by $x - 1$
Ans $x^4 - (p - 1)x^3 + (q - p + 1)x^2 - (p - 1)x + 1$

55. $mpx^3 + (mq - np)x^2 - (mr + nq)x + nr$ by $mx - n$
Ans $px^2 + qx - r$

56. $x^2-(a+b)x+ab$ by $x-b$ Ans $x-a$

57. $x^2+(2a+b)x+2ab$ by $x+b$ Ans $x+2a$

58. $x^2-ax-(a+b)b$ by $x+b$ Ans $x-(a+b)$

59. $x^3+(1+x)xy+y^2$ by x^2+y Ans $x+y$

60. $x^3+(ab-1)x^2-(ab+1)x-(a^2b^2-1)$ by $x+(ab-1)$
Ans $x^2-(ab+1)$

61. $a^2(b+c)+b^2(a-c)+c^2(a-b)+abc$ by $a+b+c$
Ans $a(b+c)-bc$

62. $a^2x^4+(2ac-b^2)x^2+c^2$ by ax^2-bx+c Ans ax^2+bx+c.

63. $x^3-2ax^2+(a^2+ab-b^2)x-a^2b+ab^2$ by $x^2-x(a+b)+ab$
Ans $x-a+b$

64. $(x+y)^2-2(x+y)z+z^2$ by $x+y-z$ Ans $x+y-z$

65. $x^4+(n-m)x^3-m^2nx$ by x^2-mx Ans $x^2+nx+mn$

66. $x^4-(a^2+b^2)x^2+a^2b^2$ by $x^2+(a+b)x+ab$
Ans $x^2-(a+b)x+ab$

67. $(a^3+b^3)x^3+(2a^2-ab)x^2+2ax+1$ by $(a+b)x+1$
Ans $(a^2-ab+b^2)x^2+(a-b)x+1$

68. $x^3+(a-b-c)x^2+(bc-ab-ac)x+abc$ by $x-b$
Ans $(x+a)(x-c)$ or $x^2+(a-c)x-ac$

69. $(r^2+s^2)x^2+c^2(x^2+y^2)+(r^2+s^2)y^2$ by x^2+y^2
Ans $c^2+r^2+s^2$

70. $x^5-(1+m)x^4+(1+m+n)x^3-(m+n+p)x^2+(p+n)x-p$ by $x^2-(x-1)$ Ans x^3-mx^2+xn-p

71. $x^2(x-2a)+(a^2+ab-b^2)x-ab(a-b)$ by $x^2-(a+b)x+ab$
Ans $x-a+b$

72. $(2x+a)^2(5x+b)^2-(3x+a)^2(4x+b)^2$ by $(2x+a)(5x+b)$ $+(3x+a)(4x+b)$ Ans $-2x^2+ax-bx$

73. $(x+3)^4-1$ by $x+4$ Ans $(x+3)^3-(x+3)^2+(x+3)-1$

74. $(3x+2)^3+(7x+5)^3$ by $10x+7$
Ans $(3x+2)^2-(3x+2)(7x+5)+(7x+5)^2$

75. $(4x^2+6y)^3-(3x^2+2y)^3$ by x^2+4y
Ans $(4x^2+6y)^2+(3x^2+2y)(4x^2+6y)+(3x^2+2y)^2$

76. $x^4+2qx^3-(p^2-q^2)x^2-2rpx-r^2$ by $x^2-(p-q)x-r$
Ans $x^2+(p+q)x+r$

77. $x^{pq}-1$ by x^p-1 and write down the last three terms of the quotient when q denotes an integer.
Ans $x^{(p-1)q}+x^{(p-2)q}+x^{(p-3)q}+\dots\dots\dots x^{2q}+x^q+1$

78. $a^n b-ab^n-a^n c+ac^n+b^n c-bc^n$ by $(a-b)(a-c)$
Ans $(b-c)a^{n-2}+(b^2-c^2)a^{n-3}+(b^3-c^3)a^{n-4}+\&c$
$+(b^{n-2}-c^{n-2})a+(b^{n-1}-c^{n-1})$

79. $(x-y)^3-2y(x-y)^2+y^2(x-y)$ by $(x-2y)^2$ Ans $x-y$

80. $2x^{2m(n+1)}+7x^{2mn}+3x^{2m(n-1)}$ by $2x^{m(n+1)}+x^{m(n-1)}$
Ans $x^{m(n+1)}+3x^{m(n-1)}$

81. $x^{mn}-x^{(m-1)n}y^m-x^n y^{(n-1)m}+y^{mn}$ by x^n-y^m
Ans $x^{(m-1)n}-y^{(n-1)m}$

82. $x^{-4}-2x^{-2}y^{-2}+y^{-4}$ by $x^{-2}-y^{-2}$ Ans $x^{-2}-y^{-2}$

83. $4x^{-2}+16+64x^2$ by $2x^{-1}+4+8x$ Ans $2x^{-1}-4+8x$

84. x^2-x^{-2} by $x-x^{-1}$ Ans $x+x^{-1}$

85. x^3-x^{-3} by $x-x^{-1}$ Ans x^2+1+x^{-2}

86. $4x^{-3}+8x^{-4}-2x^{-6}-x^{-7}+6x^{-8}$ by $4x^{-1}-2x^{-4}+3x^{-5}$
Ans $x^{-2}+2x^{-3}$

87. $a^{-\frac{2}{3}}-x^{-1}$ by $a^{-\frac{1}{3}}+x^{-\frac{1}{2}}$ Ans $a^{-\frac{1}{3}}-x^{-\frac{1}{2}}$

88. $x^{-4}+x^{-2}+1$ by $x^{-2}-x^{-1}+1$ Ans $x^{-2}+x^{-1}+1$

89. $64x^{-1}+27y^{-2}$ by $4x^{-\frac{1}{3}}+3y^{-\frac{2}{3}}$

Ans $16x^{-\frac{2}{3}}-12x^{-\frac{1}{3}}y^{-\frac{2}{3}}+9y^{-\frac{4}{3}}$

90. $(x+x^{-1})^2-4(x-x^{-1})$ by $x-2-x^{-1}$ Ans $x-2-x^{-1}$

91. $a^{-\frac{2}{3}}+a^{-\frac{1}{3}}+1$ by $a^{-\frac{1}{3}}-1$ Ans $a^{-\frac{1}{3}}+2$, rem. 3

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92. $x^{\frac{1}{5}}y^{\frac{2}{5}}$ by $x^{\frac{1}{2}}y^{\frac{4}{5}}$ Ans $x^{\frac{3}{10}}y^{-\frac{2}{5}}$

93. $x^{\frac{2}{3}}-y^{\frac{2}{3}}$ by $x^{\frac{1}{6}}-y^{\frac{1}{6}}$ Ans $x^{\frac{1}{2}}+x^{\frac{1}{3}}y^{\frac{1}{6}}+x^{\frac{1}{6}}y^{\frac{1}{3}}+y$, rem. $y^{\frac{7}{6}}-y^{\frac{2}{3}}$

94 $a-b$ by $a^{\frac{1}{3}}-b^{\frac{1}{3}}$ Ans $a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$

95. $a+b$ by $a^{\frac{1}{5}}+b^{\frac{1}{5}}$ Ans $a^{\frac{4}{5}}-a^{\frac{3}{5}}b^{\frac{1}{5}}+a^{\frac{2}{5}}b^{\frac{2}{5}}-a^{\frac{1}{5}}b^{\frac{3}{5}}+b^{\frac{4}{5}}$

96. $x^{\frac{3}{2}}-xy^{\frac{1}{2}}+x^{\frac{1}{2}}y-y^{\frac{3}{2}}$ by $x^{\frac{1}{2}}-y^{\frac{1}{2}}$ Ans $x+y$

97. $x^{\frac{2}{3}}+y^{\frac{2}{3}}-z^{\frac{2}{3}}+2x^{\frac{1}{3}}y^{\frac{1}{3}}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$ Ans $x^{\frac{1}{3}}+y^{\frac{1}{3}}-z^{\frac{1}{3}}$

98. $a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$ by $a^{\frac{1}{3}}+a^{\frac{1}{6}}b^{\frac{1}{6}}+b^{\frac{1}{3}}$ Ans $a^{\frac{1}{3}}-a^{\frac{1}{6}}b^{\frac{1}{6}}+b^{\frac{1}{3}}$

99. $x^{\frac{5}{4}}-4x+6x^{\frac{3}{4}}-3x^{\frac{1}{2}}$ by $x^{\frac{1}{2}}-x^{\frac{1}{4}}$ Ans $x^{\frac{3}{4}}-3x^{\frac{1}{2}}+3x^{\frac{1}{4}}$

100. $x^{\frac{5}{6}}-x^{\frac{2}{3}}+2x^{\frac{1}{2}}-x^{\frac{1}{3}}+2$ by $x^{\frac{1}{2}}-x^{\frac{1}{3}}+1$ Ans $x^{\frac{1}{3}}+2$

101. $x+3x^{\frac{3}{4}}+2x^{\frac{1}{2}}+x^{\frac{1}{4}}+2$ by $x^{\frac{1}{4}}+2$ Ans $x^{\frac{3}{4}}+x^{\frac{1}{2}}+1$

102. $x^{3x}+y^{3x}$ by $x^{x}+y^{x}$ Ans $x^{2x}-x^{x}y^{x}+y^{2x}$

103. $\frac{1}{6}x^3+\frac{1}{9}x^4$ by $\frac{1}{3}x$ Ans $\frac{1}{2}x^2+\frac{1}{3}x^3$

104. $\frac{4}{21}x^7+\frac{1}{2}x^4$ by $\frac{2}{3}x^3$ Ans $\frac{2}{7}x^4+\frac{3}{4}x$

105. $\frac{1}{3}x^8-\frac{1}{8}x^5$ by $\frac{1}{2}x^3$ Ans. $\frac{2}{3}x^5-\frac{1}{4}x^2$

106. $\frac{2}{3}x^2-\frac{57}{60}x+\frac{1}{3}$ by $\frac{4}{5}x-\frac{1}{2}$ Ans $\frac{5}{6}x-\frac{2}{3}$

107. $\frac{5}{12}x^3-\frac{57}{80}x^2+\frac{123}{160}x-\frac{9}{20}$ by $\frac{5}{8}x-\frac{3}{5}$ Ans $\frac{2}{3}x^2-\frac{1}{2}x+\frac{3}{4}$

108. $\frac{3}{4}x^5 - 4x^4 + \frac{77}{8}x^3 - \frac{43}{4}x^2 - \frac{33}{4}x + 27$ by $\frac{x^2}{2} - x + 3$

Ans $\frac{3}{2}x^3 - 5x^2 + \frac{x}{4} + 9$.

109. $1 + 2x$ by $1 - 3x$ to 5 terms in the quotient

Ans $1 + 5x + 15x^2 + 45x^3 + 135x^4 +$ &c.

110. $1 - 3x - 2x^2$ by $1 - 4x$ to 5 terms in the quotient

Ans $1 + x + 2x^2 + 8x^3 + 32x^4 +$ &c

Exercise 7.

ADAPTATION TO FORMULÆ.

(1) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

(2) $(a + b)(a - b) = a^2 - b^2$

Apply the above formulæ to the following examples.

1. $(2x + 4)^2$ Ans $4x^2 + 16x + 16$
2. $(4x^2 - 8)^2$ Ans $16x^4 - 64x^2 + 64$
3. $(x + a + 2b)^2$ Ans $x^2 + a^2 + 4b^2 + 2ax + 4bx + 4ab$
4. $(x^2 - 2x + 1)^2$ Ans $x^4 - 4x^3 + 6x^2 - 4x + 1$.
5. $(a^2 + b^2 - 2c)^2$ Ans $a^4 + b^4 + 4c^2 + 2a^2b^2 - 4a^2c - 4b^2c$
6. $(a - b)^2 + (b - c)^2 + (c - a)^2$ Ans $2(a^2 + b^2 + c^2 \overset{-ab}{\wedge} - ac - bc)$
7. $(a^2 + ax + x^2)(a^2 - ax + x^2)$ Ans $a^4 + a^2x^2 + x^4$
8. $(a^2 + ax - x^2)(a^2 + ax + x^2)$ Ans $a^4 + 2a^3x + a^2x^2 - x^4$
9. $(4x + 3y)(4x - 3y)(16x^2 + 9y^2)$ Ans $256x^4 - 81y^4$
10. $(2x + 3y)^2(2x - 3y)^2$ Ans $16x^4 - 72x^2y^2 + 81y^4$
11. $(a + b + c)(a + b - c)$ Ans $a^2 + 2ab + b^2 - c^2$

12. $(a-b+c)(b+c-a)$ Ans $c^2-b^2+2ab-a^2$

13. $(a+b+c)(a+b-c)(a-b+c)(b+c-a)$
Ans $2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4$

14. $(x+y-a)(x-y+a)$ Ans $x^2-y^2+2ay-a^2$

Exercise 8.

FACTORS.

Resolve the following expressions into factors :–

1. $2ax+x^2$ ✓ Ans $x(2a+x)$
2. $8a^2x^2+2a^2x^4$ ✓ Ans $2a^2x^2(4+x^2)$
3. $9a^3x^3-3ax$ ✓ Ans $3ax(3a^2x^2-1)$
4. $4b^4+8b^2c^2+10b^3c$ ✓ Ans $2b^2(2b^2+4c^2+5bc)$
5. $3x^4-9x^2-12x$ ✓ Ans $3x(x^3-3x-4)$
6. x^2+2x ✓ Ans $(x+2)x$
7. x^2+3x+2 ✓ Ans $(x+2)(x+1)$
8. $x^2-3x-28$ Ans $(x-7)(x+4)$
9. x^2+x-30 Ans $(x-5)(x+6)$
10. x^2-x-72 Ans $(x+8)(x-9)$
11. $x^2-2x-24$ Ans $(x+4)(x-6)$
12. x^4+4x^2+3 Ans $(x^2+1)(x^2+3)$
13. x^2+6x+5 Ans $(x+1)(x+5)$
14. $x^2+9x+20$ Ans $(x+4)(x+5)$
15. x^2-5x+6 Ans $(x-2)(x-3)$
16. $x^2-8x+15$ Ans $(x-3)(x-5)$

17. $x^2 - x - 6$ Ans $(x - 3)(x + 2)$

18. $x^2 - 2x - 3$ Ans $(x - 3)(x + 1)$

19. $4x^2 + 8x + 3$ Ans $(2x + 1)(2x + 3)$

20. $4x^2 + 13x + 3$ Ans $(4x + 1)(x + 3)$

21. $4x^2 + 11x - 3$ Ans $(4x - 1)(x + 3)$

22. $4x^2 - 4x - 3$ Ans $(2x - 3)(2x + 1)$

23. $3x^2 + 4x - 4$ Ans $(3x - 2)(x + 2)$

24. $6x^2 + 5x - 4$ Ans $(3x + 4)(2x - 1)$

25. $12x^2 - 5x - 2$ Ans $(4x + 1)(3x - 2)$

26. $12x^2 - 14x + 2$ Ans $2(6x - 1)(x - 1)$

27. $12x^2 - x - 1$ Ans $(4x + 1)(3x - 1)$

28. $3x^2 - 2x - 5$ Ans $(3x - 5)(x + 1)$

29. $a^2x^2 - 3a^3x + 2a^4$ Ans $a^2(x - a)(x - 2a)$

30. $a^3 - a^2x - 6ax^2$ Ans $a(a - 3x)(a + 2x)$

31. $3a^3b + a^2b^2 - 2ab^3$ Ans $ab(3a - 2b)(a + b)$

32. $12a^4 + a^2x^2 - x^4$ Ans $(2a + x)(2a - x)(3a^2 + x^2)$

33. $2x^3y + 5x^2y^2 + 2xy^3$ Ans $xy(2x + y)(x + 2y)$

34. $9x^2y^2 - 3xy^3 - 6y^4$ Ans $3y^2(3x + 2y)(x - y)$

35. $6a^4x^2 + a^3x - a^2$ Ans $a^2(3ax - 1)(2ax + 1)$

36. $6b^2x^2 - 7bx^3 - 3x^4$ Ans $x^2(2b - 3x)(3b + x)$

37. $x^3 + y^3$ Ans $(x + y)(x^2 - xy + y^2)$

38. $1 + x^3y^3$ Ans $(1 + xy)(1 - xy + x^2y^2)$

39. $x^3 + 8$ Ans $(x + 2)(x^2 - 2x + 4)$

40. $x^{3p} + 1$ Ans $(x^p + 1)(x^{2p} - x^p + 1)$

41. $x^3 - y^3$ Ans $(x - y)(x^2 + xy + y^2)$

42. $a^3 - 8b^3$ Ans $(a - 2b)(a^2 + 2ab + 4b^2)$

43. $a^3x^2y + 27x^2y^4$ Ans $x^2y(a + 3y)(a^2 - 3ay + 9y^2)$

44 $x^5 + 32$ Ans $(x + 2)(x^4 - 2x^3 + 4x^2 - 8x + 16)$

45. $a^3x^3 + 27a$ [illegible] Ans $x^3(a + 3x)(a^2 - 3ax + 9x^2)$

46 $8x^3 + y^6$ Ans $(2x + y^2)(4x^2 - 2xy^2 + y^4)$

47. $x^4 - 1$ Ans $(x^2 + 1)(x + 1)(x - 1)$

48. $4x^2 - y^2$ Ans $(2x + y)(2x - y)$

49. $x^6 + a^6$ Ans $(x^2 + a^2)(x^4 - a^2x^2 + a^4)$

50. $x^6 - a^6$ Ans $(x + a)(x^2 - ax + a^2)(x - a)(x^2 + ax + a^2)$

51. $1 - 4x^2$ Ans $(1 + 2x)(1 - 2x)$

52. $a^2 - 9n^2$ Ans $(a + 3n)(a - 3n)$

53. $9m^2 - 4n^2$ Ans $(3m - 2n)(3m + 2n)$

54. $(a + b)^2 - (c + d)^2$ Ans $(a + b + c + d)(a + b - c - d)$

55. $a^2xy^3 - x^5y$ Ans $xy(ay + x^2)(ay - x^2)$

56. $a^2 - b^2 - c^2 + 2bc$ Ans $(a + b - c)(a - b + c)$

57. $a^2 + b^2 - c^2 - d^2 - 2(ab - cd)$ Ans $(a - b + c - d)(a - b - c + d)$

58. $4x^2 + 9y^2 - 9 - 12xy - 16c^2 + 24c$

Ans $(2x - 3y + 4c - 3)(2x - 3y - 4c + 3)$

59. $a^4b^{12} - c^8$ Ans $(ab^3 + c^2)(ab^3 - c^2)(a^2b^6 + c^4)$

60. $8x^3 - 27$ Ans $(2x - 3)(4x^2 + 6x + 9)$

61. $81q^4 - 1$ Ans $(3x - 1)(3x + 1)(9x^2 + 1)$

62. $x^6 - 64$ Ans $(x - 2)(x + 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$

63. $(3x - 2)^2 - (x - 3)^2$ Ans $(4x - 5)(2x + 1)$

64. $125x^3 + 8y^3$ Ans $(5x + 2y)(25x^2 - 10xy + 4y^2)$

65 $(4x + 3y)^2 - (3x + 4y)^2$ Ans $7(x - y)(x + y)$

66. $(x^2+y^2)^2-4x^2y^2$ Ans $(x-y)^2(x+y)^2$

67. $(a+b)^3+c^3$ Ans $(a+b+c)\{(a+b)^2-(a+b)c+c^2\}$

68. $x^3+y^3+3xy(x+y)$ Ans $(x+y)^3$

69. $m^3-n^3-m(m^2-n^2)+n(m-n)^2$ Ans $mn(m-n)$

70. $4(a+b)^2-4(a+b)-3$ Ans $\{2(a+b)-3\}\{2(a+b)+1\}$

71. $3(x^2-y^2)-5(x-y)^2$ Ans $2(x-y)(4y-x)$

72. $(x+y)^2+2(x^2+xy)-3(x^2-y^2)$ Ans $4y(x+y)$

73. $2(a^3+a^2b+ab^2)-(a^3-b^3)$ Ans $(a+b)(a^2+ab+b^2)$

74. $(1+a)^2(1+c^2)-(1+c)^2(1+a^2)$ Ans $2(a-c)(1-ac)$

75. $x^4-(a+b)x^3+(a+b)abx-a^2b^2$

Ans $(x-a)(x-b)(x^2-ab)$

76. $6(x+y)^2+5(x+y)-4$ Ans $\{3(x+y)+4\}\{2(x+y)-1\}$

77. $2y^2-5xy+2x^2-ay-ax-a^2$ Ans $(2y-x+a)(y-2x-a)$

78. x^4+x^3+x+1 Ans $(x+1)^2(x^2-x+1)$

79. x^4+4 Ans $(x^2+2x+2)(x^2-2x+2)$

80. $x^4+a^2x^2+a^4$ Ans $(x^2+ax+a^2)(x^2-ax+a^2)$

81. $x^3+6x^2+13x+12$ Ans $(x+3)(x^2+3x+4)$

82. $x^3+7x^2+16x+16$ Ans $(x+4)(x^2+3x+4)$

83. $x^3+9x^2+11x-21$ Ans $(x+7)(x+3)(x-1)$

84. $x^3-10x^2+28x-15$ Ans $(x-5)(x^2-5x+3)$

85. $x^3-29x+42$ Ans $(x+6)(x^2-6x+7)$

86. $x^3+x^2-35x+49$ Ans $(x+7)(x^2-6x+7)$

87. $9x^3+48x^2+52x+16$ Ans $(3x+2)(3x^2+14x+8)$

88. x^4+x^2-6 Ans $(x^2-2)(x^2+3)$

89. x^4-3x^2+2 Ans $(x^2-2)(x+1)(x-1)$

90. $x^2 - y^2 - z^2 + 2yz + x + y - z$ Ans $(x+y-z)(x-y+z+1)$

91. $35x^3 + 47x^2 + 13x + 1$ Ans $(7x+1)(x+1)(5x+1)$

92. $1 - a^{\frac{1}{2}} - a + a^{\frac{3}{2}}$ Ans $\left(1 - a^{\frac{1}{2}}\right)^2 \left(1 + a^{\frac{1}{2}}\right)$

93. $x^5 - x^4 - x^3 - 7x^2 - x + 15$ Ans $(x^2 - x - 3)(x^3 + 2x - 5)$

94. $2xyz + x^2(y+z) + y^2(z+x) + z^2(x+y)$
Ans $(x+y)(x+z)(y+z)$

95. $x^3 + ax^2 - axy - y^3$ Ans $(x-y)(x^2 + xy + y^2 + ax)$

96. $x^4 + 2x^3y - a^2x^2 + x^2y^2 - 2axy^2 - y^4$
Ans $(x^2 + xy + ax + y^2)(x^2 + xy - ax - y^2)$

97. $x^3 + y^3 + z^3 - 3xyz$ Ans $(x+y+z)(x^2 + y^2 + z^2 - xz - yz - xy)$

98. $x + y - z + 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$
Ans $\left(x^{\frac{1}{3}} + y^{\frac{1}{3}} - z^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}} + y^{\frac{2}{3}} + z^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + x^{\frac{1}{3}}z^{\frac{1}{3}} + y^{\frac{1}{3}}z^{\frac{1}{3}}\right)$

99. $x^3 + y^3 + 3xy - 1$ Ans $(x+y-1)(x^2 + y^2 + 1 - xy + x + y)$

100. $2(x+y+z)^2 - 7(a+b+c)(x+y+z) + 3(a+b+c)^2$
Ans $(2x + 2y + 2z - a - b - c)(x + y + z - 3a - 3b - 3c)$

101. $(1-x^2)(1-y^2)(1-z^2) - (z+xy)(y+xz)(x+yz)$
Ans $(1 - x^2 - y^2 - z^2 - 2xyz)(1 + xyz)$

102. $(a+b+c)(ab+ac+bc) - abc$ Ans $(a+b)(a+c)(b+c)$

103. $(a^2 + b^2 - c^2)^2 - 4a^2b^2$
Ans $(a+b+c)(a+b-c)(a-b+c)(a-b-c)$

104. $x^4 - y^4 + 2y(x^3 + y^3) - (x+y)^2(x-y)^2$ Ans $2x^2y(x+y)$

105. $x^3 + 3x^2 - 20$ Ans $(x^2 + 5x + 10)(x - 2)$

Exercise 9.

GREATEST COMMON MEASURE.

Find the greatest common measure in the following examples.

1. $25x^4$, $15x^2$ Ans $5x^2$

2. $14a^3b^2$, $21a^2b^4$ Ans $7a^2b^2$

3. $18x^4y^4z^8$, $27x^8y^8z^{12}$ Ans $9x^4y^8z^8$

4. $50x^2y^{-1}z^4$, $75x^4y^{-2}z^5$ Ans $25x^2y^{-1}z^4$

5. $8(x-1)^2$, $42(x^2-1)$ Ans $2(x-1)$

6. $15(x^2-1)$, $9(x^2+2x+1)$ Ans $3(x+1)$

7. $x^2-\left(a+\frac{1}{a}\right)x+1$, $2\left(x-\frac{1}{a}\right)^2$ Ans $x-\frac{1}{a}$

8. $16(x^4-y^4)$, $24(x^2+y^2)(x+y)$ Ans $8(x^2+y^2)(x+y)$

9. $x^{16}-y^{16}$, x^4-y^4 Ans x^4-y^4

10. $x^2+11x+30$, $x^2+13x+42$ Ans $x+6$

11. $x^2-13x+12$, $x^2-20x+96$ Ans $x-12$

12. $x^2-8x-33$, $x^2-7x-44$ Ans $x-11$

13. $x^2-13x+42$, x^2+x-42 Ans $x-7$

14. $x^3-9x^2+23x-12$, $x^3-10x^2+28x-15$ Ans x^2-5x+3

15. x^3+x^2+x+21, $x^3+3x^2-3x+35$ Ans x^2-2x+7

16. x^3-x^2-4x-6, x^4+4 Ans x^2+2x+2

17. x^4-1, $3x^5+2x^4+4x^3+2x^2+x$ Ans x^2+1

18. $6x^2+x-2$, $9x^3+48x^2+52x+16$ Ans $3x+2$

19. $x^3-3a^2x-2a^3$, $x^3-ax^2-4a^3$ Ans $x-2a$

20. $3x^4-11x^3-2x^2-4x-16$, $2x^4-7x^3-4x^2+x-4$

Ans x^2-3x-4

21. $x^4 + ax^3 - 9a^2x^2 + 11a^3x - 4a^4$, $x^4 - ax^3 - 3a^2x^2 + 5a^3x - 2a^4$
Ans $x^3 - 3ax^2 + 3a^2x - a^3$

22 $3x^4 + 2x^3 + 3x + 2$, $4x^3 + 10x^2 + 4x - 2$ Ans $x + 1$

23. $x^4 - x^3 - x + 1$, $x^3 - 2x + 1$ Ans $x - 1$

24. $(ax + by)^2 + (a - b)(x + z)(ax + by) + (a - b)^2xz$
and $(ax - by)^2 - (a + b)(x + z)(ax - by) + (a + b)^2xz$
Ans $b(x + y)$

25. $4(x^3 + a^3)$, $6(x^2 - 2ax - 3a^2)$ Ans $2(x + a)$

26. $x^2 - 7x + 10$, $4x^3 - 25x^2 + 20x + 25$ Ans $x - 5$

27. $8x^2 + 14x - 15$, $8x^3 + 30x^2 + 13x - 30$ Ans $8x^2 + 14x - 15$

28. $2y^3 - 10y^2 + 12y$ and $3y^4 - 15y^3 + 24y^2 - 24$ Ans $y - 2$

29. $3a^6 + 15a^5b - 3a^4b^2 - 15a^3b^3$, $10a^5 - 30a^4b - 10a^3b^2 + 30a^2b^3$
Ans $a(a^2 - b^2)$

30. $6x^2 + 13x + 6$, $8x^2 + 6x - 9$ Ans $2x + 3$

31. $15x^2 - x - 6$, $9x^2 - 3x - 2$ Ans $3x - 2$

32. $5x^3 + 2x^2 - 15x - 6$, $-7x^3 + 4x^2 + 21x - 12$ Ans $x^2 - 3$

33 $6x^4 - x^3y - 3x^2y^2 + 3xy^3 - y^4$, $9x^4 - 3x^3y - 2x^2y^2 + 3xy^3 - y^4$
Ans $3x^2 - 2xy + y^2$

34. $6x^4 - 25a^2x^2 - 9a^4$ and $3x^3 - 15ax^2 + a^2x - 5a^3$
Ans $3x^2 + a^2$

35. $m^{2x}a^2 + m^{2x} - a^2 - 1$, $m^{2x}a^2 + 2m^xa^2 - m^{2x} - 2m^x + a^2 - 1$
Ans $am^x + m^x + a + 1$

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36. $x^2 - 2x - 3$, $x^2 - 7x + 12$, $x^2 - x - 6$ Ans $x - 3$

37. $x^2 + 7x + 12$, $x^2 - x - 12$, $x^2 + 2x - 3$ Ans $x + 3$

38. $x^2 - 1$, $(x + 1)^2$, $x^3 + 1$ Ans $x + 1$

39. $x^4 - 16, x^2 - 4, x^3 + 8$ Ans $x + 2$

40. $x^2 - 5x + 4, x^2 - 3x - 4, x^3 - 4x^2 + x - 4$ Ans $x - 4$

41. $x^4 - mx^3 + (n - 1)x^2 + mx - n$
and $x^4 - nx^3 + (m - 1)x^2 + nx - m$ Ans $x^2 - 1$

42. $3x^2 - (4a + 2b)x + 2ab + a^2$ and
$x^3 - (2a + b)x^2 + (2ab + a^2)x - a^2b$ Ans $x - a$

43. $x^2 + 2y^2 + (x + 2y)\sqrt{xy}$ and $x^2 - y^2 + (x - y)\sqrt{xy}$
Ans $\sqrt{x} + \sqrt{y}$

44 $x^3 + (a + 1)x^2 + (a + 1)x + a$ and $x^3 + (a - 1)x^2 - (a - 1)x + a$
Ans $x + a$

45. $ax^3 - (a - b)x^2 - (b - c)x - c$ and $2ax^3 + (a + 2b)x^2$
$+ (b + 2c)x + c$ Ans $ax^2 + bx + c$

46. $x^4 - 2a(a - b)x^2 + (a^2 + b^2)(a - b)x - a^2b^2$ and
$x^4 - (a - b)x^3 + (a - b)b^2x - b^4$ Ans $x^2 + (b - a)x + b^2$

47. $a^2 - b^2 + c^2 - 2ac$ and $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$
Ans $a - b - c$

Exercise 10.

LEAST COMMON MULTIPLE.

Find the least common multiple in the following examples.

1. $4a^3x, 18ax^3$; ab, ac, bc; $12a^2c, 14bc^2, 36ab^2$
Ans $36a^3x^3$; abc; $252a^2b^2c^2$

2. $4a^3x$ and $6a^2x^2$ Ans $12a^3x^2$

3. $3x^2y$ and $12xy^2$ Ans $12x^2y^2$

4. $4a^3b$ and $8a^2b^2$ Ans $8a^3b^2$

5. ax, a^3x and a^4x^2 Ans a^4x^2

6. $2ax$, $4ax^2$ and x^3 Ans $4ax^3$

7. ab, a^2c and b^2c^3 Ans $a^2b^2c^3$

8. a^2x^2, a^3y^2 and xy^3 Ans $a^3x^2y^3$

9. $51a^2x^2$, $34ax^3$ and ax^4 Ans $102a^2x^4$

10 $5m^2n$, $15mn^2$ and mnr Ans $15m^2n^2r$

11. $a-x$, a^2-x^2 and a^2+ax Ans a^3-ax^2

12. x and $ax+x^2$ Ans $ax+x^2$

13. x^2-1 and x^2-x Ans x^3-x

14. a^2-b^2 and a^2+ab Ans a^3-ab^2

15. $2x-1$ and $4x^2-1$ Ans $4x^2-1$

16. $x+y$ and x^3+y^3 Ans x^3+y^3

17. $x+1$, $x-1$ and x^2-1 Ans x^2-1

18. $x-1$, x^3-1 and x^2+x+1 Ans x^3-1

19. $x+1$, x^2+1 and x^3+1 Ans $x^5+x^3+x^2+1$

20. $x-1$, x^2-1 and x^3-1 Ans x^4+x^3-x-1

21. x^2-1, x^2+1 and x^4-1 Ans x^4-1

22. x^2-1, x^2-x and x^3+1 Ans $x^5-x^4+x^2-x$

23. $(x+y)^2$ and x^2-y^2 Ans $x^3+x^2y-xy^2-y^3$

24. $4(1+x)$, $5(1-x)$ and $8(1-x^2)$ Ans $40(1-x^2)$

25 $(a-b)(a-c)$ and $(a-c)(b-c)$

Ans $a^2(b-c)-b^2(a-c)+c^2(a-b)$

26. $(x+1)(x+2)$, $(x+2)(x+3)$ and $(x+1)(x+3)$

Ans $x^3+6x^2+11x+6$

27. $x^2 - y^2$, $(x+y)^2$ and $(x-y)^2$. Ans $x^4 - 2x^2y^2 + y^4$

28. $(a+3)(a+1)$, $(a+3)(a-1)$ and a^2-1 Ans a^3+3a^2-a-3

29. $x^2(x-y)$, $x(x^2-y^2)$ and $x+y$ Ans $x^4-x^2y^2$

30. $(x+1)(x+3)$, $(x+2)(x+3)(x+4)$ and $(x+1)(x+2)$
Ans $x^4+10x^3+35x^2+50x+24$

31. x^2-y^2, $3(x-y)^2$ and $12(x^3+y^3)$ Ans $12(x-y)^2(x^3+y^3)$

32. x^2+5x+6 and x^2+6x+8 Ans $(x+2)(x+3)(x+4)$

33. x^2-x-20 and x^2+x-12 Ans $(x-5)(x+4)(x-3)$

34. x^2+3x+2 and x^2+4x+3 Ans $(x+1)(x+2)(x+3)$

35. $x^2+11x+30$ and $x^2+12x+35$ Ans $(x+5)(x+6)(x+7)$

36. $x^2-9x-22$ and $x^2-13x+22$ Ans $(x-11)(x+2)(x-2)$

37. $2x^2+3x+1$ and x^2-x-2 Ans $(2x+1)(x+1)(x-2)$

38. $21x^2-26x+8$ and $7x^3-4x^2+21x+12$
Ans $(7x-4)(3x-2)(x^2-3)$

39 x^2-3x+2, x^2-4x+3 and x^2-5x+4
Ans $(x-2)(x-1)(x-3)(x-4)$

40. x^2+5x+4, x^2+4x+3 and $x^2+7x+12$
Ans $(x+4)(x+1)(x+3)$

41. $x^2-9x+20$, $x^2-12x+35$, and $x^2-11x+28$
Ans $(x-4)(x-5)(x-7)$

42 $6x^2-x-2$, $21x^2-17x+2$ and $14x^2+5x-1$
Ans $(3x-2)(2x+1)(7x-1)$

43. x^2-1, x^2+2x-3 and $6x^2-x-2$
Ans. $(x+1)(x-1)(x+3)(3x-2)(2x+1)$

44. x^3-27, $x^2-15x+36$ and x^4-3x^3-2x+6
Ans $(x-3)(x^2+3x+9)(x-12)(x^3-2)$

45. $x^2+5x+10$, $x^2-19x-30$ and $x^2-15x-50$
Ans $(x^2-19x-30)(x^2+5x+10)(x-5)$

46. $7x^3-4x^2-21x+12$ and $21x^2-26x+8$
Ans $(7x-4)(3x-2)(x^2-3)$

47. $3x^2-11x+6$, $2x^2-7x+3$ and $6x^2-7x+2$
Ans $6x^3-25x^2+23x-6$

48. x^3-3x^2+3x-1, x^3-x^2-x+1, x^4-2x^3+2x-1, and $x^4-2x^3+2x^2-2x+1$
Ans $x^6-2x^5+x^4-x^2+2x-1$

49. $18(x^2-y^2)$, $12(x-y)^2$ and $36(x^3+y^3)$
Ans $36(x^5-2x^4y+x^3y^2+x^2y^3-2xy^4+y^5)$

50. x^4-x^2+1, x^2-x+1, and x^2+x+1 Ans x^8+x^4+1

Exercise 11.

EASY IDENTITIES.

Shew that

1. $x(y+z)(y^2+z^2-x^2)+y(x+z)(z^2+x^2-y^2)+z(x+y)(x^2+y^2-z^2)=2xyz(x+y+z)$

2. $(x+y+z)^3+(x-y-z)^3+(y-z-x)^3+(z-y-x)^3=24xyz$

3. $a^2(b+c)^2+b^2(a+c)^2+c^2(a+b)^2+2abc(a+b+c)=2(ab+bc+ac)^2$

4. $x(y+z)^2+y(z+x)^2+z(x+y)^2=(x+y)(x+z)(y+z)+4xyz$

5. $(x+y+z)^2-(x+y)^2=(y+z)^2+(x+z)^2-x^2-y^2-z^2$

6. $(a-b)^3+b^3=a^3-3ab(a-b)$

7. $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$

8. $(x+y+z)^2=x^2+y^2+z^2+2xy+2xz+2yz$

9. $(x+y+z)(xy+xz+yz)-xyz=(x+y)(y+z)(x+z)$

10. $(a+b+c)^3 = a^3+b^3+c^3+3a^2(b+c)+3b^2(a+c)+3c^2(a+b)$
$+6abc$

11. $(x+y+z)^3-(y+z-x)^3-(x+z-y)^3-(x+y-z)^3 = 24xyz$

12. $\{2(a+b+c)\}^2+\{2(a+b-c)\}^2+\{2(a-b+c)\}^2+\{2(b+c-a)\}^2$
$=16(a^2+b^2+c^2)$

13. $(a+b)^2+(a-b)^2+(a-c)^2+(a+c^2)^2 = 4a^2+2b^2+2c^2$

14. $(x+y)^2-2(y^2-x^2)+(x-y)^2 = 4x^2$

Exercise 12.

REDUCTION OF FRACTIONS.

Express the following fractions as mined quantities.

1. $\frac{61x}{11}$; $\frac{47x}{8}$; $\frac{53x}{7}$; $\frac{110x^2}{19}$

Ans $5x+\frac{6x}{11}$; $5x+\frac{7x}{8}$; $7x+\frac{4x}{7}$; $5x^2+\frac{15x^2}{19}$

2. $\frac{21ax+4x}{7}$ Ans $3ax+\frac{4x}{7}$

3. $\frac{12b^2c+2c}{3b^2c}$ Ans $4+\frac{2}{3b^2}$

4 $\frac{6xy-4x}{y}$ Ans $6x-\frac{4x}{y}$

5. $\frac{10x^2+40x+6}{x+4}$ Ans $10x+\frac{6}{x+4}$

6. $\frac{8x^2+64x-4}{x+8}$ Ans $8x-\frac{4}{x+8}$

7. $\frac{x^3-x+a^3+a}{x+a}$ Ans $x^2-ax+a^2+\frac{a-x}{a+x}$

8. $\frac{2x^3 - x^2 - 5x}{x^2 + x + 1}$ Ans $2x - 3 - \frac{4x - 3}{x^2 + x + 1}$

9. $\frac{x^5 + 3x^4 + 4x^3 + 4x^2 - x + 4}{x + 2}$ Ans $x^4 + x^3 + 2x^2 - 1 + \frac{6}{x + 2}$

10. $\frac{6x^3 - 2x^2y - 6xy^2 + 2y^3 - 2x + y}{x^2 - y^2}$ Ans $6x - 2y - \frac{x^2 - y^2}{2x - y}$

11. $\frac{a^2 - 2ab + b^2}{a + b}$ Ans $(a + b) - \frac{4ab}{a + b}$

12. $\frac{x^5 + 2x^4 - x^3 + x^2 + x}{x^3 + 1}$ Ans $x^2 + 2x - 1 - \frac{x + 1}{x^3 + 1}$

Reduce the following fractions to their lowest terms.

13. $\frac{16x^4y^4}{24x^2y^6}$; $\frac{25a^4b^2}{30a^6b^4}$; $\frac{x^2 - xy}{xy}$; $\frac{2x^2 - 4x^3}{8x^2}$

Ans $\frac{2x^2}{3y^2}$; $\frac{5}{6a^2b^2}$; $\frac{x}{y} - 1$; $\frac{1 - 2x}{4}$

14. $\frac{8x^2y - 4xy^2}{4x^3y - 2x^2y^2}$ Ans $\frac{2}{x}$

15. $\frac{4(a^3 + b^3)}{12(a + b)^2}$ Ans $\frac{a^2 - ab + b^2}{3(a + b)}$

16. $\frac{a^4 - b^4}{a^2 + 2ab + b^2}$ Ans $\frac{(a^2 + b^2)(a - b)}{a + b}$

17. $\frac{x^2 + xy + y^2}{(x^3 - y^3)xy}$ Ans $\frac{1}{(x - y)xy}$

18. $\frac{x^2 + 13x + 42}{x^2 + 17x + 70}$ Ans $\frac{x + 6}{x + 10}$

19. $\frac{x^3 - 11x^2 - x + 11}{x^3 - 11x^2 + 2x - 22}$ Ans $\frac{x^2 - 1}{x^2 + 2}$

20 $\frac{x^3 - 29x + 42}{x^3 + x^2 - 35x + 49}$ Ans $\frac{x + 6}{x + 7}$

21. $\frac{x^3 - 9x^2 + 23x - 12}{x^3 - 10x^2 + 28x - 15}$ Ans $\frac{x - 4}{x - 5}$

22. $\dfrac{24(x^2-x+1)}{18(x^4+x^2+1)}$ Ans $\dfrac{4}{3(x^2+x+1)}$

23. $\dfrac{x^3-4x^2+2x+3}{2x^4-9x^3+12x^2-7}$ Ans $\dfrac{x-3}{2x^2-7x+7}$

24. $\dfrac{x^6+a^6}{x^4+2a^2x^2+a^4}$ Ans $\dfrac{x^4-a^2x^2+a^4}{x^2+a^2}$

25. $\dfrac{x^2+(a-c)x-ac}{x^2-(b-a)x-ab}$ Ans $\dfrac{x-c}{x-b}$

26. $\dfrac{(x+b)^2-(a+c)^2}{(x+c)^2-(a+b)^2}$ Ans $\dfrac{x-a+b-c}{x-a-b+c}$

27 $\dfrac{x^4+4}{x^4-2x^3+4x^2-4x+4}$ Ans $\dfrac{x^2+2x+2}{x^2+2}$

28. $\dfrac{a+b-2\sqrt{ab}}{a-b}$ Ans $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

29. $\dfrac{(a-c)^2-b^2}{a^2-(b+c)^2}$ Ans. $\dfrac{a+b-c}{a+b+c}$.

30. $\dfrac{x^4+a^2x^2+a^4}{(x+a)(x^2+ax+a^2)}$ Ans $\dfrac{x^2-ax+a^2}{x+a}$

31. $\dfrac{x^5-1}{x^6-1}$ Ans $\dfrac{x^4+x^3+x^2+x+1}{(x+1)(x^2-x+1)(x^2+x+1)}$

✓ 32. $\dfrac{6x^3-19x^2-4x+1}{8x^3-30x^2+11x-1}$ Ans $\dfrac{3x+1}{4x-1}$

✓ 33. $\dfrac{8x^3-2x^2+5x+4}{28x^3-33x^2+37x-12}$ Ans $\dfrac{2x+1}{7x-3}$

✓ 34. $\dfrac{x^6+x^5+x^4+x^3+x^2+x+1}{x^7-1}$ Ans $\dfrac{1}{x-1}$

✓ 35. $\dfrac{28x^4+4x^3-7x-1}{32x^4-4x^3-8x+1}$ Ans $\dfrac{7x+1}{8x-1}$

✓ 36. $\dfrac{x^3+y^3+3xy-1}{x^2+2xy+y^2-1}$ Ans $\dfrac{x^2+y^2-xy+x+y+1}{x+y+1}$

37. ✓ $\frac{x^3+9x^2+26x+24}{x^3+5x^2-2x-24}$ Ans $\frac{x+2}{x-2}$.

38. ✓ $\frac{2x^3+x^2-x+3}{2x^3+3x^2-3x+5}$ Ans $\frac{2x+3}{2x+5}$

39. ✓ $\frac{30x^3-4x^2+3x+1}{30x^3+2x^2+x+2}$ Ans $\frac{5x+1}{5x+2}$

40. ✓ $\frac{x^8-a^8}{x^6-a^6}$ Ans $\frac{(x^4+a^4)(x^2+a^2)}{x^4+a^2x^2+a^4}$

Reduce the following fractions to their lowest common denominator:—

41. $\frac{6}{7x}$, $\frac{5}{21x^2}$, $\frac{2}{14x^3}$ Ans $\frac{36x^2}{42x^3}$, $\frac{10x}{42x^3}$, $\frac{6}{42x^3}$

42. $\frac{x}{x-1}$, $\frac{2x}{x+1}$, $\frac{a}{x^2-1}$, $\frac{ax}{(x-1)^2}$

Ans $\frac{x(x^2-1)}{(x+1)(x-1)^2}$, $\frac{2x(x-1)^2}{(x+1)(x-1)^2}$, $\frac{a(x-1)}{(x+1)(x-1)^2}$, $\frac{ax(x+1)}{(x+1)(x-1)^2}$

43. $\frac{-1}{a+1}$, $\frac{a+1}{2}$, $\frac{a^3}{a^2-1}$

Ans $\frac{2(a^2-1)(a-1)}{2(a^2-1)}$, $\frac{(a+1)(a^2-1)}{2(a^2-1)}$, $\frac{2a^3}{2(a^2-1)}$

44. $\frac{1}{x-a}$, $\frac{2}{x^2-a^2}$, $\frac{3x}{x^2+a^2}$

Ans $\frac{(x+a)(x^2+a^2)}{x^4-a^4}$, $\frac{2(x^2+a^2)}{x^4-a^4}$, $\frac{3x(x^2-a^2)}{x^4-a^4}$

45. $\frac{4}{x^2-ax+a^2}$, $\frac{3}{x^3+a^3}$, $\frac{3}{x^2-a^2}$

Ans $\frac{4(x^2-a^2)}{(x^3+a^3)(x-a)}$, $\frac{3(x-a)}{(x^3+a^3)(x-a)}$, $\frac{3(x^2-ax+a^2)}{(x^3+a^3)(x-a)}$

46. $\frac{1}{x^2-4x+3}$, $\frac{1}{x^2-7x+12}$, $\frac{1}{x^2+2x-15}$

Ans $\frac{(x-4)(x+5)}{\mathrm{D}}$, $\frac{(x-1)(x+5)}{\mathrm{D}}$, $\frac{(x-1)(x-4)}{\mathrm{D}}$,

if $\mathrm{D}=(x-3)(x-1)(x-4)(x+5)$

47. $\frac{1}{x^2+(a+c)x+ac}$, $\frac{1}{x^2+(a+b)x+ab}$, $\frac{1}{x^2+(b+c)x+bc}$

Ans $\frac{x+b}{(x+a)(x+b)(x+c)}$, $\frac{x+c}{(x+a)(x+b)(x+c)}$,

$\frac{x+a}{(x+a)(x+b)(x+c)}$

48. $\frac{1}{1-x^n}$, $\frac{1}{1-x^{2n}}$, $\frac{1}{1-x^{3n}}$

Ans $\frac{(1+x^n)(1+x^n+x^{2n})}{\mathrm{D}}$, $\frac{(1+x^n+x^{2n})}{\mathrm{D}}$, $\frac{1+x^n}{\mathrm{D}}$

if $\mathrm{D}=(1+x^n)(1-x^{3n})$

Exercise 13.

ADDITION AND SUBTRACTION OF FRACTIONS.

Simplify the following expressions

1. $\frac{2x}{5}+\frac{7x}{9}+\frac{2x}{3}$ Ans $\frac{83x}{45}$

2. $2x+\frac{3x}{5}+4+\frac{x}{10}$ Ans $\frac{40+27x}{10}$

3. $\frac{4x+6}{8}-\frac{2x-5}{4}$ Ans 2

4. $\frac{6x^2+8xy}{6}-\frac{xy-4x^2}{3}$ Ans $\frac{7x^2+3xy}{3}$

5. $\dfrac{4a^2+c}{6}+\dfrac{3a^2+7b^2}{18}$ Ans $\dfrac{15a^2+7b^2+3c}{18}$

6. $\dfrac{x+5}{4x}+\dfrac{5x+1}{6x}+\dfrac{2}{3}$ Ans $\dfrac{21x+17}{12x}$

7. $\dfrac{3a^2}{2b}+\dfrac{4a}{5}+\dfrac{7b}{3a}$ Ans $\dfrac{45a^3+24a^2b+70b^2}{30ab}$

8. $\dfrac{4x+9}{4}-\dfrac{6x-5}{6x}$ Ans $\dfrac{12x^2+15x+10}{12x}$

9. $\dfrac{x}{a^2+b^2}+\dfrac{2x}{a^2-b^2}$ Ans $\dfrac{x(3a^2+b^2)}{a^4-b^4}$

10. $\dfrac{x}{y}+\dfrac{x+y}{x-2y}$ Ans $\dfrac{x^2-xy+y^2}{y(x+2y)}$

11. $\dfrac{2x-3}{x}+\dfrac{x}{x-1}$ Ans $\dfrac{3x^2-5x+3}{x(x-1)}$

12. $\dfrac{x}{x-7}-\dfrac{x}{x-6}$ Ans $\dfrac{x}{x^2-13x+42}$

13. $\dfrac{x+a+b}{a+b}-\dfrac{x}{a+b}-1$ Ans 0

14. $\dfrac{2+x}{1+x+x^2}-\dfrac{1-x}{1-x+x^2}$ Ans $\dfrac{1-x+x^2+2x^3}{1+x^2+x^4}$

15. $\dfrac{2x}{3y}-\dfrac{xz}{y(y+z)}$ Ans $\dfrac{x(2y-z)}{3y(y+z)}$

16. $\dfrac{1}{a-1}-\dfrac{1}{2(a+1)}-\dfrac{a+3}{2(a^2+1)}$ Ans $\dfrac{a+3}{a^4-1}$

17. $\dfrac{2x+3y}{2x-3y}+\dfrac{2x-3y}{2x+3y}$ Ans $\dfrac{2(4x^2+9y^2)}{4x^2-9y^2}$

18. $\dfrac{3(a-b)}{5(a+b)}-\dfrac{a^2-b^2}{a^2+2ab+b^2}$ Ans $\dfrac{2(b-a)}{5(a+b)}$

19. $\dfrac{x}{(4a)^x+1}-\dfrac{x}{(4a)^x-1}$ Ans $\dfrac{2x}{1-(4a)^{2x}}$

20. $\frac{5x-3}{2x+11} - \frac{4x^2-10x+7}{4x^2-121}$ Ans $\frac{6x^2-51x+26}{4x^2-121}$

21. $\frac{1}{x^{m-n}+1} + \frac{1}{x^{n-m}-1}$ Ans $\frac{1+x^{2(m-n)}}{1-x^{2(m-n)}}$

22 $\frac{1}{2x+\sqrt{4x^2-y^2}} + \frac{1}{2x-\sqrt{4x^2-y^2}}$ Ans $\frac{4x}{y^2}$

23. $\frac{2x(4+2x)}{4-2x} + \frac{4x^2-5ax}{2x-4}$ Ans $\frac{x(8+5a)}{4-2x}$

24. $\frac{x}{2(a-b)} + \frac{y}{2(b-a)}$ Ans $\frac{x-y}{2(a-b)}$

25 $\frac{2x-\sqrt{y}}{2z+\sqrt{a}} + \frac{2x+\sqrt{y}}{2z-\sqrt{a}}$ Ans $\frac{2(4xz+\sqrt{ay})}{4z^2-a}$

26. $\frac{\sqrt{2x+y}}{\sqrt{2x-2y}} - \frac{\sqrt{2x-2y}}{\sqrt{2x+y}}$ Ans $\frac{3y}{\sqrt{4x^2-2xy-2y^2}}$

27. $\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} - \frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}$ Ans $\frac{4a\sqrt{a^2-b^2}}{b^2}$

28. $\sqrt{\frac{4x}{5x+2}} + \sqrt{\frac{5x+2}{4x}}$ Ans $\frac{9x+2}{\sqrt{4x(5x+2)}}$

29. $\frac{1}{(2x)^{-2}} - \frac{(2x)^{-2}}{(2x)^{-4}}$ Ans 0

30. $\frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}-\sqrt{a^2-1}} + \frac{\sqrt{a^2+1}-\sqrt{a^2-1}}{\sqrt{a^2+1}+\sqrt{a^2-1}}$ Ans $2a^2$

31 $\frac{1}{2x} + \frac{3}{2x+1} - \frac{2}{2x+2}$ Ans $\frac{4x^2+7x+1}{2x(x+1)(2x+1)}$

32. $\frac{1}{(x+1)(x+3)} - \frac{4}{(x+1)(x+3)(x+5)}$ Ans $\frac{1}{(x+3)(x+5)}$

33. $(x^2+5x+6)^{-1} - (x^2+4x+4)^{-1} + (x+3)^{-2}$

Ans $(x^2+3x+1)(x+2)^{-2}(x+3)^{-2}$ or $\frac{x^2+3x+1}{(x+2)^2(x+3)^2}$

34. $\frac{2a}{(a+b)^2}-\frac{a-2b}{a^2-b^2}$ Ans $\frac{a^2-ab+2b^2}{(a+b)^2(a-b)}$

35. $\left(\frac{x+y}{x-y}\right)^2-\frac{4xy}{(x-y)^2}$ Ans 1

36. $\frac{1}{(a-b)(a-c)}+\frac{1}{(c-a)(c-b)}$ Ans $\frac{1}{(a-b)(b-c)}$

37. $\frac{a^3}{a-1}-\frac{2a}{a-1}+\frac{1}{a-1}$ Ans $a-1$

38. $\frac{\text{[illegible]}a-4b}{6}-\frac{2a-b-c}{2}+\frac{13a-3b}{12}$ Ans $\frac{9a-5b+6c}{12}$

39. $\frac{2a}{a-x}-\frac{4a}{a+x}+\frac{8ax}{a^2-x^2}$ Ans $\frac{2a(7x-a)}{a^2-x^2}$

40. $\frac{x^2}{1-x^2}+\frac{x}{x-1}+\frac{4x}{x+1}$ Ans $\frac{x(3-4x)}{1-x^2}$

41. $\frac{2}{a-x}-\frac{3}{a+x}+\frac{5a}{a^2+x^2}$ Ans $\frac{4a^3+5a^2x-6ax^2+5x^3}{a^4-x^4}$

42. $\frac{5x^2+1}{x-1}-\frac{2x^2-5x+2}{1-x^2}+\frac{2x^2+1}{x+1}$ Ans $\frac{7x^3+5x^2+5}{x^2-1}$

43. $\frac{1}{4(1+y)}-\frac{1}{4(y-1)}+\frac{1}{2(1+y^2)}-\frac{1}{1-y^4}$ Ans 0

44. $\frac{1}{x^2+x+1}+\frac{2x}{x^4-x^2+1}-\frac{1}{x^2-x+1}$ Ans $\frac{4x^3}{x^8+x^4+1}$

45. $\frac{4}{2(x^2-1)}+\frac{6x}{4(x+1)}-\frac{10}{5(x-1)}$ Ans $\frac{3x^2-7x}{2(x^2-1)}$

46. $\frac{a-2x}{x-b}-\frac{a+2x}{x+b}+\frac{4x-a}{b^2-x^2}$ Ans $\frac{2ab-4x^2-4x+a}{x^2-b^2}$

47. $\frac{4x}{8}+\frac{7}{1-x}+\frac{3-2x}{x^2-16}$ Ans $\frac{x^3-25x-34}{2(x^2-16)}$

48. $\dfrac{a^3}{(a-b)^3}+\dfrac{ab}{(a-b)^2}-\dfrac{2a}{a-b}$ Ans $\dfrac{5a^2b-3ab^2-a^3}{(a-b)^3}$

49. $\dfrac{x-1}{x+1}+\dfrac{x+1}{x-1}+\dfrac{x+2}{x^2-1}$ Ans $\dfrac{2x^2+x+4}{x^2-1}$

50. $\dfrac{6}{1+x}-\left\{\dfrac{5}{1+x}+\dfrac{4x}{1-x^2}\right\}$ Ans $\dfrac{1-5x}{1-x^2}$

51. $\dfrac{a+x}{a-x}+\dfrac{a-x}{a+x}-\dfrac{a^2}{a^2-x^2}$ Ans $\dfrac{a^2+2x^2}{a^2-x^2}$

52. $\left(\dfrac{2}{x-1}-\dfrac{1}{x+1}\right)-\left(\dfrac{6}{x^2+1}+\dfrac{x}{x^2-1}\right)$ Ans $\dfrac{3(3-x^2)}{x^4-1}$

53. $\dfrac{2a+c}{(x-a)(b-a)}-\dfrac{b+c}{(x-b)(b-a)}$

Ans $\dfrac{2ax-2ab-bc-bx+ab+ac}{(x-a)(x-b)(b-a)}$

54. $\dfrac{1}{(x+1)(x+2)}-\dfrac{1}{(x+1)(x+3)}-\dfrac{1}{(x+1)(x+2)(x+3)}+\dfrac{1}{x-1}$

Ans $\dfrac{1}{x-1}$

55. $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-a)(b-c)}$ Ans $\dfrac{1}{(a-c)(c-b)}$

56. $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(c-a)(c-b)}+\dfrac{1}{(b-a)(b-c)}$ Ans 0

57. $\dfrac{1}{x(x-y)(x-z)}-\dfrac{1}{xyz}+\dfrac{1}{y(y-x)(y-z)}$ Ans $\dfrac{1}{z(x-z)(z-y)}$

58. $\dfrac{1}{1-x}-\dfrac{1}{1+x}-\dfrac{2x}{1+x^2}-\dfrac{4x^3}{1+x^4}-\dfrac{8x^7}{1+x^8}-\dfrac{16x^{15}}{1-x^{16}}$ Ans 0

59. $\dfrac{bc(x-a)^2}{(a-b)(a-c)}+\dfrac{ac(x-b)^2}{(b-c)(b-a)}+\dfrac{ab(x-c)^2}{(c-a)(c-b)}$ Ans x^2

60. $\dfrac{1}{(x-2)(x-3)}+\dfrac{1}{(x-1)(x-2)}$ Ans $\dfrac{2}{(x-1)(x-3)}$

61. $\frac{1}{x^2-y^2}+\frac{1}{(x+y)^2}-\frac{1}{(x-y)^2}$ Ans $\frac{x^2-4xy-y^2}{x(^2-y^2)^2}$

62. $\frac{1}{x-1}+\frac{1}{x+1}+\frac{2x-3}{2(1-x^2)}$ Ans $\frac{2x+3}{2(x^2-1)}$

63. $\frac{1}{x^2-x+1}+\frac{1}{x^4-x^3+1}+\frac{1}{x^2+x+1}$ Ans $\frac{2x^6+x^4+x^2+3}{x^8+x^4+1}$

64. $\frac{1-x^2}{(1-x)^3}+\frac{4-x^2}{x^4-16}-\frac{x^4+x^3+5x^2+9x}{1-x^2}$

Ans $\frac{3}{(1-x)^2(x^2+4)(1+x)}$

65 $\frac{1}{2(3-\sqrt{x})}+\frac{2}{4(3+\sqrt{x})}-\frac{3}{9+x}$ Ans $\frac{6x}{81-x^2}$

66. $\frac{1}{1-x+x^2}+\frac{1}{1+x}+\frac{2}{1+x^3}$ Ans $\frac{x^2+4}{1+x^3}$

67. $\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-x)(y-z)}+\frac{z^2}{(z-x)(z-y)}$ Ans 1

68. $\frac{m}{a^m-1}+\frac{m}{a^{-m}-1}$ Ans $-m$

69. $\frac{4x}{(x+2)(x+1)}-\frac{x+2}{x^2-1}+\frac{x+3}{(x+2)(x-1)}$

Ans $\frac{4x^2-4x-1}{(x+2)(x+1)(x-1)}$

70. $\frac{x^2+y^2+2xy}{2(x+y)^3}+\frac{x+xy}{x^2-y^2}+\frac{x}{(x+y)^2}$

Ans $\frac{3x^2+y^2+2x^2+4xy+2xz-2yz}{2(x-y)(x+y)^2}$

71. $\frac{1}{1+x^{m-n}+x^{m-p}}+\frac{1}{1+x^{n-m}+x^{n-p}}+\frac{1}{1+x^{p-m}+x^p}$

Ans 1

72. $\frac{x^{3n}}{x^n-1}-\frac{x^{2n}}{x^n+1}-\frac{1}{x^n-1}+\frac{1}{x^n+1}$ Ans $x^{2n}+2$

73. $\frac{2-x+x^2}{1+x-x^2}+\frac{6x^2+4x}{1+x-x^2}-2$ Ans $\frac{9x^2+x}{1+x-x^2}$

74. $\frac{x^3-1}{x^2+x+1}+\frac{x^2-1}{x-1}$ Ans $2x$

75. $\frac{x^3-5x^2-x+14}{x^2-3x-7}+\frac{x^3-5x^2-46x-40}{x+4}$ Ans $x^2-8x-12$

76. $\frac{1-x}{1+x}+\frac{1-x-x^2}{1+x+x^2}+\frac{1-x-x^2-x^3}{1+x+x^2+x^3}$

Ans $\frac{3-x^2-5x^3-4x^4-3x^5}{1+2x+3x^2+3x^3+2x^4+x^5}$

77 $\frac{x^3+y^3+3xy-1}{x^2-xy+x+y^2+y+1}-y+1$ Ans x

78. $\frac{x(a+x)}{a-x}+\frac{5ax-x^2}{x-a}+\frac{2a^2}{a-x}-2(a-x)$ Ans 0

79. $\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}-\frac{c^2}{(a-c)(c-b)}$ Ans 1

80. $\frac{2a+b}{2ab}(4a^2+b^2-c^2)+\frac{b+c}{bc}(b^2+c^2-4a^2)+\frac{2a+c}{2ac}(c^2+4a^2-b^2)$

Ans $2(2a+b+c)$

81. $\frac{2x^2+3}{x^4+x^2+1}+\frac{1}{x+\sqrt{x}+1}+\frac{1}{x-\sqrt{x}+1}-\frac{2}{x^2+x+1}$

Ans $\frac{2x^3+2x+3}{x^4+x^2+1}$

82. $\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}$ Ans 1

83. $\dfrac{1}{x(x-y)(x-z)} + \dfrac{1}{y(y-z)(y-x)} + \dfrac{1}{z(z-x)(z-y)}$

Ans $\dfrac{1}{xyz}$

84. $\dfrac{1}{(a-b)(a-c)(x+a)} - \dfrac{1}{(a-b)(b-c)(x+b)} + \dfrac{1}{(a-c)(b-c)(x+c)}$

Ans $\dfrac{1}{(x+a)(x+b)(x+c)}$

85. $\dfrac{1}{x^2+3x+2} + \dfrac{2x}{x^2+2x+1} + \dfrac{3x}{x^2+5x+6}$

Ans $\dfrac{5x^3+17x^2+19x+3}{(x+1)^2(x+2)(x+3)}$

86. $\dfrac{(a-b)^3}{(x+a)^2(x+b)^2} - \dfrac{a-b}{(x+a)^2} - \dfrac{2(b-a)}{(x+a)(x+b)}$ Ans $\dfrac{a-b}{(x+b)^2}$

87. $\dfrac{a}{a^2-1} + \dfrac{a^2}{a^4-1} + \dfrac{a^4}{a^2-1} - \dfrac{a^6+a^4-1}{a^4-1}$ Ans $\dfrac{1}{a-1}$

88. $\dfrac{(1+xy)(1+xz)}{(x-y)(z-x)} - \dfrac{(1+yz)(1+xy)}{(x-y)(z-y)} - \dfrac{(1+xz)(1+zy)}{(x-z)(y-z)}$ Ans 1

89. $\dfrac{2(x^2-1)(y^2-1)xy+4x^2y^2-(x^2+y^2)(1+x^2y^2)}{2(x^2+1)(y^2+1)xy-4x^2y^2-(x^2+y^2)(1+x^2y^2)}$ Ans $\left(\dfrac{xy+1}{xy-1}\right)^2$

90 $\dfrac{2}{a-b} + \dfrac{2}{b-c} + \dfrac{2}{c-a} + \dfrac{(a-b)^2+(b-c)^2+(c-a)^2}{(a-b)(b-c)(c-a)}$ Ans 0.

91. $\dfrac{d(a-b)(b-c)+b(a-d)(c-d)}{c(a-b)(a-d)+a(b-c)(c-d)}$ Ans $\dfrac{b-d}{a-c}$

92. $\dfrac{d^2(a-b)(b-c)+b^2(a-d)(c-d)}{c^2(a-b)(a-d)+a^2(b-c)(c-d)}$ Ans $\dfrac{b-d}{a-c}$

Exercise 14.

MULTIPLICATION OF FRACTIONS.

Find the vaules of

1. $\frac{2x}{3}\times\frac{6}{8x}$ Ans. $\frac{1}{2}$

2. $\frac{2x}{a}\times\frac{a^2}{mx}\times\frac{m^2}{x}$ Ans $\frac{2am}{x}$

3. $\frac{ax}{m+1}\times\frac{m^2-1}{a^2x^4}\times\frac{a}{x}$ Ans $\frac{m-1}{x^2}$

4. $\frac{4x^2}{5y^2}\times\frac{20y^2}{6z^2}\times\frac{6z^2}{4x^2}$ Ans 4

5. $\frac{x^2+xy}{x^3-y^3}\times\frac{x^2-y^2}{(x+y)^2}\times\frac{x^2+xy+y^2}{x}$ Ans 1.

6. $\frac{a^2+2ax}{a^2-4x^2}\times\frac{x^{-1}}{(a-2x)}\times(a^2-4ax+4x^2)$ Ans $\frac{a}{x}$

7. $\frac{a^2}{a-x}\times\frac{a-x}{a-2x}\times\frac{a^2-4x^2}{a^3-x^3}\times\frac{a^2-x^2}{a^2}$ Ans $\frac{(a+2x)(a+x)}{a^2+ax+x^2}$

8. $\frac{a^{3m}-x^{3m}}{a^m-x^m}\times\frac{a^m x^{m+2}}{a^{2m}+a^m x^m+x^{2m}}\times\frac{1}{x^m x^2}$ Ans a^m

9. $\frac{x^2+5x+6}{x+2}\times\frac{x+5}{x^2+10x+21}\times\frac{x+7}{x^2}$ Ans $\frac{x+5}{x^2}$

10 $\frac{\sqrt{x}-\sqrt{y}}{x-y}\times\frac{x+2\sqrt{xy}+y}{\sqrt{xy}}\times 4$ Ans $4\sqrt{(xy)}(\sqrt{x}+\sqrt{y})$

11 $\frac{x+y}{(x-y)^3}\times\frac{x^3-y^3}{x^3+y^3}\times\frac{(x-y)^2+xy}{(x+y)^2-xy}\times(x-y)^3$ Ans 1

12. $\frac{\frac{a+x}{a-x}+\frac{a-x}{a+x}}{a^2+x^2}\times\frac{c+x}{2}\times 6x$ Ans $\frac{6x}{a-x}$

13. $\left(\frac{\sqrt{(a+x)}}{\sqrt{(a-x)}} - \frac{\sqrt{(a-x)}}{\sqrt{(a+x)}}\right) \times (a^2 - x^2) \times \sqrt{x}$ Ans $2x^{\frac{3}{2}}\sqrt{(a^2 - x^2)}$

14. $\frac{x^2 - 3x - 10}{x^2 + 6x - 7} \times \frac{x^2 + 10x + 21}{x^2 - 3x - 10} \times \frac{x^2 + 3x - 4}{x^2 + 7x + 12}$ Ans 1

15. $\frac{a^4 + a^2 + 1}{a^2} \times \left(a - \frac{1}{a}\right)$ Ans $a^3 - \frac{1}{a^3}$

16. $x^{m+n+p} \times (2x)^{4n-m-p} \times 2^{m+p-4n} \times 4$ Ans $4x^{5n}$

17. $(x^{m+n})^2 \times (x^{n-p})^2 \times (x^{-m+p})^2$ Ans x^{4n}

18. $\left(\frac{2x^2}{7} - \frac{99}{49}x + \frac{1}{7}\right) \times \frac{7}{x - 7}$ Ans $2x - \frac{1}{7}$

19. $\frac{(a - x)^2}{x(a + x)} \times \frac{x^2}{a^2 - x^2}$ Ans $\frac{x(a - x)}{(a + x)^2}$

20. $\left(x^3 - x + \frac{1}{x} - \frac{1}{x^3}\right) \times \left(x + \frac{1}{x}\right)$ Ans $x^4 - \frac{1}{x^4}$

21. $\frac{a + b + c}{a + b - c} \times (a^2 - b^2 + c^2 - 2ac)$ Ans $a^2 - b^2 - c^2 - 2bc$

22. $(x + \frac{1}{x} - 1) \times \frac{x^2 + x + 1}{x}$ Ans $x^2 + \frac{1}{x^2} + 1$

23. $\frac{(a + x)^2}{x^2 + xy + y^2} \times \frac{a + x}{x - y}$ Ans $\frac{(a + x)^3}{x^3 - y^3}$

24. $\left(\frac{a + b}{c - d} + \frac{a - b}{c + d}\right) \times \frac{ac - bd}{ac + bd}$ Ans $\frac{2(ac - bd)}{c^2 - d^2}$

25. $\frac{a^3 - b^3}{x^3 + y^3} \times \frac{x + y}{a - b} \times 9\left(\frac{x^2 - xy + y^2}{a^2 + ab + b^2}\right)$ Ans 9

Exercise 15.

DIVISION OF FRACTIONS.

Find the values of

1. $\frac{5x}{7} \div \frac{25x}{21}$ Ans $\frac{3}{5}$

2. $\frac{4x^2}{5y^2} \div \frac{2x}{6y^2}$ Ans $\frac{12x}{5}$

3. $\frac{4x^2-4}{18} \div \frac{6(x+1)}{36}$ Ans $\frac{4(x-1)}{3}$

4. $\frac{x^2+xy}{xy-y^2} \div \frac{x^2(x+y)^2}{(x-y)^2}$ Ans $\frac{x-y}{xy(x+y)}$

5. $\frac{a-b}{ab+b^2} \div \frac{5(a^2-b^2)}{a^2+ab}$ Ans $\frac{a}{5b(a+b)}$

6. $\frac{x^3-y^3}{x^2-y^2} \div \frac{(x-y)x}{x+y}$ Ans $\frac{x^2+xy+y^2}{x(x-y)}$

7. $\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+1\right) \div \left(\frac{x}{y}+\frac{y}{x}+1\right)$ Ans $\left(\frac{x}{y}+\frac{y}{x}-1\right)$

8. $\frac{(a-c)^2-b^2}{abc} \div \left(\frac{a}{bc}-\frac{b}{ac}-\frac{c}{ab}-\frac{2}{a}\right)$ Ans $\frac{a+b-c}{a+b+c}$

9. $\frac{x^2+(a+b)x+ab}{x^2+(a-c)x-ac} \div \frac{x^2-b^2}{x^2-c^2}$ Ans $\frac{x+c}{x-b}$

10. $\frac{a^4-b^4}{(a+b)^2} \div \frac{(a-b)}{a(a+b)}$ Ans $a(a^2+b^2)$

11. $1-\frac{1}{1+x}$ by $1+\frac{x^2}{1-x^2}$ Ans $x(1-x)$

12. $\left(23x^2-3y+\frac{y}{8}\right) \div \left(6x-\frac{x}{4}\right)$ Ans $4x-\frac{y}{2x}$

13. $\left(x+\frac{2x}{x-3}\right) \div \left(x-\frac{2x}{x-3}\right)$ Ans $\frac{x-1}{x-5}$

14. $\frac{7a(a^2-x^2)}{3b(c^2-x^2)} \div \frac{a^2-ax}{bc+bx}$ Ans $\frac{7(a+x)}{3(c-x)}$

15. $\left(x^6+\frac{1}{x^6}+x^4+\frac{1}{x^4}+x^2+\frac{1}{x^2}+2\right)\div\left(x^3+\frac{1}{x^3}+x+\frac{1}{x}\right)$

Ans $x^3+\frac{1}{x^3}$

16. $\frac{b-3a}{2ab}\div\frac{6a-2b}{b^2-2ab}$ Ans $\frac{2a-b}{4a}$

17. $\left(\frac{2a+x}{2a-x}+\frac{2a-x}{2a+x}\right)\div\left(\frac{2a+x}{2a-x}-\frac{2a-x}{2a+x}\right)$ Ans $\frac{4a^2+x^2}{4ax}$

18. $\left(\frac{a-1}{a}+\frac{b-1}{b}+\frac{c-1}{c}\right)\div\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ Ans $\frac{3abc}{ab+ac+bc}-1$

19. $\left(\frac{x+y}{x-y}+\frac{x^2+y^2}{x^2-y^2}\right)\div\left(\frac{x-y}{x+y}-\frac{x^3-y^3}{x^3+y^3}\right)$

Ans $\frac{x^4+x^2y^2+y^4}{xy(2xy-x^2-y^2)}$

20. $\left(\frac{x^2+y}{x^2-y}-\frac{x^2-y}{x^2+y}\right)\div\left(\frac{x+\sqrt{y}}{x-\sqrt{y}}-\frac{x-\sqrt{y}}{x+\sqrt{y}}\right)$ Ans $\frac{x\sqrt{x}}{x^2+y}$

21. $\left(\frac{2x+3y}{2x-3y}+\frac{2x-3y}{2x+3y}\right)\div\left(\frac{4x^2+9y^2}{(2x)^2-(3y)^2}-\frac{(2x)^2-(3y)^2}{3x^2+6y^2}\right)$

Ans $\frac{(4x^2+9y^2)^2}{72x^2y^2}$

Simplify

22. $\frac{\left(\frac{x-a}{x+a}-\frac{x+a}{x-a}\right)}{\left(\frac{x+a}{x-a}+\frac{x-a}{x+a}\right)}+\frac{x}{x-a}-\frac{x}{x+a}$ Ans $\frac{4a^3x}{x^4-a^4}$

23. $\cfrac{1}{a+\cfrac{1}{b+{}^{1}}}$. Ans $\frac{bc+1}{abc+a+c}$

24. $\dfrac{1}{x-4+\dfrac{2}{x-\dfrac{2}{x}}}$ Ans $\dfrac{x^2-2}{x^3-4x^2+8}$

25. $\dfrac{\dfrac{x^2+\sqrt{(x^4-a^4)}}{x^2-\sqrt{(x^4-a^4)}}-\dfrac{x^2-\sqrt{(x^4-a^4)}}{x^2+\sqrt{(x^4-a^4)}}}{4\sqrt{\left(\dfrac{x^2-a^2}{x^2+a^2}\right)}} \div \dfrac{x^2(x^2+a^2)}{a^4}$ Ans 1

Find the values of the following expressions

26 $\dfrac{x-m}{y-m}$ when $m=\dfrac{xy}{x+y}$ Ans $\dfrac{x^2}{y^2}$

27. $\dfrac{1}{x+a}+\dfrac{1}{x-a}$ when $x^2=a^2+x$ Ans 2

28. $a+\dfrac{x}{m}+\dfrac{x}{b-m}-\dfrac{m}{b+m}$ when $x=\dfrac{m^2(b-m)}{b(b+m)}$ Ans a

29. $\left(\dfrac{x-a}{x-b}\right)^2-\dfrac{x-2a+b}{x+a-2b}+1$ when $x=\dfrac{a+b}{2}$ Ans 1

30. $\dfrac{1}{m(n-z)}+\dfrac{1}{n(p-z)}+\dfrac{1}{m(z-p)}$, when $z=\dfrac{n(m-n+p)}{m}$ Ans 0

Exercise 16.

INVOLUTION.

Find the values of

1. $(4a^2bx^4)^3$ Ans $64a^6b^3x^{12}$

2. $(-2a^4b^2)^2$ Ans $4a^8b^4$

3. $(-5b^2c^4d^3)^4$ Ans $625b^8c^{16}d^{12}$

4. $\left(\dfrac{4x^3}{7y^4}\right)^2$ Ans $\dfrac{16x^6}{49y^8}$

5. $\left(-\frac{2x^2}{3y^3}\right)^5$ Ans $-\frac{32x^{10}}{243y^{15}}$

6. $\left(-\frac{2x^3}{y^2z^3}\right)^3$ Ans $-\frac{8x^9}{y^6z^9}$

7. $(x+y)^4$ Ans $x^4+4x^3y+6x^2y^2+4xy^3+y^4$

8. $(x-y)^7$

Ans $x^7-7x^6y+21x^5y^2-35x^4y^3+35c^3y^4-21x^2y^5+7xy^6-y^7$

9. $(x+2)^3(x-2)^3$ Ans $a^6-12x^4+48x^2-64$

10. $(x-2)^3$ Ans $x^3-6x^2+12x-8$

11. $(2+x)^4$ Ans $16+32x+24x^2+8x^3+x^4$

12. $(1+x)^4$ Ans $1+4x+6x^2+4x^3+x^4$

13 $(2-3x)^3$ Ans $8-36x+54x^2-27x^3$

14. $(x-2)^5$ Ans $x^5-10x^4+40x^3-80x^2+80x-32$

15. $(4x+1)^3$ Ans $64x^3+48x^2+12x+1$

16. $(2x+3y)^3+(2x-3y)^3$ Ans $4x(4x^2+27y^2)$

17. $(4x+1)^4+(4x-1)^4$ Ans $2(256x^4+96x^2+1)$

18. $(1-x)^5+(1+x)^5$ Ans $2(1+10x^2+5x^4)$

19. $(1+x)^3(1-x)^3$ Ans $1-3x^2+3x^4-x^6$

20. $(a+b+e)^2$ Ans $a^2+b^2+c^2+2ab+2ac+2bc$

21. $(x+a+2)^2$ Ans $x^2+a^2+4+2ax+4x+4a$

22. $(2x-a-1)^2$ Ans $4x^2+a^2+1-4ax-4x+2a$

23. $(1-x+x^2)^2$ Ans $1+x^4-2x+3x^2-2x^3$

24. $(a+b+c+d)^2$

Ans $a^2+b^2+c^2+p^2+2ab+2ac+2ad+2bd+2bc+2cd$

25. $(x^2+x+a+1)^2$

Ans $x^4+x^2+a^2+1+2x^3+2ax^2+2x^2+2ax+2x+2a$

26. $(2x^2-3x+1)^2$ Ans $4x^4+13x^2+1-12x^3-6x$

27. $(4x^2-2x+1)^2-(4x^2+2x+1)^2$ Ans $-32x^3-8x$

28. $(a+b+c)^3$

Ans $a^3+b^3+c^3+3a^2(b+c)+3b^2(a+c)+3c^2(a+b)+6abc$

29. $(x^2+x-2)^3$ Ans $x^6+3x^5-3x^4-11x^3+6x^2+12x-8$

30. $(1+x+x^2)^3$ Ans $1+3x+6x^2+7x^3+6x^4+3x^5+x^6$

31. $(x^2-x-2)^3$ Ans $x^6-3x^5-3x^4-13x^3+6x^2-12x-8$

32. $(a+x^2+z^3)^3$

Ans $a^3+x^6+z^9+3a^2(x^2+z^3)+3x^4(a+z^3)+3z^6(a+x^2)+6ax^2z^3$

33. $(2x^2+3x+4)^3$

Ans $8x^6+36x^5+102x^4+171x^3+204x^2+144x-64$

34. $(1+x^2+x^4+x^6)^2$ Ans $1+2x^2+3x^4+4x^6+3x^8+2x^{10}$

35. $(1-2x+3x^2-4x^3)^2$

Ans $1+10x^2-14x^3+25x^4-24x^5+16x^5$

36. $(x+y+z+a)^2-(x-y+z-a)^2$ Ans $4(xy+yz+ax+az)$

37. $(x-y-z+2)^2+(x-y+z-2)^2$

Ans $2(x^2+y^2+z^2+4-2xy-4z)$

38. $(1+2x+3x^2+4x^3)^2$ Ans $1+10x^2+20x^3+25x^4+24x^5+16x^6$

39. $(1-2x+4x^2-6x^3)^2$

Ans $1+12x^2-28x^3+40x^4-48x^5+36x^6$

40. $(a+b+c+d+e)^2$ Ans $a^2+b^2+c^2+d^2+e^2+2ab+2ac$
$+2ad+2ae+2bc+2bd+2be+2cd+2ce+2de$

41. $(1+x+2x^2+3x^3+4x^4)^2$

Ans $1+2x+5x^2+10x^3+18x^4+20x^5+25x^6+24x^7+16x^8$

✓42. $(1+x)^3(1-x+x^2)^3$ Ans $1+3x^3+3x^6+x^9$

43. $(1 - x + x^2)^3 (1 + x + x^2)^3$

Ans $1 + 3x^2 + 6x^4 + 7x^6 + 6x^8 + 3x^{10} + x^{12}$

44. $(x + 1)^8$ Ans $x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$

45. $(1 - 3x + 4x^2)^3$

Ans $1 - 9x - 39x^2 - 99x^3 + 156x^4 - 144x^5 + 64x^6$

46. $(3x - 1)^6$

Ans $729x^6 - 1458x^5 + 1215x^4 - 540x^3 + 135x^2 - 18x + 1$

47. $\left(x^{\frac{1}{2}} - a^{\frac{1}{2}}\right)^2$ Ans $x - 2a^{\frac{1}{2}}x^{\frac{1}{2}} + a$

48. $\left(\frac{x}{2} + \frac{a}{3}\right)^2$ Ans $\frac{x^2}{4} + \frac{ax}{3} + \frac{a^2}{9}$

49. $(x^{-2} - x^{-1})^2$ Ans $x^{-4} - 2x^{-3} + x^{-2}$

50. $\left(\frac{a}{b} - \frac{b}{a}\right)^2$ Ans $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

Exercise 17.

EVOLUTION.

Extract the square roots of the following expressions

1. $16x^2y^4$ Ans $4xy^2$

2. $81a^4b^{10}$ Ans $9a^2b^5$

3. $121m^{10}n^{12}r^{14}$ Ans $11m^5n^6r^7$

4. $64a^6b^8c^4$ Ans $8a^3b^4c^2$

5. $\frac{169a^4b^6}{121x^8y^{10}}$ Ans $\frac{13a^2b^3}{11x^4y^5}$

6. $\frac{625a^2b^3}{324c^4d^5}$ Ans $\frac{25ab^{\frac{3}{2}}}{18c^2d^{\frac{5}{2}}}$

7. $x^2 - 4ax + 4a^2$ Ans $x - 2a$

8. $9x^2 - 12x + 4$ Ans $3x - 2$

9. $4x^6 + 12x^4 + 9x^2$ Ans $2x^3 + 3x$

10. $9x^2 - 30x + 25$ Ans $3x - 5$

11. $x^4 - 6x^3 + 19x^2 - 30x + 25$ Ans $x^2 - 3x + 5$

12. $9x^4 + 12x^3 + 10x^2 + 4x + 1$ Ans $3x^2 + 2x + 1$

13. $1 - 6x + 13x^2 - 42x^3 + 4x^4$ $1 - 3x + 2x^2$

14. $x^6 - 4x^5 + 10x^4 - 12x^3 + 9x^2$ Ans $x^3 - 2x^2 + 3x$

15. $4y^4 - 12y^3z + 25y^2z^2 - 24yz^3 + 16z^4$ Ans $2y^2 - 3yz + 4z^2$

16. $a^2 + 4ab + 4b^2 + 9c^2 + 6ac + 12bc$ Ans $a + 2b + 3c$

17. $a^6 + 2a^5b + 3a^4b^2 + 4a^3b^3 + 3a^2b^4 + 2ab^5 + b^6$
Ans $a^3 + a^2b + ab^2 + b^3$

18. $x^6 - 4x^5 + 6x^3 + 8x^2 + 4x + 1$ Ans $x^3 - 2x^2 - 2x - 1$

19. $4x^4 + 8ax^3 + 4a^2x^2 + 16b^2x^2 + 16ab^2x + 16b^4$
Ans $2x^2 + 2ax + 4b^2$

20. $9 - 24x + 58x^2 - 116x^3 + 129x^4 - 140x^5 + 100x^6$
Ans $3 - 4x + 7x^2 - 10x^3$

21. $16a^4 - 40a^3b + 25a^2b^2 - 80ab^2x + 64b^2x^2 + 64a^2bx$
Ans $4a^2 - 5ab + 8bx$

22. $9a^4 - 24a^3p^3 - 30a^2t + 16a^2p^6 + 40ap^3t + 25t^2$
Ans $3a^2 - 4ap^3 - 5t$

23. $16y^4x^2 - 12y^3x^3 - 8y^5x + 9y^2x^4 + 4y^6$
Ans $2y^2x - 3yx^2 - 2y^3$

24. $16x^4 - 24x^3y + 25x^2y^2 - 12xy^3 + 4y^4$ Ans $4x^2 - 3xy + 2y^2$

25. $9a^2 - 12ab + 24ac - 16bc + 4b^2 + 16c^2$ Ans $3a - 2b + 4c$

26. $x^4 + 9x^2 + 25 - 6x^3 + 10x^2 - 30x$ Ans $x^2 - 3x + 5$

27. $25x^2 - 20xy + 4y^2 + 9z^2 - 12yz + 30xz$ Ans $5x - 2y + 3z$

28. $4x^2(x^2 - y) + y^3(y - 2) + y^2(4x^2 - 1)$ Ans $2x^2 - y + y^2$

29. $(a - b)^4 - 2(a^2 + b^2)(a - b)^2 + (a^2 + b^2)^2$ Ans $(a - b)^2 - (a^2 + b^2)$

30. $4x + 12\sqrt{x} + 9$ Ans $2\sqrt{x} + 3$

31. $(x + \frac{1}{x})^2 - 4(x - \frac{1}{x})$ Ans $x - \frac{1}{x} - 2$

32 $a^4 + 2(2b - c)a^3 + (4b^2 - 4bc + 3c^2)a^2 + 2c^2(2b - c)a + c^4$

Ans $a^2 + a(2b - c) + c^2$

33. $(a - 2b)^2x^4 - 2a(a - 2b)x^3 + (a^2 + 4ab - 6a - 8b^2 + 12b)x^2 -$
$(4ab - 6a)\,x + 4b^2 - 12b + 9$ Ans $(a - 2b)x^2 - ax + (2b - 3)$

34. $4a - 12a^{\frac{1}{2}}b^{\frac{1}{3}} + 9b^{\frac{2}{3}} + 16a^{\frac{1}{2}}c^{\frac{1}{4}} - 24b^{\frac{1}{3}}c^{\frac{1}{4}} + 16c^{\frac{1}{2}}$

Ans $2a^{\frac{1}{2}} - 3b^{\frac{1}{3}} + 4c^{\frac{1}{4}}$

35. $256x^{\frac{4}{3}} - 512x + 640x^{\frac{2}{3}} - 512x^{\frac{1}{3}} + 304 - 128x^{-\frac{1}{3}} + 40x^{-\frac{2}{3}}$
$- 8x^{-1} + x^{-\frac{4}{3}}$ Ans $16x^{\frac{2}{3}} - 16x^{\frac{1}{3}} + 12 - 4x^{-\frac{1}{3}} + x^{-\frac{2}{3}}$

36. $a + b + c + 2\{\sqrt{ab} + \sqrt{ac} + \sqrt{bc}\}$ Ans $\sqrt{a} + \sqrt{b} + \sqrt{c}$

37. $(x + x^{-1}) - 2(x^{\frac{1}{2}} - x^{-\frac{1}{2}}) - 1$ Ans $x^{\frac{1}{2}} - 1 - x^{-\frac{1}{2}}$

38 $a^{\frac{1}{3}} - 2a^{\frac{1}{4}} + 3a^{\frac{1}{6}} - 2a^{\frac{1}{12}} + 1$ Ans $a^{\frac{1}{6}} - a^{\frac{1}{12}} + 1$

39. $1 - ax^{\frac{1}{2}} - \frac{15}{4}a^2x + 2a^3x^{\frac{3}{2}} + 4a^4x^2$ Ans $1 - \frac{1}{2}ax^{\frac{1}{2}} - 2a^2x$

40. $a^2b^{-2} + \frac{1}{4}a^{-2}b^2 - a^{-1}b + 2ab^{-1}$ Ans $ab^{-1} - \frac{1}{2}a^{-1}b + 1$

41. $x^{\frac{4}{3}} - 4x + 8x^{\frac{1}{3}} + 4$ Ans $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 2$

42. $(x + x^{-1})^2 - 4(x - x^{-1})$ Ans $x - 2 - x^{-1}$

43. $x + 1 - 2\sqrt[4]{x}(1 + \sqrt{x}) + 3\sqrt{x}$ Ans $x^{\frac{1}{2}} - x^{\frac{1}{4}} + 1$

44. $2+a^{2\sqrt{2}}+a^{-2\sqrt{2}}$ Ans $a^{\sqrt{2}}+a^{-\sqrt{2}}$

45. $a^2+b^2+c^2$ [illegible] $b+2ac+2be-2\sqrt{ab}(a+b+c)$

Ans $a+b+c-2\sqrt{ab}$

46. $x^{-4}-6x^{-3}y^{-1}+\frac{29}{3}x^{-2}y^{-2}-2x^{-1}y^{-3}+\frac{1}{9}y^{-4}$

Ans $x^{-2}-3x^{-1}y^{-1}+\frac{1}{3}y^{-2}$

47. $4x^4-4x^3+(2\sqrt{2}+1)x^2-\sqrt{2}x+\frac{1}{2}$ Ans $2x^2-x+\frac{1}{\sqrt{2}}$

48. $1+x$ to 5 terms Ans $1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4$

✓49. Find for what value of x the expression

$x^4+6x^3+11x^2+3x+31$ is a perfect square

Find the squre roots of Ans $x=10$

50. $16x^8+24x^6+25x^4+12x^2+4$ Ans $4x^4+3x^2+2$

51. $x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}-2x^{-\frac{1}{6}}+x^{-\frac{1}{3}}-1$ Ans. $x^{\frac{1}{3}}+x^{\frac{1}{6}}+1-x^{-\frac{1}{6}}$

52. $x^4-x^3+\frac{x^2}{4}+4x-2+\frac{4}{x^2}$ Ans $x^2-\frac{x}{2}+\frac{2}{x}$

53. $\frac{a^2}{b^2}+\frac{b^2}{a^2}-2$ Ans $\frac{a}{b}-\frac{b}{a}$

✓54. $x^2+\frac{1}{x^2}+2\left(x-\frac{1}{x}\right)-1$ Ans $x-\frac{1}{x}+1$

55. $\frac{a^2}{x^2}-\frac{4a}{3b}+\frac{4x^2}{9b^2}$ Ans $\frac{a}{x}-\frac{2x}{3b}$

✓56. $\frac{x}{a}\left(\frac{x}{a}-2\right)+\frac{b}{x}\left(\frac{b}{x}-2\right)+\frac{2b+a}{a}$ Ans $\frac{x}{a}-1+\frac{b}{x}$

✓57. $4\{(a^2-b^2)cd+(c^2-d^2)ab\}^2+\{(a^2-b^2)(c^2-d^2)-4abcd\}^2$

Ans $(a^2+b^2)(c^2+d^2)$

58. $\frac{a^2c}{b}+cx-2ac\sqrt{\frac{x}{b}}$ Ans $a\sqrt{(cb^{-1})}-\sqrt{(cx)}$

59. $x^2+9+4a-6x-12\frac{a}{x}+4\frac{a^2}{x^2}$ Ans $x-3+\frac{2a}{x}$

60. $\frac{y^2}{4}-xy+3x^2-\frac{4x^3}{y}+\frac{4x^4}{y^2}$ Ans $\frac{y}{2}-x+\frac{2x^2}{y}$

61. $\frac{x^4}{9}+\frac{2x^3}{3}+\frac{4x^2}{3}+x+\frac{1}{4}$. Ans $\frac{x^2}{3}+x+\frac{1}{2}$

62. $x^2-ax-bx+\frac{a}{2}\left(\frac{a}{2}+b\right)+\frac{b^2}{4}$ Ans $x-\frac{a}{2}-\frac{b}{2}$

63. $\frac{a^2}{4}+\frac{b^2}{9}+\frac{c^2}{16}-\frac{ab}{3}+\frac{ac}{4}-\frac{bc}{6}$ Ans $\frac{a}{2}-\frac{b}{3}+\frac{c}{4}$

64. $\frac{a^2}{b^2}+\frac{b^2}{a^2}+\frac{2a}{b}+\frac{2b}{a}+3$ Ans $\frac{a}{b}+\frac{b}{a}+1$

65. $9\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)-24\left(\frac{x}{y}+\frac{y}{x}\right)+34$ Ans $3\left(\frac{x}{y}+\frac{y}{x}\right)-4$

66. $\frac{a^4}{b^2}+\frac{c^2}{a^4}+\frac{bc^2}{a^2}\left(b+\frac{2}{a}\right)+2c\left(a+\frac{1}{b}\right)$ Ans $\frac{a^2}{b}+\frac{bc}{a}+\frac{c}{a^2}$

67. $\frac{4x^2}{49y^2}-\frac{20x}{7y}+\frac{178}{7}-\frac{15y}{2x}+\frac{9y^2}{16x^2}$ Ans $\frac{2x}{7y}-5+\frac{3y}{4x}$

68. $ax^2+by^2-2xy\sqrt{(ab)}$ Ans $x\sqrt{a}-y\sqrt{b}$

69. $\frac{x^2}{y^2}+\frac{y^2}{x^2}-\left(\frac{x}{y}+\frac{y}{x}\right)\sqrt{2}+2\frac{1}{2}$ Ans $\frac{x}{y}-\frac{1}{\sqrt{2}}+\frac{y}{x}$

70. $1+\frac{41a}{16}-\frac{3(1+a)}{2}\sqrt{a}+a^2$ Ans $1-\frac{3}{4}\sqrt{a}+a$

71. $4a^2+bx-y^2-2\sqrt{(4a^2bx-bxy^2)}$ Ans $\sqrt{(4a^2-y^2)}-\sqrt{(bx)}$

Extract the cube roots of the following expressions.

72. $a^3b^6x^{12}$ Ans ab^2x^4

73. $a^5x^{10}y^9$ Ans $a^{\frac{5}{3}}x^{\frac{10}{3}}y^3$

74. $a^3-3a^2b+3ab^2-b^3$ Ans $a-b$.

75. $8a^3 + 12a^2 + 6a + 1$ Ans $2a + 1$

76. $a^3 + 24a^2b + 192ab^2 + 512b^3$ Ans $a + 8b$

77. $a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3$
Ans $a + b + c$

78. $x^3 - 3x^2y + 3xy^2 - y^3 + 3x^2z - 6xyz + 3y^2z + 3xz^2 - 3yz^2 + z^3$
Ans $x - y + z$.

79. $27x^6 - 54x^5 + 63x^4 - 44x^3 + 21x^2 - 6x + 1$ Ans $3x^2 - 2x + 1$

80. $1 - 3a + 6a^2 - 7a^3 + 6a^4 - 3a^5 + a^6$ Ans $1 - a + a^2$

81. $x^3 - 3x^2y + 3xy^2 - y^3 + 8z^3 + 6x^2z - 12xyz + 6y^2z + 12xz^2 - 12yz^2$
Ans $x - y + 2z$

82. $a^6 - 12a^5 + 54a^4 - 112a^3 + 108a^2 - 48a + 8$ Ans $a^2 - 4a + 2$

83. $8m^6 - 36m^5 + 66m^4 - 63m^3 + 33m^2 - 9m + 1$ Ans $2m^2 - 3m + 1$

84. $x^3 + 6x^2y + 12xy^2 + 8y^3 - 3x^2z - 12xyz - 12y^2z + 3xz^2 + 6yz^2 - z^3$
Ans $x + 2y - z$

85. $8x^6 + 48x^5 + 60x^4 - 80x^3 - 90x^2 + 108x - 27$ Ans $2x^2 + 4x - 3$

86. $x^6 - 3ax^5 + 5a^3x^3 - 3a^5x - a^6$ Ans $x^2 - ax - a^2$

87. $x^{\frac{3}{2}} - 8y^{\frac{3}{2}}z^{\frac{3}{2}} - 6\sqrt{(xyz)}(\sqrt{x} - 2\sqrt{(yz)})$ Ans $x^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}}$

88. $\frac{1}{27}a^{-3} - \frac{1}{6}a^{-2}b^{-1} + \frac{1}{4}a^{-1}b^{-2} - \frac{1}{8}b^{-3}$ Ans $\frac{1}{3}a^{-1} - \frac{1}{2}b^{-1}$

89. $x^3 - 3x^2\sqrt{-1} - 3x + \sqrt{-1}$ Ans $x - \sqrt{-1}$

90. $1 - x$ to 4 terms Ans $1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 -$ &

91. $m^3 + 3m^2 - 5 + \dfrac{3}{m^2} - \dfrac{1}{m^3}$ Ans $m + 1 - \dfrac{1}{m}$

Extract the fourth roots of the following expressions

92. $\left(x^2 + \dfrac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$ Ans $x - \frac{1}{x}$

93. $81a^4-108a^3b+54a^2b^2-12ab^3+b^4$ Ans $3a-b$

94. $1-4x+6x^2-4x^3+x^4$ Ans $1-x$

95. $a^4-8a^3+24a^2-32a+16$ Ans $a-2$

96. $16a^4-96a^3b+216a^2b^2-216ab^3+81b^4$ Ans $2a-3b$

Extract the 8th roots of the following expressions.

97. $x^8-16x^7+112x^6-448x^5+1120x^4-1792x^3+1792x^2-1024x$
$+256$ Ans $x-2$

98. $a^8-8a^7b+28a^6b^2-26a^5b^3+70a^4b^4-56a^3b^5+28a^2b^6$
$-8ab^7+b^8$ Ans $a-b$

Exercise 18.

INDICES.

Find the values of

1. $36^{\frac{1}{2}}$; $4^{-\frac{1}{2}}$; $8^{\frac{1}{3}}$; $27^{-\frac{2}{3}}$; $100^{\frac{3}{2}}$; $81^{\frac{5}{4}}$

Ans 6; $\frac{1}{2}$; 2; $\frac{1}{9}$; 1000; 243

Simplify

2. $(x^2)^4$; $(x^{-2})^6$; $\{(2x)^{-1}\}^5$; $[\{(3x^2)^{-2}\}]^{-3}$

Ans x^8; x^{-12}; $\frac{1}{32x^5}$; 3^6x^{12}

3. $\sqrt{(a^{-6})}$; $\sqrt[5]{(a^{-10})}$; $\sqrt[m]{(a^{2m}b^m)}$; $\sqrt[2n]{(a^n b^2 c^{2n})}$

Ans a^{-3}; a^{-2}; a^2b; $a^{\frac{1}{2}}b^{\frac{1}{n}}c$

4. $[\{(-x^2)\}^3]^{-5}$; $\{(2x^{\frac{1}{2}})^{-\frac{1}{3}}\}^{-\frac{6}{5}}$; $[\{(x^m)^n\}^{2p}]^{\frac{1}{2mnp}}$

Ans x^{-30}; $(4x)^{\frac{1}{5}}$; x

5. $\{-(x^3)^{\frac{1}{2}}\}^{-\frac{1}{4}} \times \{-(-x^2)^{-3}\}^{\frac{1}{3}}$ Ans $-x^{-\frac{19}{8}}$

6. $\left\{\left(\frac{x^{-m}}{y^{-n}}\right)^{-1}\right\}^{\frac{1}{m}}$ Ans $_{y^m}$

7. $(x^{2m-n}y^{5m+1}z^{3p})^{\frac{1}{m}} \times (x^n y^{m-1} z^{m-3p})^{m}$ Ans x^2y^6z

8. $\left[\left\{\left(\frac{a^{-2m}}{b^{-2n}}\right)^{\frac{p}{m}}\right\}^{\frac{q}{n}}\right]^{\frac{1}{pq}}$ Ans $\frac{b^{\frac{2}{m}}}{a^{\frac{2}{n}}}$

9. $\left\{\frac{x^{m+n}}{x^n}\right\}^{2m} \div \left(\frac{x^n}{x^{n+m}}\right)^{2(n-m)}$ Ans x^{2mn}

10. $\frac{2^n \times (2^{n-1})^n}{2^{n+1} \times 2^{n-1}}$ Ans 2^{n^2-2n}

11. $\frac{a^x + b^y}{a^{-x} + b^{-y}} \times \frac{a^y - b^x}{a^{-y} - b^{-x}}$ Ans $-(ab)^{x+y}$

12. $\left\{\left(\frac{a-b}{a+b}\right)^{-m}\right\}^{-\frac{q}{m}} \times \left\{\left(\frac{(a+b)^2}{a^2-b^2}\right)^{-n}\right\}^{-\frac{q}{n}}$ Ans 1

13. $\left(\frac{x}{y}\right)^{-1} \times \frac{x^{-1}}{y^{-2}} \times \frac{y^{-3}}{x^{-4}}$ Ans x^2

14. $\left[\left\{\left(\frac{x^{-2a}}{y^{-2b}}\right)^{\frac{c}{a}}\right\}^{\frac{d}{b}}\right]^{\frac{1}{2}}$ Ans $\left(\frac{y^{\frac{1}{a}}}{x^{\frac{1}{b}}}\right)^{cd}$

15. $x^{m+n+p}(2x)^{m+n-p}(x^{-1})^{n-p+m}$ Ans $2^{m+n-p}x^{m+n+p}$

16. Prove that $\frac{3^{27^2}}{27^{9^2}} = 3^{2\times 3^5}$

17. $\left\{\left(\frac{x+\sqrt{(x^2-y)}}{2}\right)^{\frac{1}{2}} + \left(\frac{x-\sqrt{(x^2-y)}}{2}\right)^{\frac{1}{2}}\right\}^2$ Ans $x+\sqrt{y}$

18. Prove that $a^0=1$ and $a^{-m}=\frac{1}{{}^{m}}$

19. Find the value of $x^{-\frac{1}{2}} + (2x)^{-\frac{1}{3}} + (4x)^{-\frac{1}{4}} - (x^3)^{-\frac{1}{6}}$ when $x=4$ Ans 1

20. Simplify,

$$\left(x^{1+\frac{a}{b}}\right)^{\frac{b}{a+b}} + \left\{\frac{x^{2a}}{(x^{-1})^{-a}}\right\}^{\frac{1}{a}}$$ Ans $2x$

21. Simplify $(1+x^{m-n})^{-1} + (1+x^{n-m})^{-1}$ Ans 1

22. $\dfrac{(x^{b+c})^2(x^{c+d})^2(x^{b+d})^2}{(x^{b+c+d})^4}$ Ans 1

Exercise 19.

SURDS.

Simplify

1. $\sqrt{(18)} + 4\sqrt{8} + \sqrt{(32)}$ Ans $15\sqrt{2}$

2. $\sqrt{(12)} + 2\sqrt{(48)} - 2\sqrt{(27)}$ Ans $4\sqrt{3}$

3. $8\sqrt[3]{8} + 4\sqrt[3]{27} + 2$ Ans 6

4. $4\sqrt{5} + 3\sqrt{(2\frac{1}{4})} - 5\sqrt{(5\frac{4}{9})}$ Ans $4\sqrt{5} - 7\frac{1}{6}$

Retionalise the denominators of the following fractions.

5. $\dfrac{4+\sqrt{2}}{2+\sqrt{3}}$ Ans $8 + 2\sqrt{2} - 4\sqrt{3} - \sqrt{6}$

6. $\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ Ans $\dfrac{5+\sqrt{21}}{2}$

7. $\dfrac{2\sqrt{3}-\sqrt{5}}{3\sqrt{2}+\sqrt{3}}$ Ans $\dfrac{6\sqrt{6} - 3\sqrt{10} - 6 + \sqrt{15}}{15}$

8. $\dfrac{2\sqrt{3} - 3\sqrt{\ }}{3\sqrt{2} + 2\sqrt{\ }}$ Ans $9 + 2\sqrt{15} - 3\sqrt{6} - 3\sqrt{10}$

Extract the square roots of

9. $3+2\sqrt{2}$ Ans $1+\sqrt{2}$

10. $7-4\sqrt{3}$ Ans $2-\sqrt{3}$

11. $9+6\sqrt{2}$ Ans $\sqrt{6}+\sqrt{3}$

12. $5-2\sqrt{6}$ Ans $\sqrt{3}-\sqrt{2}$

13. $30+12\sqrt{6}$ Ans $2\sqrt{3}+3\sqrt{2}$

14. $95-30\sqrt{10}$ Ans $5\sqrt{2}-3\sqrt{5}$

Simplify

15. $\dfrac{1}{\sqrt{(7+2\sqrt{(10)})}}$ Ans $\dfrac{\sqrt{5}-\sqrt{2}}{3}$

16. $\dfrac{2}{\sqrt{(4-2\sqrt{3})}}$ Ans $\sqrt{3}+1$

17. $\dfrac{\sqrt{(28+16\sqrt{3})}}{2+\sqrt{3}}$ Ans 2

18. $\sqrt{(32+10\sqrt{7})}+\sqrt{(32-10\sqrt{7})}$ Ans 10

19. $\dfrac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$ Ans $\dfrac{8+3\sqrt{6}}{10}$

Extract the cubic roots of

20. $7+5\sqrt{2}$ Ans $1+\sqrt{2}$

21. $26+15\sqrt{3}$ $2+\sqrt{3}$

22. $\dfrac{1+y}{1+\sqrt{(1+y)}}+\dfrac{1-y}{1+\sqrt{(1-y)}}$ when $y=\dfrac{\sqrt{3}}{2}$ Ans $\frac{5}{3}\sqrt{3}-2$

23. $10+6\sqrt{3}$ Ans $1+\sqrt{3}$

24. $45-29\sqrt{2}$ Ans $3-\sqrt{2}$

Find the value of

Ex 9 to 13 and 20 to 25 are to be worked after learning the equations.

25. $\frac{1+y}{1+\sqrt{(1+y)}}+\frac{1-y}{1-\sqrt{(1-y)}}$ when $y=\frac{\sqrt{3}}{2}$ Ans 1

26. $\frac{1}{a-(a^2-x^2)^{\frac{1}{2}}}-\frac{1}{a+(a^2-x^2)^{\frac{1}{2}}}$ Ans $\frac{2(a^2-x^2)^{\frac{1}{2}}}{x^2}$

27. Shew that $\left\{\frac{-1+\sqrt{-3}}{2}\right\}^x+\left\{\frac{\sqrt{(-3)}}{-2}+\frac{1}{-2}\right\}^x$ is equal to 2 if x be a multiple of 3 and equal to -1 if x be any other integer

28. Find a factor which will rationalise $x^{\frac{1}{3}}-y^{\frac{1}{2}}$

Ans $x^{\frac{5}{3}}+x^{\frac{4}{3}}y^{\frac{1}{2}}+xy+x^{\frac{2}{3}}y^{\frac{3}{2}}+x^{\frac{1}{3}}y^2+y^{\frac{5}{2}}$

Simplify

29 $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{(2+\sqrt{3})}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{(2-\sqrt{3})}}$ Ans $\sqrt{2}$

30. $\frac{\sqrt{(a+x)}-\sqrt{(a-x)}}{\sqrt{(a+x)}+\sqrt{(a-x)}}-\frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}$ Ans $\frac{2\sqrt{(a^2-x^2)}}{x}$

31. $\frac{1}{a+\sqrt{(a^2-x^2)}}+\frac{1}{a-\sqrt{(a^2-x^2)}}$ Ans $\frac{2a}{x^2}$

32. $\frac{a-\sqrt{b}}{c+\sqrt{d}}+\frac{a+\sqrt{b}}{c-\sqrt{d}}$ Ans $\frac{2ac+2\sqrt{(bd)}}{c^2-d}$

33. $\frac{x+\sqrt{(x^2-b^2)}}{x-\sqrt{(x^2-b^2)}}-\frac{x-\sqrt{(x^2-b^2)}}{x+\sqrt{(x^2-b^2)}}$ Ans $\frac{4x\sqrt{(x^2-b^2)}}{b^2}$

34. $\sqrt{\frac{6x}{2x+1}}+\sqrt{\frac{2x+1}{6x}}$ Ans $\frac{8x+1}{\sqrt{(12x^2+6x)}}$

35 $\frac{\sqrt{(x^2+1)}+\sqrt{(x^2-1)}}{\sqrt{(x^2+1)}-\sqrt{(x^2-1)}}+\frac{\sqrt{(x^2+1)}-\sqrt{(x^2-1)}}{\sqrt{(x^4+1)}+\sqrt{(x^2-1)}}$ Ans $2x^2$

36. $\frac{1}{x+\sqrt{(x^2-1)}}+\frac{1}{x-\sqrt{(x^2-1)}}$ Ans $2x$

37. $\frac{\sqrt{(1-a)}+\frac{1}{(\sqrt{1+a})}}{1+\frac{1}{\sqrt{(1-a^2)}}}$ Ans $\sqrt{1(-a)}$

38. $\dfrac{2a^2}{(1-a^2)^{\frac{3}{2}}}+\dfrac{1}{(1-a^2)^{\frac{1}{2}}}-\dfrac{a^2+1}{(1-a^2)^{\frac{3}{2}}}$ Ans 0

39. $(a+\sqrt{-1})^2+(a-\sqrt{-1})^2$ Ans $2(a^2-1)$

40. $\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$ Ans $\sqrt{2}$

41. $\dfrac{x+y\sqrt{-1}}{x-y\sqrt{-1}}+\dfrac{x-y\sqrt{-1}}{x+y\sqrt{-1}}$ Ans $\dfrac{2(x^2-y^2)}{x^2+y^2}$

42. $\sqrt[3]{(x-1)}\cdot\sqrt[3]{(x+1)}\cdot\sqrt[3]{(x^2+x+1)}\cdot\sqrt[3]{(x^2-x+1)}\,(x^6-1)^{\frac{2}{3}}$
Ans x^6-1

43. $\dfrac{1}{(4x^3-3x)^2}-\left\{\dfrac{3\dfrac{\sqrt{(1-x^2)}}{x}-\dfrac{(1-x^2)^{\frac{3}{2}}}{x^3}}{1-3\left(\dfrac{1-x^2}{x^2}\right)}\right\}^2$ Ans 1
(Entrance 1880)

Exercise 20.

SIMPLE EQUATIONS OF ONE UNKNOWN QUANTITY.

Solve

1. $4x+8=3x+12$ $x=4$
2. $8x-6=5x+3$ $x=3$
3. $3x+4=4x-1$ $x=5$
4. $5x+3a=4x+7b$ $x=7b-3a$
5. $8x+19=6x+29$ $x=5$
6. $4x-8-(3x+4)=0$ $x=12$
7. $6x-18=81-3x$ $x=11$
8. $5x+24=34+3x$ $x=5$
9. $3x-5-(5x+17)=2$ $x=-12$
10. $8x-10=5x+20$ $x=10$
11. $5x-9=7x+15$ $x=-12$

12. $7x + 5 - (6x + 10) = 0$ $\quad x = 5$

13. $8x - 5 = 13 - 7x$ $\quad x = 1\frac{1}{5}$

14. $2ax - b = 2m - nx$ $\quad x = \dfrac{2m + b}{2a + n}$

15. $7x - 4 - (3x - 11) = 0$ $\quad x = -1\frac{3}{4}$

16. $13x - 20 = 15 + 20x$ $\quad x = -5$

17. $3{\cdot}75x + {\cdot}5 = 2{\cdot}25x + 8$ $\quad x = 5$

18. ${\cdot}5x - 7 = {\cdot}2x - 1{\cdot}6$ $\quad x = 18$

19. $x + {\cdot}\dot{3} \times (x - 2) = {\cdot}5(x - 1) + 4$ $\quad x = 5$

20. ${\cdot}5(x - 7\frac{1}{2}) = {\cdot}25(3x - 9) + {\cdot}\dot{3}(27 - 5x)$ $\quad x = 7\frac{7}{17}$

21. $x - 3\frac{1}{2} = 3\frac{1}{2} + 15x$ $\quad x = -\frac{1}{2}$

22. $ax + m = bx + n$ $\quad x = \dfrac{n - m}{a - b}$

23. $52 - 5(2x - 1) = 27$ $\quad x = 3$

24. $4(x - 3) - 7(x - 4) = 6 - x$ $\quad x = 5$

25. $20x - 50(3x - 4) = 200 - 20(4x - 5) - (3x - 4)50,$ $\quad x = 3$

26. $2x = bx + 4ab$ $\quad x = \dfrac{4ab}{2 - b}$

27. $2x - 5(1 + x) - 7 = 0$ $\quad x = -4$

28. $3(x - p) - 4(p - x) = 0$ $\quad x = p$

29. $4(5x - 3) - 64(3 - x) - 3(12x - 4) = 96$ $\quad x = 6$

30. $4x + 11 - 11(15 - x) = 11$ $\quad x = 11$

31. $x - a = (b - a)x$ $\quad x = \dfrac{a}{x - b + 1}$

32. $x - 3 - (3 - x)(x + 1) = x(x - 3) + 8$ $\quad x = 7$

33. $6(x - 1) - 3(x - 3) + 2(x - 5) = 48$ $\quad x = 11$

34. $ax^2 + a^3 = (ax + b^2)(a + x)$ $\qquad x = \frac{a(a^2 - b^2)}{a^2 + b^2}$

35. $12x + 7(x - 1) = 42(x - 4)$ $\qquad x = 7$

36. $15x - 3(3x - 2) = 45 - 5(2x - 5)$ $\qquad x = 4$

37. $22(2x + 7) - 14(9x - 8) = 77(x - 11)$ $\qquad x = 7$

38. $15(3x + 3) + 16(7x - 4) - 12(7x + 1) = 480$ $\qquad x = 7$

39. $5(5x - 1) - (\cdot 7x - \cdot 2) = 6\cdot 6 - \cdot 5x$ $\qquad x = \frac{57}{241}$

40. $a(x - a) + b(x - b) + c(x - c) - 2ab - 2bc - 2ac = 0$ $\qquad x = a + b + c$

41. $a^2x - a^3 - abx = b^3 - b^2x$ $\qquad x = a + b$

42. $a^2x + b^2c = b^2x + a^2c$ $\qquad x = c$

43. $2(x - 4) - 3(x - 12) = 33 - 2x$ $\qquad x = 5$

44. $3(3x - 4) + 7(5x + 3) = 21(43 - 5x)$ $\qquad x = 6$

45. $9(x - 1) + 6(2x + 7) - 2(x + 2) = 162$ $\qquad x = 7$

46. $15(3x + 5) - 15(2x + 7) + 1050 - 63x = 0$ $\qquad x = 10$

47. $(x - a)(x - b) - (c - x)(d - x) = 0$ $\qquad x = \frac{cd - ab}{c + d - a - b}$

48. $(2a + b)x = 4a^2 - 3a(x - b)$ $\qquad x = \frac{4a^2 + 3ab}{5a + b}$

49. $(x - a)^2 = 5a^2 + (x + a)^2$ $\qquad x = -\frac{5a}{4}$

50. ~~$(2x + 3)^2 - 32 = (4x - 5)^2$~~ $\qquad$ ~~$x = 3$~~

51. $(2b + 2c - x)^2 + (2b - 2c + x)^2 - (2b + 2d + x)^2 = (2b - 2d - x)^2$ $\qquad x = c - d$

52. $(2x^2 - 3x + 1)(2x + 1) = 4x^3 - 4x^2 - 5x + 10$ $\qquad x = 2\frac{1}{4}$

53. $22 - x - 2(x - 1)(x + 2) = (3x - 2)(2x - 1)$ $\qquad x = 2$

54. $(x + 2)(x + 4) + 5 = (x + 3)^2 + x$ $\qquad x = 4$

55. $(x+1)(x+2)(x+3)=(x-1)(x-2)(x-3)+3(4x-2)(x+1)$ $x=3$

56. $(x-8)(x-7)(x-5)(x-6)=5(x-6)(x-4)$ $x=10$

57. $(8x-3)^2(x-1)=(4x-1)^2(4x-5)$ $x=\frac{4}{13}$

58. $\cdot5x+\cdot6x+\cdot8=\cdot75x+\cdot25$ $x=-\frac{11}{7}$

59. $x(x+a)+x(x-b)=2(x+a)(x-b)$ $x=-\frac{2ab}{b-a}$

60. $(x-a)(x+b)(x+2a-2b)=(x+2a)(x-2b)(x-a+b)$ $x=2(a-b)$

61. $(x+a)(x+b)=(x+a+b)^2$ $x=-\frac{a^2+ab+b^2}{a+b}$

62. $(a+b)(x-c)+(b+c)(x-a)=(c-a)(x+b)$ $x=c$

63. $(x-1)^3+(x-2)^3+(x-3)^3=3(x-1)(x-2)(x-3)$ $x=2$

64. $(x+a+b+c)(a+b-c+x)=x^2+a^2+b^2+c^2$ $x=\frac{c^2-ab}{a+b}$

Equations involving fractions

65. $x+\frac{x}{3}+\frac{x}{5}=23$ $x=15$

66. $\frac{x}{5}+\frac{x}{6}+\frac{x}{3}=21$ $x=30$

67. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=x+17$ $x=60$

68. $\frac{x}{2}+\frac{2x}{3}-\frac{3x}{4}=\frac{1}{2}$ $x=1\frac{1}{5}$

69. $\frac{3x}{17}+3=\frac{2x}{34}+5$ $x=17$

70. $2x+\frac{1}{2}x+\frac{7}{8}x-\frac{2}{7}x=3x+5$ $x=56$

71. $\frac{5x}{2}+2+\frac{3x}{4}=4x-1$ $x=4$

72. $13\frac{3}{4}-\frac{1}{2}x=2x-8\frac{3}{4}$ $x=9$

73. $\frac{4x+6}{11}=\frac{8x+8}{20}$ $x=4$

74. $\frac{56-x}{2} = 4x + 1$ $x = 6$

75. $\frac{3x}{17} + \frac{11x}{17} - 1 = \frac{4x}{17} + \frac{7}{17} + \frac{7x}{17}$ $x = 8$

76. $\frac{x-1}{2} + \frac{x-2}{3} + \frac{x-3}{4} = \frac{5x-1}{6}$ $x = 7$

77. $\frac{2x-6}{5} - \frac{x-4}{9} - \frac{3x}{13} = 0$ $x = 13$

78. $\frac{x}{3} - \frac{1}{3} - \frac{x}{4} + \frac{1}{4} = \frac{x}{5} - \frac{1}{5} - \frac{x}{6} + \frac{1}{6}$ $x = 1$

79. $\frac{5x-7}{2} - \frac{2x+7}{3} = 3x - 14$ $x = 7$

80. $\frac{7x+9}{4} = 7 + x - \frac{2x-1}{9}$ $x = 5$

81. $\frac{2x-5}{3} - \frac{5x-3}{4} + 2\frac{2}{3} = 0$ $x = 3$

82. $2x - \left(\frac{19-2x}{2} + \frac{2x-11}{3}\right) = 0$ $x = 2\frac{1}{2}$

83 $\frac{2x-5}{6} + \frac{6x+3}{4} = 5x - 17\frac{1}{2}$ $x = 5\frac{1}{2}$

84. $\frac{1}{7}(3x-4) + \frac{1}{3}(5x+3) = 43 - 5x$ $x = 6$

85. $\frac{1}{3}(2x+4) + \frac{3}{4}x = x - 3$ $x = 4$

86. $\frac{1}{2}(27-2x) = \frac{9}{2} - \frac{1}{10}(7x-54)$ $x = 12$

87. $5x - [6x - 3\{14 - 3x - (6-2x)\}] = 4x$ $x = 3$

88. $\frac{1}{6}(8-x) + x - 1\frac{2}{3} = \frac{1}{2}(x+6) - \frac{x}{3}$ $x = 5$

89. $\frac{2-x}{3} + \frac{3-x}{4} + \frac{4-x}{5} + \frac{5-x}{6} + \frac{3}{4} = 0$ $x = 4$

90. $\frac{12}{x} - \left(1 - \frac{1}{12x}\right) = \frac{5}{24}$ $x = 10$

91 $17 - \frac{3x-5}{7} = 2x - \left(\frac{125}{77} + \frac{x-7}{11}\right)$ $x = 8$

92. $\frac{x+10}{3}-\frac{2}{5}\left(3x-4\right)+\frac{(3x-2)(2x-3)}{6}=x^2-\frac{2}{15}$ $x=2$

93. $\frac{1}{2}(\frac{2}{3}x+4)-\frac{7\frac{1}{2}-x}{3}=\frac{x}{2}(\frac{6}{x}-1)$ $x=3$

94. $\frac{x-3\frac{2}{3}}{2}-\frac{5-4x}{7}=x+12-\frac{3x-\left(\frac{9-2x}{3}-2\right)}{6}$ $x=21\frac{24}{43}$

95. $\frac{2x+a}{b}-\frac{x-b}{a}=\frac{3ax+(a-b)^2}{ab}$ $x=\frac{2ab}{a+b}$

96. $\frac{x-\frac{2x-36}{9}}{8}-\frac{x-18}{6}=x+9-\frac{5x-\frac{2(x-10)}{13}}{4}$ $x=36$

97. $\frac{x-1}{4}-\frac{x-5}{32}+\frac{15-2x}{40}+\frac{7}{8}=\frac{9-x}{2}$, $x=5$

98. $\frac{x+1}{7}+x(x-2)=(x-1)^2$ $x=6$

99. $\frac{x-4}{3}+(x-1)(x-2)=x^2-2x-4$ $x=7$

100. $\frac{3x^2-2x-8}{5}=\frac{(7x-2)(3x-6)}{35}$ $x=2$

101. $\frac{\frac{7x}{5}-2}{4}-\frac{5x-\frac{1}{2}}{3}=\frac{\frac{x}{4}-2}{5}-3\frac{1}{3}$; $x=3$

102. $\frac{4x-17}{9}-\frac{3\frac{2}{3}-22x}{33}=x-\frac{6}{x}\left(1-\frac{x^2}{54}\right)$ $x=3$

103. $\frac{x-4\frac{2}{3}}{3}-\frac{2x-3\frac{2}{3}}{4}=\frac{3}{2}\left\{x-\frac{x-1\frac{1}{2}}{2}\right\}+\frac{4x}{3}\left\{x-3-\frac{(x-1)(x-2)}{x}\right\}$ $x=\frac{65}{66}$

104. $\frac{3x^2-2x+1}{5}=\frac{(7x-2)(3x-6)}{35}+\frac{18}{20}$ $x=1\frac{5}{68}$

105. $\frac{3x-1}{2}-\frac{2x-5}{3}+\frac{x-3}{4}-\frac{x}{6}=x+1$ $x=-7$

106. $\frac{\frac{1}{2}x-3}{5}+\frac{\frac{3}{4}x-10}{2}+\frac{4-x}{4}=\frac{10-x}{6}$ $x=16$

107. $\frac{5}{6}(x-\frac{1}{3})+\frac{7}{6}(\frac{x}{5}-\frac{1}{7})=4\frac{8}{9}$ $x=5$

108. $x+\frac{5x-8}{3}=6-\frac{3x-8}{5}$ $x=3\frac{1}{7}$

109. $\frac{x}{7}-\frac{3x}{2}+\frac{71}{7}=\frac{3x+1}{2}+1\frac{1}{14}$ $x=3$

110. $\frac{x+10}{3}-\frac{2}{5}(3x-4)+\frac{(3x-2)(2x-3)}{6}=x^2-\frac{2}{15}$ $x=2$

111. $\frac{x+5}{6}+\frac{1}{9}(\frac{x}{2}+\frac{2}{5})-\frac{2}{3}(3+2x)=\frac{4x-14}{3}+\frac{x+10}{10}$ $x=1$

112. $\frac{42}{x-2}=\frac{70}{2x-6}$ $x=8$

113. $\frac{20}{x}+\frac{1}{3x}=\frac{61}{12}$ $x=4$

114. $\frac{10x+17}{18}-\frac{12x+2}{11x-8}=\frac{5x-4}{9}$ $x=4$

115. $\frac{4}{x-8}=\frac{10}{x-5}$ $x=10$

116. $\frac{2x-6}{3x-8}=\frac{6x-15}{9x-21}$ $x=2$

117. $\frac{x-2}{x+2}+\frac{1}{x}=\frac{3}{3}$ $x=\frac{2}{3}$

118. $\frac{3x-1}{2x-1}-\frac{4x-2}{3x-2}=\frac{1}{6}$ $x=2$

119. $\frac{1}{x-1}-\frac{2}{x+7}=\frac{1}{7(x-1)}$ $x=7$

120. $\frac{2}{x-4}+\frac{3}{x-6}=\frac{5}{x-2}$ $x=4\frac{1}{2}$

121. $\frac{x+1}{7}+x(x-2)=(x-1)^2$ $x=6$

122. $\frac{3x-1}{2x-1}=\frac{1}{6}+\frac{4x-2}{3x-2}$ $x=2$

123. $x+1-\frac{x^2+3}{x+2}=2$ $x=5$

124. $\frac{x-1}{x-2}=\frac{7x-21}{7x-26}$ $x=8$

✓125. $\frac{a}{b+cx}=\frac{d}{e+fx}$ $x=\frac{bd-ae}{af-cd}$

✓126. $\frac{1}{x+3}-\frac{1}{x+5}=\frac{1}{6}\frac{1}{x+3}$ $x=7$

✓127. $\frac{6x+8}{2x+1}=1+\frac{2x+38}{x+12}$ $x=2$

✓128. $3x^2-2x-8=\frac{1}{7}(7x-2)(3x-6)$ $x=2$

✓129. $\frac{2}{2x-3}+\frac{1}{x-2}=\frac{6}{3x+2}$ $x=\frac{50}{29}$

✓130. $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$ $x=4\frac{1}{2}$

131. $\left(\frac{8x-3}{4x-1}\right)^2=\frac{4x-5}{x-1}$ $x=\frac{4}{13}$

132. $\frac{x-4}{x-5}-\frac{x-5}{x-6}=\frac{x-7}{x-8}-\frac{x-8}{x-9}$ $x=7$

133. $\frac{2x+3}{3x+9}=\frac{2x-8}{3x-13}$ $x=3$

134. $\frac{9x+20}{36}=\frac{4x-12}{5x-4}+\frac{x}{4}$ $x=8$

135. $\frac{6x+7}{3x+1}=\frac{2x+19}{x+7}$ $x=3$

136. $\frac{10x+17}{18}-\frac{12x+2}{13x-16}=\frac{5x-4}{9}$ $x=4$

137. $\frac{x-7}{x+7}-\frac{2x-15}{2x-6}=-\frac{1}{2(x+7)}$ $x=8$

138. $\frac{1}{x-1}-\frac{1}{2(x+1)}-\frac{x+3}{2(x^2+1)}=\frac{6}{x^4-1}$ $x=3$

139. $\frac{x^2+1}{4x^2-1}+\frac{1}{4}=\frac{x}{1+2x}$ $x=-\frac{3}{4}$

140. $\frac{2x+1}{2x-1}-\frac{1}{4x^2-1}=\frac{2x-1}{2x+1}$ $x=1$

141. $\frac{x^3+1}{x-1}-(x-1)=\frac{10-x^2}{x+1}$ $x=2$

142. $x^2+2x=\frac{20}{x^2-2x+2}-2$ $x=2$

143. $\frac{x+\frac{1}{x}}{x^2+1}-\frac{1}{x+1}=\frac{3}{(x+1)^2}$ $x=\frac{1}{2}$

144. $\frac{3-2x}{1-2x}-\frac{2x-5}{2x-7}=1-\frac{4x^2-1}{7-16x+4x^2}$ $x=-1$

145. $\frac{a}{x+a}+\frac{b}{x+b}=\frac{a+b}{x+c}$ $x=\frac{ab(a+b-2c)}{(a+b)c-a^2-b^2}$

146. $\frac{1}{ab-ax}+\frac{1}{bc-bx}=\frac{1}{ac-ax}$ $x=\frac{b}{a}(a-b+c)$

147. $\frac{1}{5}\frac{(x+1)(x-3)}{(x+2)(x-4)}+\frac{1}{9}\frac{(x+3)(x-5)}{(x+4)(x-6)}-\frac{2}{13}\frac{(x+5)(x-7)}{(x+6)(x-8)}=\frac{92}{585}$

$x=1+\sqrt{19}$

148. $\frac{a}{x}+\frac{x}{c}+\frac{a(x-a)}{x(x+a)}-\frac{x(x+a)}{a(x-a)}=\frac{ax}{a^2-x^2}-2$ $x=4a$

149. $2x^3+4x^2+8x+16=\frac{1440}{3x-6}$ $x=4$

150. $\frac{2x}{x^4-x^2+1}-\frac{1}{x^2-x+1}+\frac{1}{x^2+x+1}=\frac{4x^3}{x^8+17}$ $x=2$

151. $\frac{x^2-2x+3}{x^3+1}+\frac{x-2}{x^2-x+1}-\frac{1}{x+1}=\frac{x(2x-5)}{x^3+1}$ $x=3$

152. $\frac{1}{(a-b)(a-c)(x-a)}+\frac{1}{(b-a)(b-c)(x-b)}+\frac{1}{(c-a)(c-b)(x-c)}$

$=\frac{1}{(x-a)(x-b)(2x-d)}$ $x=d-c$

153. $\frac{a}{x-a}-\frac{b}{x-b}=\frac{a-b}{x-c}$ $x=\frac{ab}{a+b-c}$

154. $\frac{1}{x-a}-\frac{1}{x-a+c}=\frac{1}{x-b-c}-\frac{1}{x-b}$ $x=\frac{a+b}{2}$

155 $\frac{x-a}{x-a-1}-\frac{x-a-1}{x-a-2}=\frac{x-b}{x-b-1}-\frac{x-b-1}{x-b-2}$ $x=\frac{1}{2}(a+b+3)$

156 $\frac{x^4-8x^3+21x^2-1}{x^4+x^2+1}=1-\frac{4(2x-3)}{x^2+x+1}$ Ans $x=\frac{7}{10}$

157. $\frac{2}{x^2-9x-10}+\frac{3}{x^2-7x-30}-\frac{5}{x^2+4x+3}$

$=\frac{118}{(x-10)(x+1)(3x+4)}$; $x=2$

158. $\frac{x^2+3x+2}{x+1}+\frac{x^2-x-6}{x+2}=\frac{5x}{2}$ $x=2$

158. $\frac{1}{x}+\frac{1}{2x}-\frac{1}{3x}=\frac{7}{3}$; $x=\frac{1}{2}$

160. $\left(\frac{x-2a}{x+2b}\right)^2=\frac{x-2a-2b}{x+2a+2b}$ $x=\frac{2(a^2+b^2)}{a-b}$

161. $\frac{x+a}{x+b}=\frac{(2x+a)^2}{(2x+b)^2}$ $x=-\frac{(a+b)}{ab}$

162. $\frac{(2x+3)x}{2x+1}+\frac{1}{3x}=x+1$ $x=1$

163. $\dfrac{66x+1}{1\cdot5x+1}+\dfrac{4x+5}{\cdot5x-1}=52$ $\qquad x=28$

164. $\dfrac{6-5x}{15}-\dfrac{7-2x^2}{14(x-1)}=\dfrac{3x+1}{21}-\dfrac{2x-2\frac{1}{5}}{6}+\dfrac{1}{105}$ $\qquad x=4$

165. $\dfrac{x}{2}-\dfrac{\frac{1}{3}(2x-3)-\frac{1}{4}(3x-1)}{\frac{1}{2}(x-1)}=\frac{3}{2}\left(\dfrac{x^2+2}{3x-2}\right)$ $\qquad x=4\frac{1}{3}$

166. $\dfrac{3+2x}{1+2x}-\dfrac{5+2x}{7+2x}=1-\dfrac{4x^2-2}{7+16x+4x^2}$ $\qquad x=\frac{7}{8}$

167. $\dfrac{x^2-x+1}{x-1}+\dfrac{x^2+x+1}{x+1}=2x$ $\qquad x=0$

168. $\dfrac{x^2+2x+2}{x+1}+\dfrac{x^2+8x+20}{x+4}=\dfrac{x^2+4x+6}{x+2}+\dfrac{x^2+6x+12}{x+3}$ $\qquad x=-2\frac{1}{2}$

169. $\dfrac{x^2-7x-30}{2(x-10)}+\dfrac{x^2+x-12}{3x-9}+\dfrac{x^2+x-20}{4x-16}=16$ $\qquad x=11$

170. $\dfrac{1}{(x+a)^2-b^2}+\dfrac{1}{(x+b)^2-a^2}=\dfrac{1}{x^2-(a+b)^2}+\dfrac{1}{x^2-(a-b)^2}$,

$x=\dfrac{a^2+b^2}{a+b}$

171. $31\left\{\dfrac{24-5x}{x+1}+\dfrac{5-6x}{x+4}\right\}=29\left\{\dfrac{17-7x}{x+2}+\dfrac{8x+55}{x+3}\right\}-370$ $\qquad x=-2\frac{1}{2}$

172. $x\cdot\dfrac{x+3a}{c+3x}=\sqrt{(ac)}\cdot\dfrac{a+3x}{x+3c}$ $\qquad x=\sqrt{(ac)}$

173. $\dfrac{1}{x-2}-\dfrac{1}{x-4}=\dfrac{1}{x-6}-\dfrac{1}{x-8}$ $\qquad x=5$

174. $\dfrac{(x+1)^2+x}{(x+1)^2-x}=\dfrac{9+x}{9-x}$ $\qquad x=2$

175. $\left(\dfrac{x+2a}{x+2b}\right)^3=\dfrac{x-4a-2b}{x+2a+4b}$ $\qquad x=a-b$

176. $\dfrac{(5x^4+10x^2+1)(5a^4+10a^2+1)}{(x^4+10x^2+5)(a^4+10a^2+5)}=ax$ $\qquad x=\dfrac{1}{a}$

177. $\frac{x+a}{x+b}=\left(\frac{2x+a+c}{2x+b+c}\right)^2$ $x=\frac{c^2-ab}{a+b-2c}$

178. $\frac{x^3+12x}{6x^2+8}=\frac{126}{124}$ $x=3$

179. $\frac{ax^2+bx+c}{px^2+qx+r}=\frac{ax+b}{px+q}$ $x=\frac{br-cq}{cp-ar}$

180. $\frac{3abc}{a+b}+\frac{a^2b^2}{(a+b)^3}+\frac{(2a+b)b^2x}{a(a+b)^2}=3cx+\frac{bx}{a}$ $x=\frac{ab}{a+b}$

181. $\frac{m(x+a)}{x+b}+\frac{n(x+b)}{x+a}=m+n$ $x=\frac{bn-am}{m-n}$

182. $\frac{4x^3+4x^2+8x+1}{2x^2+2x+3}=\frac{2x^2+2x+1}{x+1}$ $x=2$

183. $\frac{x^3+ax^2-bx+c}{x^3-ax^2+bx+c}=\frac{x^2+ax-b}{x^2-ax+b}$ $x=\frac{b}{a}$

184. $\frac{x^4-1}{x^2-1}\times\frac{x+1}{x^4+2x^3+2x^2+2x+1}=\frac{1}{6}$ $x=5$

185. $\frac{x}{x-2}+\frac{x-9}{x-7}=\frac{x+1}{x-1}+\frac{x-8}{x-6}$ Ans 4

Equations involving surds

186. $\sqrt{(x-9)}=2$ $x=13$

187. $\sqrt{(x+9)}=2\sqrt{x}-3$ $x=16$

188. $\sqrt{(4x)}+\sqrt{(4x-7)}=7$ $x=4$

189. $\sqrt{(x+12)}=2+\sqrt{x}$ $x=4$

190 $\sqrt{(x-16)}+\sqrt{x}=8$ $x=25$

191. $\sqrt{(x+14)}+\sqrt{(x-14)}=14$ $x=50$

192. $(3x+4)^{\frac{1}{4}}=2$ $x=4$

193. $x+\sqrt{(2ax+x^2)}=b-a$ $x=\frac{(b-a)^2}{2b}$

194. $\sqrt{(x+11)}=10-\sqrt{(x-9)}$ $x=25$

195. $\sqrt{(x-5)}=6-\sqrt{(x+7)}$ $x=9$

196 $\sqrt{(4m+x)}=2\sqrt{(n+x)}-\sqrt{x}$ $x=\dfrac{(n-m)^2}{2m-n}$

197. $(\sqrt{x}+28)(\sqrt{x}+6)=(\sqrt{x}+4)(\sqrt{x}+38)$ $x=4$

198. $\sqrt{(5+x)}+\sqrt{x}=\dfrac{15}{\sqrt{(5+x)}}$ $x=4$

199. $\sqrt{(x+4mn)}=2m-\sqrt{x}$ $x=(m-n)^2$

200. $\dfrac{3x-1}{\sqrt{(3x)}+1}-1=\dfrac{\sqrt{(3x)}-1}{2}$ $x=3$

201. $\sqrt{x}-\sqrt{2}=\sqrt{(x-2)}$ $x=2$

202 $\sqrt{(x-m)}+\sqrt{(x-n)}=\sqrt{(m-n)}$ $x=m$

203. $(12+x)^{\frac{1}{2}}=2+x^{\frac{1}{2}}$ $x=4$

204. $\sqrt{(x+m)}+\sqrt{(x+n)}=p$ $x=\left(\dfrac{p^2+n-m}{2p}\right)^2$

205, $\sqrt{\dfrac{b}{a+x}}=\sqrt[4]{\left(\dfrac{4bc}{a^2-x^2}\right)}-\sqrt{\left(\dfrac{c}{a-x}\right)}$ $x=\dfrac{ab+ac}{b-c}$

206. $\sqrt{(x^2+40)}=x+4$ $x=3$

207. $(5x+10)^{\frac{1}{2}}=(5x)^{\frac{1}{2}}+2$ $x=\frac{9}{20}$

208. $\sqrt{(3+x)}+\sqrt{x}=\dfrac{6}{\sqrt{(3+x)}}$ $x=1$

209. $(3x^{\frac{1}{2}}+5)^2=108+9x$ $x=(\frac{83}{30})^2$

210. $\sqrt{\dfrac{x+1}{x-1}}+\sqrt{\dfrac{x-1}{x+1}}=\sqrt{5}$ $x=\sqrt{5}$

211. $\sqrt{p^2+px}+\sqrt{p^2-px}=p$ $x=\dfrac{p}{2}\sqrt{3}$

212. $\sqrt{x-5}-7-\sqrt{x-12}=0$ $x=21$

213. $13 + \sqrt{4x+1} = 16$ $\qquad x = 2$

214. $\sqrt{x+16} - \sqrt{x} - 2 = 0$ $\qquad x = 9$

215. $\sqrt{4x+9} - 2\sqrt{x} = 1$ $\qquad x = 4$

216. $x = 7 - \sqrt{(x^2 - 7)}$ $\qquad x = 4$

217. $x + \sqrt{(8x + x^2)} = 4$ $\qquad x = 1$

218. $\sqrt[m]{(4 + x)} = \sqrt[2m]{(x^2 + 32x + 4)}$ $\qquad x = \frac{1}{2}$

219. $\frac{5x - 9}{\sqrt{5x} + 3} - 1 = \frac{1}{2}(\sqrt{5x} - 3)$ $\qquad x = 5$

220. $\sqrt{x} = \sqrt{(x + 1)} + \frac{1}{\sqrt{x}}$ $\qquad x = \frac{1}{3}$

221. $\sqrt{x} - \sqrt{\frac{2}{x}} = \sqrt{(2 + x)}$ $\qquad x = \sqrt{2} - 1$

222. $\sqrt{(x - 2)^2} + 4\sqrt{x} = 13$ $\qquad x = 9$

223. $\frac{\sqrt{5}\cdot\sqrt{(x + 2)} - \sqrt{5x}}{2} = 1$ $\qquad x = \frac{9}{20}$

224. $\sqrt{(x^2 + m^2)} + x = b$ $\qquad x = \frac{b^2 - m^2}{2b}$

225. $(5 + x)^{\frac{1}{2}} + (5 - x)^{\frac{1}{2}} = 2\sqrt{x}$ $\qquad x = 4$

226. $\sqrt{x} - 4 = \frac{259 - 10x}{\sqrt{x} + 4}$ $\qquad x = 25$

227. $\sqrt{(x - 9)} = \frac{36}{\sqrt{(x - 9)}} - \sqrt{x}$ $\qquad x = 25$

228. $\sqrt{x} + \sqrt{(x + m)} = \frac{2m}{\sqrt{(x + m)}}$ $\qquad x = \frac{m}{3}$

229. $\sqrt{5 + \sqrt{(x - 4)}} = 3$ $\qquad x = 20$

230. $\frac{\sqrt{x} - 2}{\sqrt{x} + 3} = \frac{\sqrt{x} + 1}{\sqrt{x} + 21}$ $\qquad x = 9$

31. $\sqrt{(4a + x)} + \sqrt{(a + x)} = 2\sqrt{(x - 2a)}$ $\qquad x = \frac{17a}{8}$

232. $\sqrt{x} + \sqrt{x - \sqrt{(1-x)}} = 1$ $\qquad x = \frac{16}{25}$

233. $\sqrt[3]{x} - 6 = \frac{2}{3}(1 - \sqrt[3]{x})$ $\qquad x = 64$

234. $(a+x)^{\frac{1}{m}} = (x^2 + 5ax + b^2)^{\frac{1}{2m}}$ $\qquad x = \dfrac{a^2 - b^2}{3a}$

235. $\sqrt{p - \sqrt{(px + x^2)}} = \sqrt{p} - \sqrt{x}$ $\qquad x = \frac{9}{16}p$

236. $\sqrt{(5x-1)} - 1 = \sqrt{(5x-2)}$ $\qquad x = \frac{2}{5}$

237. $\sqrt{(x+9)} - \sqrt{x} = 1$ $\qquad x = 16$

238. $\sqrt{(x+11)} - \sqrt{x} = 1$ $\qquad x = 25$

239. $\sqrt{(x+13)} = 13 - \sqrt{x}$ $\qquad x = 36$

240. $\sqrt{x} + 6 = \dfrac{(\sqrt{x} + 38)(\sqrt{x} + 4)}{\sqrt{x} + 28}$ $\qquad x = 4$

241. $\sqrt{(n+x)} + \sqrt{(n-x)} = m\sqrt[4]{(n^2 - x^2)}$ $\qquad x = \dfrac{mn}{m^2 - 2}\sqrt{(m^2 - 4)}$

242. $\sqrt{4 + \sqrt{(x^4 - x^2)}} = x - 2$ $\qquad x = 2\frac{1}{8}$

243. $\sqrt[3]{\left(\dfrac{ax + b}{mx + n}\right)} = 1$ $\qquad x = \dfrac{n - b}{a - m}$

244. $\sqrt{(3+x)} - \sqrt{2} = \sqrt{x}$ $\qquad x = \frac{1}{8}$

245. $\sqrt{(x^2 + 9)} = 4 + \sqrt{34} - \sqrt{(x^2 - 9)}$ $\qquad x = 5$

246. $\sqrt{(x+m)} = \dfrac{n}{\sqrt{(x+m)}} - \sqrt{(x-m)}$ $\qquad x = \dfrac{2m^2 - 2mn + n^2}{2(n-m)}$

247. $\sqrt{(1 + x + x^2)} = 2 - \sqrt{(1 - x + x^2)}$ $\qquad x = 0$

248. $\sqrt{1 - x + \sqrt{(1+x)} - \sqrt{(1-x)}} = 1$ $\qquad x = \frac{24}{25}$

249. $\sqrt{(x+m)} = 2n - \sqrt{(x-m)}$ $\qquad x = \dfrac{m^2 + 4n^4}{4n^2}$

250. $\sqrt{(x^2 + 4x)} - 1 = x$ $\qquad x = \frac{1}{2}$

251. $\dfrac{5x - 1}{\sqrt{(5x)} + 1} = \frac{1}{2}\{3 + \sqrt{(5x)}\}$ $\qquad x = 5$

252. $\sqrt{(1 + \sqrt{x})} - \sqrt{(1 - \sqrt{x})} = m\sqrt{\left(\dfrac{x}{1 + \sqrt{x}}\right)}$ $\qquad x = \dfrac{4}{(1 - m)^2}$

253. $\sqrt{(1+x+x^2)}=3-(1-x+x^2)^{\frac{1}{2}}$ $\qquad x=\frac{3}{2}\sqrt{\frac{5}{8}}$

254. $\frac{1}{m}\sqrt{(m+x)}+\frac{1}{x}\sqrt{(m+x)}=\frac{1}{n}\sqrt{x}$ $\qquad x=\frac{mn^{\frac{2}{3}}}{m^{\frac{2}{3}}-n^{\frac{2}{3}}}$

255. $\sqrt{(m^2+x^2)}-\sqrt{(m^2-x^2)}=p$ $\qquad x=p\sqrt[4]{\left(\frac{4m^2-p^2}{4p^2}\right)}$

256. $\frac{ax-b^2}{\sqrt{(ax)}+b}=\frac{\sqrt{(ax)}-b}{n}-p$ $\qquad x=\frac{1}{a}\left\{\frac{(n-1)b-np}{n-1}\right\}^2$

257. $\sqrt{(1+x+x^2)}+\sqrt{(1-x+x^2)}=px$ $\qquad x=\frac{2}{p}\sqrt{\left(\frac{p^2-1}{p^2-4}\right)}$

258. $\frac{px-1}{\sqrt{(px)}+1}-4=\frac{\sqrt{(px)}-1}{2}$ $\qquad x=\frac{81}{p}$

259. $\sqrt{(2m+x)^2+n^2}=2m-\sqrt{(2m-x)^2+n^2}$ $\qquad x=\left(m^2+\frac{n^2}{3}\right)^{\frac{1}{2}}$

260. $\frac{\sqrt{(4+x)}+\sqrt{(4-x)}}{\sqrt{(4+x)}-\sqrt{(4-x)}}=m$ $\qquad x=\frac{8m}{1+m^2}$

261. $\left(\frac{m^2}{x}+n\right)^{\frac{1}{2}}=p^{\frac{1}{2}}+\left(\frac{m^2}{x}-n\right)^{\frac{1}{2}}$ $\qquad x=\frac{4m^2p}{4n^2+p^2}$

262. $\frac{y-ay}{\sqrt{y}}=\frac{\sqrt{(y)}}{y}$ $\qquad y=\frac{1}{1-a}$

263. $\frac{\sqrt{ax}-b}{\sqrt{ax}+b}=\frac{3\sqrt{ax}-5b}{3\sqrt{ax}+8b}$ $\qquad x=\frac{9b^2}{49a}$

364. $\frac{1-\sqrt{(1-x)}}{1+\sqrt{(1-x)}}=p$ $\qquad x=\frac{4p}{(p+1)^2}$

265. $\frac{\sqrt{(12x+1)}+\sqrt{(12x)}}{\sqrt{(12x+1)}-\sqrt{(12x)}}=15$ $\qquad x=\frac{289}{864}$

266. $\frac{\sqrt{m}+\sqrt{(m-x)}}{\sqrt{m}-\sqrt{(m-n)}}=\frac{1}{m}$ $\qquad x=\left(\frac{2m}{m+1}\right)^2$

267. $\dfrac{\left(\dfrac{m+x}{m-x}\right)^{\frac{1}{2}}+n}{\left(\dfrac{m+x}{m-x}\right)^{\frac{1}{2}}-n}=\dfrac{6}{5}$ $\qquad x=\dfrac{m(121n^2-1)}{121n^2+1}$

268. $(p+\sqrt{x})^{\frac{1}{3}}+(p-\sqrt{x})^{\frac{1}{3}}=q^{\frac{1}{3}}$ $\qquad$ ~~$x=\dfrac{8p^3+3p^2q+6pq^2-q^3}{27q}$~~ 13

269. $\dfrac{(m+x^{\frac{1}{2}})^{\frac{1}{2}}}{x^{\frac{1}{3}}}+\dfrac{(m-x^{\frac{1}{2}})^{\frac{1}{2}}}{x^{\frac{1}{3}}}=x^{\frac{1}{6}}$ $\qquad x=4(m-1)$

270. $(p+x)^{\frac{1}{2}}-(2p+x)^{\frac{1}{2}}=\left(\dfrac{p}{p+x}\right)^{\frac{1}{2}}$ $\qquad x=p^{\frac{1}{2}}\left(\dfrac{1-2p^{\frac{1}{2}}-p}{2+p^{\frac{1}{2}}}\right)$

271. $\sqrt{\left(\dfrac{1+x}{1-x}\right)}=\dfrac{1+ax}{1-ax}$ $\qquad x=\dfrac{(2a-1)^{\frac{1}{2}}}{a}$

P 272. $(x^{\frac{1}{2}}+3m^2)^{\frac{1}{2}}-(x^{\frac{1}{2}}-3m^2)^{\frac{1}{2}}=\dfrac{2m^{\frac{1}{2}}x^{\frac{1}{4}}}{n^{\frac{1}{2}}}$ $\qquad x=\dfrac{9m^3n^2}{4(n-m)}$

273. $\dfrac{3x+\sqrt{(4x-x^2)}}{3x-\sqrt{(4x-x^2)}}=\frac{1}{2}$ $\qquad x=2$

274. $\dfrac{(nx+m^2)^{\frac{1}{2}}+(nx-m^2)^{\frac{1}{2}}}{(nx+m^2)^{\frac{1}{2}}-(nx-m^2)^{\frac{1}{2}}}=r$ $\qquad x=\dfrac{m^2(r^2+1)}{2nr}$

275. $\{m(c^2-x^2)^{\frac{1}{2}}-n(c+x)\}(ma+nb)=\{m(c^2-x^2)^{\frac{1}{2}}+n(c+x)\}$
$(ma-nb)$ $\qquad x=\dfrac{c(b^2-a^2)}{a^2+b^2}$

276. $(a^2+x^{\frac{1}{2}})^{\frac{1}{3}}+(a^2-x^{\frac{1}{2}})^{\frac{1}{3}}=a^{\frac{2}{3}}$ $\qquad x=\dfrac{28a^4}{27}$

277. $\dfrac{1+\sqrt{(2x+x^2)}+x}{1+\sqrt{(2x+x^2)}-x}+ax=1$ $\qquad x=\dfrac{a^2+4a+4}{4a(a+1)}$

278. $\frac{\sqrt{(16x+1)}+\sqrt{(5x)}}{\sqrt{(16x+1)}-\sqrt{(5x)}}=\frac{7}{2}$ $x=5$

279. $(1-x)^{\frac{1}{3}}=2^{\frac{1}{3}}-(1+x)^{\frac{1}{3}}$ $x=1$

280. $\sqrt{(36x+1)}+\sqrt{(36x)}=9\{\sqrt{(36x+1)}-\sqrt{(36x)}\}$ $x=\frac{4}{81}$

281. $\frac{1+x+x^2}{1+x}=\frac{62}{63}\cdot\frac{1-x+x^2}{1-x}$ $x=\frac{1}{8}$

282. $\left(\frac{m+x}{m-x}\right)^2-1=\frac{px}{mn}$ $x=m\left(1+2\sqrt{\frac{n}{p}}\right)$

283. $\frac{1+x^3}{(1+x)^2}=m-\frac{1-x^3}{(1-x)^2}$ $x=\sqrt{\left(\frac{m-2}{m+4}\right)}$

284. $\frac{1}{(1+x)^{\frac{1}{2}}+1}+\frac{1}{(1+x)^{\frac{1}{2}}-1}=\frac{1}{x}$ $\cancel{x=\frac{1}{2}\sqrt{(\;-3)}}$

285. $\frac{x+1}{1+x+(1+x^2)^{\frac{1}{2}}}=4+\frac{x-1}{1-x+(1+x^2)^{\frac{1}{2}}}$ $x=\sqrt{3}$

286. $(1+p^2)^{\frac{1}{2}}\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}+(1-p^2)^{\frac{1}{2}}\left(\frac{1-x}{1+x}\right)^{\frac{1}{4}}=2(1-p^2)^{\frac{1}{4}}$ $x=-p^2$

287. $\frac{m+x+\sqrt{(2mx+x^2)}}{m+x-\sqrt{(2mx+x^2)}}=n^2$ $x=\frac{m}{2n}(n-1)^2$

288. $\sqrt{(m+x)}+\sqrt{(m-x)}=\sqrt[4]{(m^2+x^2)}+\sqrt[4]{(m^2-x^2)}$ $x=\frac{2m}{3}\sqrt[4]{5}$

289. $\frac{m+x}{\sqrt{m}+\sqrt{(m+x)}}+\frac{m-x}{\sqrt{m}+\sqrt{(m-x)}}=m^{\frac{1}{2}}$ $x=\frac{m}{2}\sqrt{3}$

290. $\sqrt[4]{(y^4-1)}=y^2-y^4\sqrt[4]{(y^4-1)}$ $y=\left(\frac{1+\sqrt{2}}{2}\right)^{\frac{1}{4}}$

291 $\frac{2}{9}(x^2+39x+374)^{\frac{1}{2}}=\left(\frac{x+22}{x+17}\right)^{\frac{1}{2}}+\frac{2}{19}(x^2+20x+51)^{\frac{1}{2}}$ $x=78$

292. $3(mnx)^{\frac{1}{3}} + x = n - m$ $\qquad x = (n^{\frac{1}{3}} - m^{\frac{1}{3}})^3$

293. $x = \sqrt{a^2 + x\sqrt{(b^2 + x^2)}} - a$ $\qquad x = \dfrac{b^2 - 4a^2}{4a}$

294. $\dfrac{m^2 + (m^4 - x^2)^{\frac{1}{2}}}{m^2 - (m^4 - x^2)^{\frac{1}{2}}} = p^2, \quad \dfrac{(m^2 + x)^{\frac{1}{2}} - (m^2 - x)^{\frac{1}{2}}}{(m^2 + x)^{\frac{1}{2}} + (m^2 - x)^{\frac{1}{2}}}$ $\qquad x = \dfrac{2m^2p}{1 + p^2}$

295. $\left(\dfrac{x + \sqrt{(x^2 - 9)}}{x - \sqrt{(x^2 - 9)}}\right)^2 = 243\left(\dfrac{\sqrt{(x+3)} - \sqrt{(x-3)}}{\sqrt{(x+3)} + \sqrt{(x-3)}}\right)$ $\qquad x = 5$

PROBLEMS.

1. Find a number which being multiplied by 8, and having 18 added to the product, the sum shall be 90. Ans. 9.

2. What number is that whose seventh part added to its fifth part will make 24? Ans 70

3. Twenty five is subtracted from a certain number and $\frac{2}{3}$ of the remainder is 50; What is the number? Ans 100.

4. The twelfth part of a certain number exceeds the fifteenth part by 1; find the number. Ans 60

5. Find a number from which if 8 be taken, and the remainder multiplied by 3 and then 65 added to the product, this sum divided by 4 shall give the required number. Ans 41

6. Find two numbers whose sum is 70 and whose difference is 28. Ans 21, 49

7. A father had two sons; the age of the eldest is $\frac{4}{7}$ of the age of the father, and the age of the youngest is $\frac{1}{4}$ of that of his brother, the sum of their ages is 84; What is the age of each? Ans 49, 28, 7

8. Sham is thrice as old as Ram. Fifteen years ago he was 6 times as old as Ram. What is Sham's age? Ans 75

9. Two numbers when multiplied together give 96 for the result, but when the greater is decreased by 1,the product is decreased by 8. What are the numbers ? 8, and 12

10. The cube root of a certain number is $\frac{1}{3}$ of the square root ; find the number. Ans 3^6

11. Divide Rs. 14 ans. 6 amoung 30 persons, men and boys, giving 10 as each to the men and 5 as each to the boys. How many were there of each ? Ans 16 men, 14 boys.

12. Forty-two years hence a boy will be seven times as old as be was six years since ; how old is he ? Ans 14

13. Find a number such that if 10, 40 and 25 be successively subtracted from it, the sum of the third, fourth and fifth parts of the respective results shall be equal to 60. Ans 100

14. A boy's age was thrice the number indicated by the clock, and his father's age was 3 times that of the boy. The sum of their ages was 96. What o'clock was it ? Ans 8

15. After A has received 12 Rs. from B, he finds that he has 47 Rs. more than B ; they have 147 ~~Rs~~ between them : what sum had each at first ? Ans 85 and 62

16. A and B entered into a transaction with equal sums of money, B first gave 24 ~~Rs~~ to A, but afterwards received from A $\frac{1}{4}$ of the money he then had ; A then found, that he had 14 ~~Rs~~ less than what B had : what money had each at first ? Ans 100

17. Divide the number 100 into two such parts that if $\frac{2}{3}$ of the smaller part be added to the greater the result is 89. 33 and 67

18. A person has 10 times as many eight anna pieces as two-anna pieces and together they amount to 328 ~~Rs~~. How many has he of each. Ans 640 eight anna pieces and 64 two anna pieces.

19. Divide 360 ~~Rs~~ among 3 persons, A, B, C, so that B shall have twice as much as A, and C three times as much as B.

Ans A's 40 ; B's 80 ; C's 240.

20. A debt of £3, 14*s* was paid in florins and half crowns and one more of the former was paid than of the latter. How many florins were paid ? Ans 17

21. Divide 48 into four such parts that the first increased by 3, the second diminished by 3, the third multiplied by 3 and the fourth divided by 3, shall all give the same result. Ans 6, 12, 3, 27

22. A and B have equal sums of money. A gains 180 Rs and B loses 140 Rs and then A has thrice as much as B. How much had each at first ? Ans 300 Rs

23. Find a number such that its third is as much above 80 as its fifth part is below 80. Ans 300

24. Two baskets were full of equal number of ripe mangoes ; after 75 mangoes had been taken from one basket and 50 from the other there remaind just one and a half times as many in one basket as in the other. What did each basket contain when full ?

Ans 125

25. Divide 240 into two parts such that $\frac{2}{5}$ of one part together with $\frac{3}{7}$ of the other part may be equal to 100. Ans 100, 140

26. A gentleman starts on a hunting excursion and has just 10 hours at his desposal : how far may he ride in a *gharry* which travels 12 miles an hour, so as to return home in time driving at the rate of 6 miles an hour. Ans 40 miles

27. A number is divided into both p and $p+1$ equal parts ; the product of the p parts is p times the product of the $p+1$ parts find the number. Ans $\left(\frac{p+1}{p}\right)^{p+1}$

28. A waterman finds by experience that he can with the advantage of a common tide row down a river from A to B, which is 18 miles, in an hour and a half, and that to return from B to A against an equal tide, though he rows back along the shore, where the stream is only $\frac{3}{5}$ as strong as in the middle, takes him just two

hours and a quarter. At what rate per hour the tide runs in the middle, where it is strongest. Ans $2\frac{1}{4}$ miles per hour

29. Suppose two hands of a clock (*a*) and (*b*) were together on Sunday noon at 12 oclock, and that the motion of each was such that (*a*) moved round the horary circle in one hour and (*b*) in $1\frac{1}{60}$ hour, when will they be together again for the 1st time.

Ans 61 hours.

30. The difference of two numbers is 54, and the quotient of the greater by the less is 4 ; what are the numbers. Ans 72, 18

31. Divide 1500 ~~Rs~~ among A, B, C and D in such a manner that B shall get 3 times as much as A, C shall get twice as much as A and B together and D shall get $\frac{1}{4}$ as much as A, B, and C together. Ans A, 's = 100 ; B's = 300 ; C's = 800 and D's = 300

32 A person bought a pony chaise at a certain price and paid the same price for the horse ; if the horse had cost 50 ~~Rs~~ less and the *ghary* 150 ~~Rs~~ more, the price of the pony would have been only half that of the chaise, find the cost of the chaise. Ans 250

33. A carpenter is engaged for 45 days, on condition that he receives 5 as for every day he works and pays 4 as fine for every day he is idle ; at the end of the time he receives 5 ~~Rs~~ 10 as. How many days did he work and how many was he idle.

Ans He worked 30 days, and was idle 15 days

34. Between 7 and 8 when will the two nands of a clock be (1) together (2) at right angles to each other (3) exactly opposite.

Ans Together at $38\frac{2}{11}$; right angles at $21\frac{9}{11}$; and opposite at $5\frac{5}{11}$.

35. After m o'clock the hour and minute hands of a clock are distant d of the minute divisions from each other find the time.

Ans $\frac{12}{11}(5m \pm d)$

36. A draper becoming insolvent found that the sum of money remaining unrealised from his customers was equal in amount to his liabilities, and further he finds that on 4000 ~~Rs~~ of the unrecovered money he can only get 12 as in the rupee, and the expences

of the bankruptcy are 5 per cent on his liablities ; if he pay $13\frac{1}{8}$ as. in the rupee what is the amount of his liabilities ? Ans 8000 Rs.

37. A man at a party at cards betted 3 shillings to two upon every deal. After 20 deals he won 5 shillings. How many deals did he win ? Ans 13

38. Hemadrish and Kristo were playing together, the former first won 12 ~~Rs~~ from the latter and then both had equal sums ; but Kristo on winning back his own money and 6 ~~Rs~~ more, had at last $2\frac{1}{5}$ times as much money as Hemadrish : what money had each at first. Ans 36, 60

39. A bill of 3 ~~Rs~~ 10 as. was paid in 4 as. pieces and 2 as. pieces and the whole number of coins was 18 ; how many coins were there of each kind. Ans 11 and 7

40. A confectioner buys $1\frac{1}{2}$ mds of sugar at 7 as. a seer and *chana* (curdled milk) at 6 as. a seer. What quantity of the latter must he add to the former so as to sell the mixture (sondesh) at $6\frac{3}{5}$ as. a seer. Ans A maund

41. A regiment was drawn up in a solid square ; and on a certain emergency 1500 soldiers were taken away ; it was again drawn up in a solid square and then it was found that there were 10 men fewer in a side : what was the original number of men in the regiment. Ans 6400

42. Divide 80 into 2 parts such that the first divided by 3 shall be equal to the second multiplied 3. Ans 8 and 72

43. A person sitting at a tavern distributed 3 pice each to those present there and had 7 pice left ; had he 3 as. more with him he could have given them at the rate of 4 pice each ; how many persons were there. Ans 19

44. Sham and Ram begin to travel in the same direction from Howrah by the grand Trunk Road. Sham travels 15 miles per day and after 18 days turns and goes back as far as Ram has travelled during the 9 days ; he then turns again and pursuing Ram overtakes

him at the end of 36 days from the commencement of their journey. How many miles Ram travelled in a day ? Ans 10 miles

45. A confectioner buys sugar at 6 as. a seer and *chana* at 8 as. a seer : in what proportion must he mix the two to prepare his *sondesh* so as to gain 20 per cent by selling it at 9 as. a seer ?

Ans 1 : 3

46. A person started at the rate of 3 miles an hour to walk to the Howrah railway station in order to catch a train, but after he had walked one third of the distance he was detained 15 minutes and was obliged in consequence to walk the rest of the way at the rate of 4 miles an hour. How far off was the station ? Ans $4\frac{1}{2}$ miles

47. A grocer has two sorts of sugar, one sort worth 8 as. a seer, and the other worth 10 as. a seer ; from these he wants to make a mixture of 2 maunds worth $9\frac{1}{5}$ as a seer. How many seers must be take from each sort. Ans 32 seers and 48 seers

48. Soodeen and Hurry start in different directions from a certain place in the circumference of a circular island. Soodeen travels 5 miles an hour and Hurry 7 miles per hour always keeping in the circumference. Soodeen wants 5 miles of being half way when he meets Hurry. Required the circumference of the island. Ans 60 miles.

49. A locomotive engine Alexander sets out from Howrah to go to Jamalpore, at the same time that another engine Napoleon sets out from Jamalpore to go to Howrah ; Alexander arrives at Jamalpore 4 hours and Napoleon at Howrah 9 hours after they met on the way ; how long did each take to perform the journey.

Ans. Alexander 10 hours ; Napoleon 15 hours.

50. Shoshee and Khetter play at marbles, Shoshee wins half of Khetter's and then loses 5 when he finds that he has as many as Khetter had at the commencement of the play ; how many had each at first if they had 50 between them? Ans Shoshee 20, Khetter 30.

51. The manjee (helmsman) of a boat of 8 oars weighs 1 md. 32 Srs. The average weight of the crew exclusive of the manejee

is greater than their average weight inclusive of him by 2 srs. What is their whole weight ? Ans 19 mds 32 Srs.

52. Saroda, who travels at the rate of 8 miles an hour, starts from Ooloobaria to go to Ghatal a distance of 50 miles while another Barada starts at the same time from Ghatal to come to Ooloobaria, his rate of travelling is 6 miles an hour. Saroda when went a certain distance from the starting point recollected to have left his bag in a shop at Ooloobaria and therefore turned back and on arriving at the place from which he started, again continued his journey ; they arrived both at the same time to their destination. At what distance from Ooloobaria did Saroda at first travel ? Ans $8\frac{1}{3}$ miles.

53. A steamer was seen to pass by Dacca when the minute-hand of a clock was at right angles to the hour-hand for the first time after 10 A. M.; when it arrived at Naraingunge, the minute-hand was opposite to the hour-hand after 11 A. M. The rate of the steamer is eleven miles an hour ; find the distance of Nariangunge from Dacca by water. Ans 15 miles.

54. A man had a certain number of oranges, he sells $\frac{1}{5}$ of the number and 2 more to one person ; $\frac{1}{2}$ the remainder and one more to a second person ; $\frac{2}{3}$ of the remainder and 3 more to a 3rd person, after which he had 3 with him ; how many had he altogether. Ans 50.

55. A ship's crew sailed with provisions for 30 days ; after being at sea 20 days they encountered a storm in which they lost 6 men and 2 days after the storm, they took on board 18 persons who had been wrecked and were without provisions ; they then found that to make their provisions last as long as was intended, each person's daily allowance must be reduced to $\frac{5}{6}$ of what it had been before ; how many persons were on board when the ship set sail ? Ans 51.

56. A fish was caught in the river Tista whose tail weighed 9 seers , his head weighed as much as his tail and half his body, and his body weighed as much as his head and tail together ; find the weight of the fish. Ans 1 md. 32 seers.

57. A ship sails with a supply of biscuit for 60 days at a daily allowance of one seer a head; after being at sea for 20 days she encounters a storm in which 5 men were washed overboard and damage sufferred that will cause a delay of 24 days and it is found that each man's allowance must be reduced to $\frac{5}{7}$ of a seer. Find the original number of the crew. Ans 40

58 An officer in drawing up his battalion in the form of a solid square finds that he has 6 men too many, and that he would want 19 men, to increase the side of the square by one man; how many men were there in the army? Ans 150

59. An officer can form the men of his regiment into a hollow square 8 deep. The number of men in the regiment is 1024. Find the no of men in the front of the hollow square. Ans 40

60. A colonel wished to form a solid square of his men. The first time he had 39 men over; the second time he increased the side of the square by one man, and then he found he wanted 50 men to complete it. Of how many men did the regiment consist. Ans 1975

61. A person walked out from the Oriental Seminary to the Howrah Station at the rate of $3\frac{1}{2}$ miles an hour and then walked along side of the railway to Bally at the same rate and on reaching the Bally station had to wait 6 minutes for the train which was then 3 miles off, and was proceeding towards Howrah. On arriving at the seminary which was a mile from the Howrah Station he found that he had been out 2 hours $35\frac{1}{7}$ minutes. Find the distance of Bally from Howrah. Ans 6 miles.

62. A stone is thrown into a well, and it is observed that $1\frac{1}{2}''$ elapse before the sound of its striking the bottom is heard; neglecting the time occupied by the transmission of the sound, find the depth of the well. Having given that $s = 16{\cdot}1 \times t^2$, where s = space in feet described by falling bodies from rest, on the surface of the earth, and t = time in seconds occupied for the descent. Ans 36·225 ft.

63. In passing over 180 ft. of the Chitpore Road the forewheel of a barouche had to make 3 turns more than the hind wheel and the periphery of the first is 3 ft. less than that of the 2nd ; find that of each. Ans 12 ft. , 15 ft.

64. Two workmen A and B were employed together for 50 days, at 5 shillings per day each. A spent 6d aday less than B did, and at the end of the 50 days he found he had saved twice as much as B, and the expense of 2 days over. What did each spend per day ?
Ans A 50d, B 56d a day.

65, A hare 50 of her leaps before a greyhound, takes 4 leaps to the greyhounds three ; but two of the greyhounds leaps are as much as 3 of the hare's. How many leaps must the greyhound take to catch the hare ? Ans 300 leaps.

66. Divide the no 116 into 4 such parts that if the 1st be increased by 5, the second diminished by 4, the 3rd multiplied by 3, and the 4th divided by 2, the result in each case shall be the same.
Ans 22, 31, 9 and 54.

67. Kadar and Boykanto began to trade with equal sums of money. In the 1st year Kadar gained 40 Rs. and Boykanto lost 40 ; but in the 2nd Kadar lost $\frac{1}{3}$ of what he then had and Boykanto gained a sum less by 40 Rs, than twice the sum that Kadar had lost ; when it appeared that Boykanto had twice as much money as Kadar. What money did each begin with ? Ans 320 Rs.

68. Sham and Ram being at play severally cut packs of cards so as to take off more than they left. Now it happened that Sham cut off twice as many as Ram left, and Ram cut off seven times as many as Sham left. How were the cards cut by each ?
Ans Sham 48, Ram 28,

69, A person at play won twice as much as he began with and then lost 16 Rs After this he lost $\frac{4}{5}$ of what remained, and then won as much as he began with and counting his money, found he had 80 Rs What sum did he begin with ? Ans 52 Rs

70. A man wished to inclose a garden with palisades, and found that if he put them a foot apart he should have too few by 150, but if he set them a yd asunder, he should have too many by 70. How many had he ? Ans 180

71. A constable in pursuing a thief, who was ⅛ of a mile from him runs at the rate of 5 miles an hour ; the thief runs at the rate of 4¾ miles an hour. What distance will the thief run before he is caught.
Ans 9½ miles

72. Divide the number 75 into four parts such that the first increased by 2, the second disminished by 3, the third multiplied by 4½ and the 4th divided by 6 may all be equal Ans 7, 12, 2, 54

73. A ryot made an agreement with his zemindar that his annual rent would be 35 Rs and the value of a certain number of maunds of rice ; when rice is 2 Rs a maund the whole rent becomes 10 per cent lower than when it is 3 Rs a maund. Find the number of maunds of rice which are stipulated as part of the rent. Ans 5 maunds

74. The interest on a certain sum of rupees at a certain rate of interest in one year is 405 Rs, but if the rate were increased by 1½ per cent, the interest would amount to 607 Rs 8 as. Find the principal and the rate per cent. Ans Principal 13500 Rs, rate 3 per cent

75. A train 88 yds in length overtook Naran walking along the line in the same direction at the rate of 4 miles an hour and passed him in 10 seconds ; 20 minutes afterwards the train overtook Ram and passed him in 9 seconds ; when will Naran overtake Ram. Ans 3⅓ hrs.

76. A market woman being asked how many mangoes she had replied ‘If I had as many more, half as many more and one egg and a half I should have 104 eggs ; how many had she ? Ans 41

77. A man buys a certain no of oranges at two a penny, four times as many at 5d a dozen, five times as many at 8d a score, and sells them at 3s. 8d a hundred, gaining by the transaction 3s. 6d. How many oranges did he buy ? Ans 1800

78. A gentleman walked from Calcutta to Ootterparah at the rate of 3 miles an hour and returned to Calcutta by the same route at the rate of 2 miles an hour; it took him 5 hours to perform the journey; find the distance of Ootterparah from Calcutta. Ans 6 miles

79. A gentleman walks from a village to a railway station at the rate of 3 miles an hour, runs part of the way back at the rate of $8\frac{1}{3}$ miles an hour, and then walks the remainder in 1 hr 5 min. If he is out 2 hrs 44 min, find the distance to the railway station.

Ans $4\frac{1}{2}$ miles

Exercise 22.

SIMPLE EQUATIONS OF TWO UNKNOWN QUANTITIES.

Find the values of x and y in the following equations.

1. $\left.\begin{array}{l} 4x+5y=23 \\ x+2y=8 \end{array}\right\} \begin{array}{l} x=2 \\ y=3 \end{array}$

2. $\left.\begin{array}{l} 3x+2y=30 \\ 2x+3y=25 \end{array}\right\} \begin{array}{l} x=8 \\ y=3 \end{array}$

3. $\left.\begin{array}{l} 6x+8y-74=0 \\ 10x-3y-51=0 \end{array}\right\} \begin{array}{l} x=3 \\ y=7 \end{array}$

4. $\left.\begin{array}{l} 8x-3y=2 \\ 5x+2y=9 \end{array}\right\} \begin{array}{l} x=1 \\ y=2 \end{array}$

5. $\left.\begin{array}{l} 3x+2y=39 \\ 5x-6y=37 \end{array}\right\} \begin{array}{l} x=11 \\ y=3 \end{array}$

6. $\left.\begin{array}{l} 5x+6y=12 \\ 8x-y=-2 \end{array}\right\} \begin{array}{l} x=0 \\ y=2 \end{array}$

7. $\left.\begin{array}{l} 10x-3y=54 \\ 2x-3y=6 \end{array}\right\} \begin{array}{l} x=6 \\ y=2 \end{array}$

8 $\left.\begin{array}{l} 3y-2x=21 \\ 4x-y=3 \end{array}\right\} \begin{array}{l} x=3 \\ y=9 \end{array}$

9. $\left.\begin{array}{l} 4x-3y=20 \\ 8x+3y=4 \end{array}\right\} \begin{array}{l} x=2 \\ y=-4 \end{array}$

10. $\left.\begin{array}{l} 11y-3x=-2 \\ 10x-2y=-28 \end{array}\right\} \begin{array}{l} x=-3 \\ y=-1 \end{array}$

11. $\left.\begin{array}{l} 4x-2y-50=0 \\ 70+10y=2x \end{array}\right\} \begin{array}{l} x=10 \\ y=-5 \end{array}$

12. $\left.\begin{array}{l} \frac{3x}{4}+8y=102 \\ 6x+\frac{2y}{3}=56 \end{array}\right\} \begin{array}{l} x=8 \\ y=12 \end{array}$

13. $\left.\begin{array}{l} ax+by=c \\ mx+py=d \end{array}\right\} \begin{array}{l} x=\dfrac{cp-bd}{ap-bm} \\ y=\dfrac{cm-ad}{bm-ap} \end{array}$

14. $\left.\begin{array}{l} 7x+2y=40 \\ 5xy=(x+1)(5y-6) \end{array}\right\} \begin{array}{l} x=4 \\ y=6 \end{array}$

15. $$\left.\begin{array}{c}(x+5)(y+7)=(x+1)(y-9)+112\\ 2x+10=3y+1\end{array}\right\}\begin{array}{l}x=3\\ y=5\end{array}$$

16. $$\left.\begin{array}{c}(a+c)x-by=bc\\ x+y=a+b\end{array}\right\}\begin{array}{l}x=b\\ y=a\end{array}$$

17. $$\left.\begin{array}{c}x+y=a+b\\ bx+ay=2ab\end{array}\right\}\begin{array}{l}x=a\\ y=b\end{array}$$

18. $$\left.\begin{array}{c}\frac{x}{2}+\frac{y}{3}=8\\ \frac{x}{5}+\frac{y}{3}=5\frac{3}{5}\end{array}\right\}\begin{array}{l}x=8\\ y=12\end{array}$$

19. $$\left.\begin{array}{c}\frac{x}{2}+\frac{2x-y}{7}=4\\ \frac{x+y}{11}+\frac{x-y}{2}=\frac{3}{2}\end{array}\right\}\begin{array}{l}x=6\\ y=5\end{array}$$

20. $$\left.\begin{array}{c}\frac{x+y}{3}+x=15\\ \frac{x-y}{}+y=6\end{array}\right\}\begin{array}{l}x=10\\ y=5\end{array}$$

21. $$\left.\begin{array}{c}\frac{5x}{6}+\frac{2y}{3}=24\frac{2}{3}\\ \frac{3x}{5}-y=0\end{array}\right\}\begin{array}{l}x=20\\ y=12\end{array}$$

22. $$\left.\begin{array}{c}\frac{x+y}{8}-\frac{x-y}{6}=5\\ \frac{x+y}{5}+\frac{x-y}{11}=8\end{array}\right\}\begin{array}{l}x=20\\ y=20\end{array}$$

23. $$\left.\begin{array}{c}\frac{2-3x}{6}+\frac{6y-1}{12}\\ \frac{2x+3y}{2}+3y=2\end{array}\right\}\begin{array}{l}x=\frac{1}{2}\\ y=\frac{1}{3}\end{array}$$

24. $$\left.\begin{array}{c}\cdot 08x+\cdot 21y=\overset{1.55}{\cdot 33}\\ \cdot 12x+.7y=3\cdot 54\end{array}\right\}\begin{array}{l}x=1\\ y=\end{array}$$

25. $$\left.\begin{array}{c}\frac{16}{x}-\frac{12}{y}=1\\ \frac{64}{x}+\frac{36}{y}=11\end{array}\right\}\begin{array}{l}\\ y=12\end{array}$$

$$\left.\begin{array}{c}2x-3y=5\\ \frac{x}{2y}+9=\frac{7x-4y}{y}\end{array}\right\}\begin{array}{l}x=10\\ y=5\end{array}$$

27. $$\left.\begin{array}{c}\frac{x+2}{y-1}-\frac{x-2}{y}-\frac{10}{y}\\ x-y=2\end{array}\right\}\begin{array}{l}x=4\\ y=2\end{array}$$

28. $$\left.\begin{array}{c}3x+2y=42\\ \frac{y}{2x}=\frac{5x-y}{x}-2\end{array}\right\}$$

29. $$\left.\begin{array}{c}2(x-3)-\frac{1}{5}(y-3)=3\\ 3(y-5)+\frac{1}{3}(x-2)=10\end{array}\right\}\begin{array}{l}x=5\\ y=8\end{array}$$

30. $$\begin{array}{cl}\frac{x-1}{3}=\frac{y+1}{4} & x=4\\ \frac{2x-3}{}-\frac{13-2y}{} & y=3\end{array}$$

31. $a(x+y)+b(x-y)=1$, $a(x-y)+b(x+y)=1$ | $x=\frac{1}{a+b}$, $y=0$

32. $(m+n)x=4mn+(m-n)y$, $(m-n)x=2(m+n)(m-n)-(m+n)y$;
$x=m+n\ \ y=m-n$

33. $(a+m)x+(b-m)y=c$, $(b+n)x+(a-n)y=c$ | $x=\frac{c}{a+b}$, $y=\frac{c}{a+b}$

34. $x-y=3,\ 3\left(\frac{1}{y}+\frac{1}{x}\right)=11\left(\frac{1}{y}-\frac{1}{x}\right)$ $\quad x=7$, $y=4$

35. $2(x-y)=3(x-4y)$, $14(x+y)=11(x+8)$ $\Big\}$ $x=20$, $y=2$

36. $16x+6y-1=\frac{128x^2-18y^2+217}{8x-3y+2}$, $\frac{10x+10y-35}{2x+2y+3}=5-\frac{54}{3x+2y-1}$ | $x=6$, $y=5$

37. $2x-\frac{y-3}{5}=4,\ \ 3y+\frac{x-2}{3}=9$ $\quad x=2,\ y=3$

38. $\frac{1}{11}(4x+2y)=6-\frac{1}{4}(5y-3x)$, $\frac{1}{3}(8y-10)=\frac{1}{6}(5x+3y)+5$ $\Big\}$ $x=3$, $y=5$

39. $y^{\frac{1}{2}}-(a-x)^{\frac{1}{2}}=(y-x)^{\frac{1}{2}}$, $2(y-x)^{\frac{1}{2}}=3(a-x)^{\frac{1}{2}}$ $\Big\}$ $x=\frac{4}{5}a$, $y=\frac{5}{4}a$

40. $\frac{4x-8y+1}{2}=\frac{10x^2-12y^2-14xy+2x}{5x+3y+3}$, $2\sqrt{(6+x)}=3\sqrt{(6-y)}$ $\Big\}$ $x=3$, $y=2$

41 $(x^n)^{3m}=c^m y,\ y=(a^0)^n$ $\quad x=c^{\frac{1}{3n}}\ \ y=1$

42. $x^{2p} - y^{2p} = m^2$ $\qquad x = \left(\frac{m^2 + n^2}{2n}\right)^{\frac{1}{p}}$

$x^p + y^p = n$ $\qquad y = \left(\frac{n^2 - m^2}{2n}\right)^{\frac{1}{p}}$

43. $x + y = 5$ $\qquad x = 3$
$xy = 6$ $\qquad y = 2$

44. $(a^2 - b^2)(3x + 5y) = (4a - b)2ab$

$a^2x - \frac{ab^2c}{a+b} + (a + b + c)by = b^2x + (a + 2b)ab$

$x = \frac{ab}{a-b}$, $y = \frac{ab}{a+b}$

45. $\sqrt{(x - 3)} = \frac{5}{4}\sqrt{(x + y)}$ $\qquad x = -3\frac{3}{7}$
$25y = 5x$ $\qquad y = -\frac{24}{35}$

46. $a(x^2 + y^2) - b(x^2 - y^2) = 2a$ $\qquad x = \sqrt{\left(\frac{a+b}{a-b}\right)}$

$(a^2 - b^2)(x^2 - y^2) = 4ab$ $\qquad x = \sqrt{\left(\frac{a-b}{a+b}\right)}$

47. $\sqrt{y} - \sqrt{(20 - x)} = \sqrt{(y - x)}$ $\qquad x = 16$
$2\sqrt{(y - x)} + 2\sqrt{(20 - x)} = 5\sqrt{(20 - x)}$ $\qquad y = 25$

48. $x^2 + xy = 45$ $\qquad x = 5$
$y^2 + xy = 36$ $\qquad y = 4$

49. $\left.\begin{array}{l} x + y = 5 \\ x^2 - y^2 = 5 \end{array}\right\}$ $\qquad x = 3$, $y = 2$

50. $\left.\begin{array}{r} 2x + 2y = 10 \\ x - y = 1 \end{array}\right\}$ $\qquad x = 3$, $y = 2$

51. $\left.\begin{array}{l} x^2 + y^2 = 13 \\ x + y = 5 \end{array}\right\}$ $\qquad x = 2$, $y = 3$

52. $\left.\begin{array}{l} x^2 + y^2 = 5 \\ x - y = 1 \end{array}\right\}$ $\qquad x = 2$, $y = 1$

53. $x^2 - xy = 8$ $\qquad x = 4$
$xy - y^2 = 4$ $\qquad y = 2$

54. $x^3 + y^3 = 35$ $\qquad x = 2$
$x + y = 5$ $\qquad y = 3$

55. $x^2+xy=36 \qquad x=4$

$xy+y^2=45 \qquad y=5$

56. $(x^2-xy+y^2)(x^2+y^2)=15 \qquad x=2$

$(x^2-xy+y^2)=\dfrac{21}{x^2+xy+y^2} \qquad y=1$

57. $\dfrac{5(w+z)^{\frac{1}{2}}}{w}+\dfrac{5(w+z)^{\frac{1}{2}}}{z}=6\tfrac{3}{4} \qquad w=5$

$\dfrac{3(w-z)^{\frac{1}{2}}}{z}-\dfrac{3(w-z)^{\frac{1}{2}}}{w}=\tfrac{3}{20} \qquad z=4$

Exercise 23.

SIMPLE EQUATIONS OF MORE THAN TWO UNKNOWNS.

1. $$\left.\begin{array}{l} 5x+2y+z=12 \\ 2x+3y+4z=20 \\ 3x+4y+5z=26 \end{array}\right\} \begin{array}{l} x=1 \\ y=2 \\ z=3 \end{array}$$

2. $$\begin{array}{l|l} 5x+8y-3z=59 & x=4 \\ 2x-3y+11z=23 & y=6 \\ 3x+2y-z=21 & z=3 \end{array}$$

3. $$\begin{array}{l|l} 4x+2y-z=20 & x=2 \\ 6x-7y+3z=-32 & y=8 \\ 7x+y-2z=14 & z=4 \end{array}$$

4. $$\begin{array}{l|l} 7x+8y-4z=31 & x=1 \\ 3x+2y+z=17 & y=5 \\ x+3y+4z=32 & z=4 \end{array}$$

5. $$\begin{array}{l|l} 2x+3y-z=-2 & x=-2 \\ 3x-3y+2z=-4 & y=2 \\ 5x+2y-3z=-18 & z=4 \end{array}$$

6. $$\begin{array}{l|l} x+2y+3z=6 & x=45 \\ 2x+4y+2z=8 & y=-21 \\ 3x+2y+8z=101 & z=1 \end{array}$$

7. $$\begin{array}{l|l} 3x+4z=57 & x=7 \\ 5x+3y=65 & y=10 \\ 2y-z=11 & z=9 \end{array}$$

8. $$\left.\begin{array}{l} 2x-3y-4z=21 \\ 3x-6y+3z=0 \\ 4x+6y-z=-22 \end{array}\right\} \begin{array}{l} x=-2 \\ y=-3 \\ z=-4 \end{array}$$

9. $$\begin{array}{ll} x+y=5 & x=2 \\ x+z=6 & y=3 \\ y+z=7 & z=4 \end{array}$$

10. $\frac{3x-y}{17-z}=1,\ \frac{5x-2z}{10-3y}=1,\ \frac{7x+4y}{5z+3}=1$ $\qquad x=4,\ y=0,\ z=5$

11. $\frac{x+2y}{7}=\frac{3y+4z}{8}=\frac{5x+6z}{9},\ x+y-z=126,\ x=51,\ y=76,\ z=1$

12. $\frac{1}{x}+\frac{1}{y}=\frac{3}{4},\ \frac{1}{y}+\frac{1}{z}=\frac{5}{12},\ \frac{1}{x}+\frac{1}{z}=\frac{2}{3},$ $\qquad x=2, y=4,\ z=6$

13. $x+y+z=m^3+n^2+p^2,\ x+m^2=y+n^2=z+p^2$

$$x=\tfrac{2}{3}(m^2+n^2+p^2)-m^2$$

14. $\frac{1}{x}-\frac{1}{y}=\frac{1}{12},\ \frac{1}{y}+\frac{1}{z}=\frac{9}{20},\ \frac{3}{x}+\frac{7}{y}=13\frac{3}{4},$ $\qquad x=3,\ y=4,\ z=5$

15. $x+y-z=5, x-y+z=11, y+z-x=1$ $\qquad x=8, y=3, z=6$

16. $$\left.\begin{array}{l} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1 \\[1ex] \frac{x}{2}+\frac{y}{4}+\frac{z}{3}=1 \\[1ex] \frac{x}{3}+\frac{y}{2}+\frac{z}{4}=1 \end{array}\right\} \qquad x=y=z=\frac{12}{13}$$

17. $$\left.\begin{array}{l} \frac{xy}{x+y}=1\frac{1}{5} \\[1ex] \frac{xz}{x+z}=1\frac{1}{3} \\[1ex] \frac{yz}{y+z}=1\frac{5}{7} \end{array}\right\} \begin{array}{l} x=2 \\ y=3 \\ z=4 \end{array}$$

18. $xy=2,\ xz=3,\ yz=6$ $\qquad x=1,\ y=2.\ z=3$

19. $xyz=2,\ xyw=6,\ yzw=6,\ xzw=3$ $\qquad x=1, y=2, z=1, w=3$

20. $x^2yz=6$, $xy^2z=12$, $xyz^2=18$ $\qquad x=1, y=2, z=3$

21. $y^2z=36$, $z^2x=32$, $x^2y=12$ $\qquad x=2, y=3, z=4$

22. $x^{-2}y^{-1}z=1\frac{1}{2}$, $x^{-1}yz^2=18$, $xy^2z^3=108$ $\qquad x=1, y=2, z=3$

23. $(x+y)(y+z)=35$, $(x+z)(y+z)=42$, $(x+y)(x+z)=30$

$x=2,\ y=3,\ z=4$

24. $xy=1$, $(yz)^{\frac{1}{2}}=24x$, $(xz)^{\frac{1}{2}}=4y$ $\qquad x=\frac{1}{\sqrt[3]{6}},\ y=\sqrt[3]{6},\ z=96$

25. $3x+4y=19$, $2y-z=5$, $7x+2z=13$ $\qquad x=1, y=4, z=3$

26. $2(x-y)=3z-2$, $x-3z=3y-1$, $2x+3z=4(1-y)$

$x=2,\ y=-3,\ z=4$

27. $9xy=20(x+y)$, $7xz=12(x+z)$, $8yz=15(y+z)$

$x=4,\ y=5,\ z=3$

28. $xy+20(x-y)=0$, $yz+30(y-z)=0$, $3x-2z=0$

$x=4, y=5,\ z=6$

29.
$$\begin{aligned} w+x+y+z&=10 & x&=2\\ 2w+x+y+z&=11 & y&=3\\ w+y+z&=8 & z&=4\\ w+2y+z&=11 & w&=1 \end{aligned}$$

30. $x+y+z=14$, $x^2+y^2+z^2=84$, $xz=y^2$ $\qquad x=8,\ y=4,\ z=2$

31. $x^3+y^3+z^3=3xyz$ and $3m-x+z=3n-y+x=3p-z+y$

$x=m-n,\ y=n-p,\ z=p-m$

✓ 32. $x^2+y^2+z^2=29$, $xy+xz+yz=26$, $x+y-z=5$

$x=4,\ y=3,\ z=2$

33. $xyz=\frac{24}{25}(y^2+z^2)=\frac{6}{5}(z^2+x^2)=\frac{24}{13}(x^2+y^2)$

$x=2,\ y=3,\ z=4$

34. $x+y+z=0$, $(m+n)x+(m+p)y+(n+p)z=0$, $mnx+mpy$
$+npz=0$ $\qquad x=\frac{1}{(m-p)(n-p)}, y=\frac{1}{(m-n)(n-p)}, z=\frac{1}{(m-n)(m-p)}$

35. $x - m^2y + m^4z = m^6,\ x - n^2y + n^4z = n^6,\ x - p^2y + p^4z = p^6$

$$x = m^2n^2p^2,\quad y = m^2n^2 + m^2p^2 + n^2p^2,\quad z = m^2 + n^2 + p^2$$

36. $xy = a(x+y),\ xz = b(x+z),\ yz = c(y+z)$

$$x = \frac{2abc}{ac+bc-ab},\quad y = \frac{2abc}{ab+bc-ac},\quad z = \frac{2abc}{ab+ac-bc}$$

37. $x(x+y+z) = 18,\ y(x+y+z) = 27,\ z(x+y+z) = 36$

$$x = 2,\quad y = 3,\quad z = 4$$

38. $xy = p(x+y+z),\ yz = m(x+y+z),\ xz = n(x+y+z)$

$$x = \frac{mn+mp+np}{m},\quad y = \frac{mn+mp+np}{n},\quad z = \frac{mn+mp+np}{p}$$

39. $xyz = m(yz - xz - xy) = n(xz - xy - yz) = p(xy - yz - xz)$

$$x = -\frac{2np}{n+p},\quad y = -\frac{2mp}{m+p},\quad z = -\frac{2mn}{m+n}$$

40. $(x-m) + (y-n) + (z-p) = 0,\ (n-p)x + (p-m)y + (m-n)z = 0,$

$$px + my + nz = m^2 + n^2 + p^2$$

$$x = n + p - m$$

41. $a(x+y) = 1,\ b(x+z) = 1,\ c(y+z) = 1$

$$x = \frac{1}{2a} + \frac{1}{2b} - \frac{1}{2c},\quad y = \frac{1}{2a} - \frac{1}{2b} + \frac{1}{2c},\quad z = \frac{1}{2b} + \frac{1}{2c} - \frac{1}{2a}$$

42. $x + y + z = 0,\ m^2x + n^2y + p^2z = 0$ and $n^2p^2x + m^2p^2y + m^2n^2z$ $= (n^2 - m^2)(n^2 - p^2)(p^2 - m^2),\ x = n^2 - p^2,\ y = p^2 - m^2,\ z = m^2 - n^2$

43. $x^2yz = 50,\ xy^2z = 10.\ xyz^2 = 20$

$$x = 5,\quad y = 1,\quad z = 2$$

44. $x^3 + y^3 + z^3 = 3xyz,\ x - m = y - n = z - p.\ x = m - \frac{1}{3}(m+n+p)$

$$z = p - \tfrac{1}{3}(m+n+p)\qquad y = n - \tfrac{1}{3}(m+n+p)$$

45. $3x + 4y = 2xy,\ 4y + 3z = 3yz,\ 6z + 5x = 4xz$

$$x = 4,\quad y = 3,\quad z = 2$$

46. $\frac{1}{x} + \frac{1}{y} = c,\ \frac{1}{y} + \frac{1}{z} = a,\ \frac{1}{x} + \frac{1}{z} = b,\ x = \frac{2}{b+c-a},$

$$y = \frac{2}{a-b+c},\quad z = \frac{2}{a+b-c}$$

47. $\frac{1}{xz}+\frac{1}{xy}=\frac{5}{6}$, $\frac{1}{yz}+\frac{1}{xy}=\frac{2}{3}$ and $\frac{1}{yz}+\frac{1}{xz}=\frac{1}{2}$ $\qquad x=1,\ y=2,\ z=3$

48. $x(y+z)=5,\ y(x+z)=8,\ z(x+y)=9$ $\qquad x=1,\ y=2,\ z=3$

49. $\frac{z}{x^2y}=\frac{5}{12}$, $\frac{yz^2}{x}=\frac{75}{2}$, $xy^2z^3=2250$ $\qquad x=2,\ y=3,\ z=5$

50. $x^2(y-z)=8,\ y^2(z-x)=-9,\ z^2(x-y)=-1$

$\qquad x=2,\ y=3,\ z=1$

51. $x^2(y+z)=5,\ y^2(x+z)=16,\ xyz=6$ $\qquad x=1,\ y=2,\ z=3$

52. $xyz=\frac{4}{3}(y+z)=\frac{8}{5}(x+z)=\frac{8}{3}(x+y)$ $\qquad x=1,\ y=2,\ z=4$

53. $x^2y^3z^4=324,\ x^3y^4z^2=72,\ x^4y^2z^3=432$ $\qquad x=2,\ y=1,\ z=3$

54. $x^2+y^2+z^2=14,\ x+y+z=6,\ x(y+z)=5$ $\qquad x=1,\ y=3,\ z=2$

55. $xyz=24,\ xyw=12,\ yzw=6,\ xzw=8$ $\qquad x=4,\ y=3,\ z=2, w=1$

56. $x+2y+3z+4u=27, 3x+5y+7z+u=48,\ 5x+8y+10z-2u=65$

$7x+6y+5z+4u=53$ $\qquad x=1,\ u=2\ \ y=3,\ z=4$

57. $\frac{1}{x}-\frac{1}{y}=\frac{1}{8}$, $\frac{1}{x}+\frac{1}{z}=\frac{1}{9}$, $\frac{1}{z}-\frac{1}{y}=\frac{5}{72}$ $\qquad x=12,\ y=-24,\ z=36$

58. $(m-p)x+(p-n)y+(n-m)z=0, (m-n)x+(p-m)y+(n-p)z=0$

$m(x-m^2)+p(y-p^2)+n(z-n^2)+3mnp=0$

$x=y=z=m^2+n^2+p^2-mn-mp-np$

59. $(x-m)+(y-n)+(z-p)=0,\ \ m(x-n)+n(y-p)+p(z-m)=0$

$(n-p)x+(p-m)y=(n-m)z$

$x=\frac{n+p}{2},\ \ y=\frac{m+p}{2},\ \ z=\frac{m+n}{2}$

60. $(x-m)+(y-n)+(z-p)=0 \quad (n-p)x+(p-m)y+(m-n)z=0$

$px+my+nz=mn+np+mp,$ $\qquad x=m,\ y=n,\ z=p$

Exercise 24.

PROBLEMS.

1. What fraction is that which becomes equal to $\frac{2}{3}$ if 1 be added to the numerator and equal to $\frac{1}{2}$ if 1 be added to the denominator. Ans $\frac{5}{9}$

2. If the numerator of a certain fraction be increased by 1 and the denominator be diminished by 2 the value will be 1; if the numerator be increased by the denominator and the denominator diminished by the numerator the value will be $3\frac{2}{3}$; find the fraction. Ans $\frac{4}{7}$

3. If the length and breadth of a floor be increased by 3ft and 2ft. respectively, the area would be increased by 40 sq. ft; but if the length be diminished by 2 ft. and the breadth increased by 4 ft. the area would be increased by 12 ft: find the length and breadth of the floor . Ans 8ft, 6ft

4 A certain number of two digits is equal to 3 times the sum of its digits, and if 45 be added to the number the digits are reversed: find the number. Ans 27

5. A and B can perform a piece of work together in 6 days; B and C in 7 days; and A, B and C in 4 days; how long would A and C take to do it. Ans $5\frac{1}{4}$ days

6. A boat sailing from Chandernagor with a fair wind, arrives in Calcutta in 2 hours; and on its return the wind being countary, it proceeds 6 miles an hour slower than it went; now when it is half way over, the wind changing, it sails two miles an hour faster and reaches Chandernagore sooner than it would have done had the wind not changed in the proportion of 6 : 7. Required the rates of sailing and the distance between Chandernagore and Calcutta.

Ans Distance 22 miles and in returning it sails 5 and 7 miles an hour

7. If A and B work together, they can earn 9 as in 2 days; if A and C work together, they can earn 1Re. 4as. in 4 days, and if B and C work together, they can earn 2Rs. 12as. in 8 days; find what each will earn in a day. Ans A 2as; B $2\frac{1}{2}$as; C 3as.

8. Two trains, one 80 ft long and the other 50ft, are observed to run on parallel rails; when they move in opposite directions they are found to pass each other in $1\frac{2}{7}$ seconds and when they move in the same direction they are found to pass each other in 9 seconds. Find the speeds of the trains.

Ans The faster runs 40 miles and the other 30 miles an hour.

9. A person sells 4 sheep and 5 goats for 30 Rs. and 9 sheep and 7 goats for 59 Rs: find the price of each. Ans Sheep 5Rs; Goats 2Rs

10. A cistern is filled in 24 minutes by 3 pipes, one of which conveys 8 gallons more, and another 7 gallons less than the third, every 3 minutes. The cistern holds 1050 gallons. How much flows through each pipe in a minute? Ans $17\frac{5}{36}$, $14\frac{17}{36}$, $12\frac{5}{36}$

11. Two shepherds A and B while driving their cattle on a pasture were talking in the following way; A said to B "if 5 of your sheep come to my flock, the two flocks would be equal in number whereas if I send 10 sheep to yours, my flock would have only half the no of your flock"; find the number of sheep in each flock.

Ans 40 and 50

12. A certain number of two digits is equal to 6 times the sum of the digits increased by 7, and if 27 he subtracted from the number the digits are reversed; find the number. Ans 85

13. A, B, C, D, E, play together on the condition that he who loses shall give to all the rest as much as they have already. First A loses, then B, then C, then D, then E; all lose in turn, and yet at the end of the fifth game they all have the same sum viz 32Rs. How much had each before they began to play. Ans A 81Rs, B41Rs, C21Rs, D11Rs, E6Rs

14. A and B together can perform a piece of work in *a* days, A and C together the same in *b* days, and B and C together in c days; find the time in which each can perform it separately.

Ans A in $\frac{2abc}{ac+bc-ab}$, B in $\frac{2abc}{ab+bc-ac}$, C in $\frac{2abc}{ab+ac-bc}$, days

15. To complete a certain work A requires m times as long a time as B and C together; B requires n times as long as A and C together and C requires p times as long as A and B together. Compare the times in which each would do it, and prove that $\frac{1}{m+1}+\frac{1}{n+1}=\frac{p}{p+1}$

16. A person rowed 12 miles down a river and back again in 8 hours and found that it took thrice as long a time to row against the tide as to row with the stream. Find the rate of the stream, and of the boat in still water. Ans stream 2, boat 4 miles an hour.

17. A person rowed down the river Hoogly from Serampore to Calcutta a distance of 12 miles. When he came down 4 miles of the way there occured the flood tide by which the progress of the boat was retarded. His rate of pulling in still water is 6 miles per hour and he observed that the time it occupied to row with the ebbtide was to the time it occupied to strnggle against the flood tide as 3 : 16 also that the rate of the ebbtide is $\frac{2}{3}$ of that of the flood tide. Find the rates of the ebbtide and flood tide. Ans Ebbtide 2 miles,floodtide 3 miles per hour.

18. Two vessels contain mixtures of sugar and water : in one there is twice as much sugar as water, in the other 3 times as much water as sugar. How much must be drawn off from each to fill a tumbler, in order that its contents may be half sugar and half water? It being known that the 3 vessels have thesame capacity.

Ans $\frac{3}{5}$ of the tumbler.from the 1st and $\frac{2}{5}$ from the 2nd

19. Two casks A and B contain mixtures of wine and water; in A the quantity of wine is to the water as 4 : 3; in B the like ratio is 2 : 3. If A contains 70 gallons, what must B contain so that when the two are put together, the new mixture may be half wine and half water. Ans 20 gallons of wine and 30 of water

20. There were two scholarships to be competed by 5 boys in a certain seminary, and after examination it was found that Nogender obtained $\frac{2}{5}$ths of the whole number of marks given, Jotish twice as

many as Nogender got more than Hem, who obtained 3 times as many as Jotish got more than Surrut; that Surrut obtained half as many as Nogender, Jotish and Hem together, and Preo one third more than the excess of the sum of Nogender, Jotish and Hem's marks together over Surrut. Determine the successful boys. Ans Jotish and Preo.

21. A and B run a race round a two mile course. In the first heat B reaches the winning past 2 minutes before A. In rhe second heat A increases his speed 2 miles an hour, and B diminishes his by the same quantity; and A then reaches the winning post 2 minutes before B. Find at what rate each run in the first heat.
Ans A 10 miles an hour and B 12 miles.

22. A person rows from Monghyr to Bhagulpore a distance of 20 miles and back again in 10 hours, the stream flowing unformly in the same direction all the time and he finds that he can row two miles against the stream in the same time that he rows 3 miles with it. Find the time of his going and retuning Ans, 4 hours, 6 hrs.

23. A, B, C, sit down to play; in the first game, A loses to each of B and C as much as each of them has, in the second B loses similarly to each of A and C, and in the third C loses similarly to each of A and B; and now they have each 24 Rs. What had they each at first.
Ans 39 Rs. 21 Rs. 12 Rs.

24. There is a number consisting of two digits, the 2nd of which is greater than the first; and if the number be divided by the sum of its digits the quotient is 4; but if the digits be inverted, and that number divided by a number greater by 2 than the difference of the digits, the quotient becomes 14. Required the number Ans 48.

25. A number consisting of 2 digits when divided by 4, gives a certain quotient and a remainder 3; when divided by 9 gives another quotient and remainder 8. Now the value of the digits on the left hand is equal to the quotient which was got when the number was divided by 9 and the other digit is equal to $\frac{1}{17}$ th of the quotient got when the number was divided by 4, Required the number. Ans 71.

26. If 19 lbs. of gold weigh 18 lbs. in water and 10 lbs of silver weigh 9 lbs. in water ; find the quantity of gold and silver weighing 106 lbs, in air and 99 lbs. in water Ans 76, 30.

27. A and B severally cut packs of cards, so as to cut off less than they left ; now the number of cards left by A added to the number cut off by B make 50 ; also the number of cards left by both exceed the number cut off, by 64. How many did each cut off?

Ans A 11 ; B 9.

28. A and B playing at bowls, says A to B, "if you will give me a guinea I will bet you $\frac{1}{2}$ a crown to 18*d* on each game, and will play 36 games together." B won his guinea back again and £1·17*s* besides. How many games did each win ? Ans A 8, B 28 games

29. One of the digits of a number is greater by 5 than the other. When the digits are inverted, the number becomes $\frac{3}{8}$ of the original number. Find the digits Ans 72.

30. The sum of the digits is 9. Nine times one of the numbers they form is equal to twice the other number : find the digits,

Ans 8 and 1.

31. There is a number consisting of 3 digits, the right hand one being 0. If the left hand and middle digits be interchanged the number is increased by 180. If the left hand digits be halved and the middle and right hand digits be interchanged the number is diminished by 136. Find the number. Ans 240.

32. Henry challenged Roberts to ride a bicycle of 1760 yds. Henry at the first heat gave Roberts a start of 60 ft. and beat him by $\frac{1}{4}$ a min. At the second heat Henry gave Roberts a start of 32 seconds and beat him by $\frac{104}{11}$ yrds. How many miles per hour did Henry run.

Ans 12 miles.

Exercise 25.

EXPONENTIAL EQUATIONS.

Solve the following equations,

1. $5^x = 125$ Ans $x=3$

2. $2^x = 256$ Ans $x=8$

3. $(1\frac{1}{2})^x = 5\frac{1}{16}$ Ans $x=4$

4. $3^{2x+3} = 243$ Ans $x=1$

5. $25^x = 5$ Ans $x=\frac{1}{2}$

✓6 $4^{\frac{x}{2}} + 2^{x+1} = 24$ Ans $x=3$

✓ 7. $2^{x+1} + 4^x = 80$ Ans $x=3$

✓8. $a^{-x}(a^x + b^{-x}) = \frac{a^2b^2+1}{a^2b^2}$ Ans $x=2$

✓ 9. $4^{x+1} + \frac{1}{16^{-\frac{x}{2}}} = 320$ Ans $x=3$

10. $(2^x)^2 + (4^{2x})^3 = 16 + 16 \times 2^{10x}$ Ans $x=2$

11. $7^{\frac{x}{2}+\frac{y}{3}} = 2401$ and $6^{\frac{x}{4}+\frac{y}{2}} = 1296$ Ans $x=4$, $y=6$

12. $x^y = y^x$, and $x^4 = y^2$ Ans $x=2$, $y=4$

13. $6^x = 54y$, and $4^x = 16y$ Ans $x=3$, $y=4$

14. $2^y + 3^x = 11$, and $9^x - 4^y = 77$ Ans $x=2$, $y=1$

15. $2^x 4^y = 32$, and $3^x \div 9^y = 3$ Ans $x=3$, $y=1$

16. $\sqrt{2} + 1 - \frac{1}{2^{\frac{1}{x}} - 1} = 0$ Ans $x=2$

17. $\sqrt{a^x}.\sqrt{a^y} = a^3$; $(b^x)^3 = b^6.(b^y)^3$ Ans $x=4$, $y=2$

18. $$\left.\begin{array}{l} a^{2x}.\ a^{3y+1} = a^{8m+5} \\ a^{4y}.\ a^{2x+3} = a^{4m+7} \end{array}\right\} \qquad \begin{array}{l} x=2 \\ y=m \end{array}$$

19. $5^{x+1}+\frac{1}{25^{-\frac{x}{2}}}=150$ $x=2$

20. $x^{y}=y^{x},\ x^{m}=y^{n}$ $x=\left(\frac{m}{n}\right)^{\frac{n}{m-n}},\ y=\left(\frac{m}{n}\right)^{\frac{m}{m-n}}$

21. $4^{x}=2^{x+y}\times 8$ and $\frac{x}{y}=3$, $x=\frac{9}{2},\ y=\frac{3}{2}$

22. $\frac{64^{x}}{8}=\frac{1}{4096^{x}}$ $x=\frac{1}{6}$

23. $(6{\cdot}25)\times({\cdot}4)^{x}=1$ $x=2$

24. $m^{2^{-x^{2}}}=m^{4^{-x}}.\ m^{2^{-2x}}$ $x=1$

25. $m^{x(x-1)}.\ m^{x(x+2)}=m^{5x}$ $x=2$

26. $3^{x}.\ 9^{y}=27$ and $4^{x}.\ 8^{y}=32$. $x=1,\ y=1$

Exercise 26.

RATIO.

1. Find the ratio of 12 as to 3 Rs. Ans 1 : 4.

2. Arrange the following ratios in the order of magnitude
$5:6,\ 6:9,\ 8:10,\ 5:8$ Ans $\frac{5}{6},\ \frac{6}{9},\ \frac{8}{10},\ \frac{5}{8}$.

3. Find the ratio compounded of 2 : 13 and 26 : 30 Ans $\frac{2}{15}$

4. Find the ratio compounded of $p:n,\ x:y$ and $y:p$ Ans $\frac{x}{n}$

5. What is the proportion deducible from the equation $x^{2}+y^{2}=2ax$ Ans $x:y::y:2a-x$

6. Four given numbers are represented by $a,\ b,\ x.\ y$ required the quantity which added to each will make them proportionals.

Ans $\frac{ay-bx}{b+x-a-y}$

7. Shew that the ratio $a^2 - x^2 : a^2 + x^2$ is greater than the ratio $a - x : a + x$

8. Prove that $x^3 + y^3 : x^2 + y^2$ is greater than $x^2 + y^2 : x + y$

9. Shew that the ratio $x^2 + 9x + 20 : x^3 + 7x^2 + 14x + 8$ is equal to the ratio $x + 5 : x^2 + 3x + 2$

10. What quantity must be added to each of the terms of the ratio $a : b$ that it may become equal to $m : n$ Ans $\frac{an - bm}{m - n}$

11. What is the ratio resulting from the composition of $a^2 + b^2 : a^2 - b^2$ and $(a + b)^2 : (a - b)^2$ Ans $(a + b)\ a^2 + b^2) : (o - b)^3$

12. If x be to y in the duplicate ratio of $a : b$ and a be to b in the subduplicate ratio of $a + x : a - y$ then will $2x : a = x - y : y$

12 (*a*) Which is the greater, the ratio of $2x + 3 : \frac{2x}{3} + 3$ or that of $2x + 6 : \frac{2x}{3} + 4$ Ans $2x + 6 : \frac{2x}{3} + 4 > 2x + 3 : \frac{2x}{3} + 3$

13. Compound the ratios of $41 : 6$, $6 : 11$ and $11 : 4$ and then reduce the resulting ratio to its lowest terms Ans $41 : 4$

14. Compound the subduplicate ratio of $x^2 : y^2$ with the tripliate ratio of $x^{\frac{1}{3}} . y^{\frac{1}{3}}$ Ans $x^2 : y^2$

15. Which is the greater $x^3 + y^3 : x - y$ or $x^3 - y^3 : x + y$, Ans $x^3 + y^3 : x - y$

16. Find a mean proportional to $\frac{a + b}{a - b}$ and $\frac{a - b}{a + b}$ Ans 1

17. How is the ratio $a : a - 2b$ affected by adding d to both terms

18. Find the ratio compounded of $16 : 5$, the subtriplicate ratio of $8 : 27$ and the sub-duplicate ratio of $16 : 25$. Ans $128 : 75$

19. If the ratios $a - x : a + x$, $a + x : 2 + y$ and $2 + y : a - x$ be compounded together; shew that the resulting ratio is a ratio of equality.

20. Two numbers are in the ratio of $4 : 7$ and if 4 be added to each, the ratio is that of $2 : 3$; find the numbers. Ans $8 : 14$

21. Two numbers are in the ratio of 9 : 11 and if 6 be taken from each the ratio is that of 3 : 4; find the numbers. Ans 18, 22

22. Two numbers are in the ratio of 2 : 3; if 6 be added to the less number and 3 taken from the greater number the ratio is that of 10 to 11: find the numbers. Ans 24 : 36

23. Find the number which added to each term of the ratio 9 : 7 makes it $\frac{4}{5}$ of what it would have been if the same number had been taken from each term. Ans 3

24. If the ratios of $3a+2 : 6a+1$ and of $2a+3 : a+2$ be compounded together, is the resulting ratio a ratio of greater or less inequality. Ans A ratio of greater inequality

25. Divide 18 into 3 parts such that the ratio of the first two shall be 1 : 3 and that of the last two 3 : 5. Ans 2, 6, 10

26. What is the proportion deducible from the equation $ax = x^3 + a^3$

Ans $x : x+a :: x^2 - ax + a^2 : a$

27. Prove that $a^3 + b^3 : a^2 + b^2$ is greater than $a^2 - b^2 : a + b$ unless $a = b$

28. Prove that if $a:b$ is a greater ratio than $c:d$, $a+c : b+d$ is a less ratio than $a : b$ but a greater than $c : d$.

29. Find two numbers in the ratio of 4:5 such that their difference has to the difference of their squares the ratio of 1 : 18.

Ans 8, 10

30. Find two numbers in the ratio of 5 : 7 such that their sum has to the sum of their squares the ratio of 3 : 37. Ans 10 : 14

31. Find two numbers in the ratio of 3 : 4 such that their sum has to the difference of their squares the ratio of 1 : 3

Ans 9 : 12

32. Find x so that the ratio $x : 3$ may be the duplicate ratio of $3 : x$. Ans 3

33. Find x so that the ratio $6-x : 12-x$ may be the duplicate of the ratio $6 : 12$. Ans 4

34. If $\frac{x}{a-b}=\frac{y}{b-c}=\frac{z}{c-a}$ then $x+y+z=0$

35. Find the ratio compounded of $x^6-y^6 : x^4+2x^2y^2+y^4$, $x^2+y^2 : x^2-xy+y^2$, and $x+y : x^3-y^3$. Ans $(x+y)^2 : x^2+y^2$

36. The ratio of the sum to the difference of two numbers is that of $7 : 3$. Shew that if half the less be added to the greater, and half the greater to the less, the ratio of the numbers so formed will be that of $4 : 3$.

37. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then each of these ratios

$$=\sqrt{\left(\frac{pa^2+qc^2+re^2}{pb^2+qd^2+rf^2}\right)} \text{ or } =\left(\frac{pa^3+qc^3+re^3}{pb^3+qd^3+rf^3}\right)^{\frac{1}{3}} \text{ or }$$

$$=\left(\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n}\right)^{\frac{1}{n}}.$$

38. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ prove that each of these ratios $=\frac{a+c+e}{b+d+f}$

39. Prove that a ratio of greater inequality is diminished and of less inequality increased, by adding the same quantity to each of its terms.

40. Prove that a ratio of greater inequality is increased, and a ratio of less inequality is diminished by taking from both terms of the ratio any number which is less than each of those terms.

41. If $\frac{a-m}{m-n}=\frac{b-p}{p-q}=\frac{c-r}{r-s}$ then each of these ratios

$$=\frac{ap-mb}{mq-np}=\frac{br-pc}{ps-qr}=\frac{cm-ar}{nr-ms}=\frac{a+b+c-(m+p+r)}{m+p+r-(n+q+s)}$$

42. There are two vessels A and B each containing a mixture of

water and sugar, A in the ratio of 3 : 4, B on the ratio of 2 : 3. What quantity must be taken from each in order to form a third mixture which shall contain 7 seers of water and 10 seers of sugar.

Ans 1*st* 7 seers, 2*nd* 10 seers.

Exercise 27.

PROPORTION.

Find the value of x in each of the following proportions

1. $12 : 16 :: 24 : x$ Ans $x = 32$
2. $6 : 8 :: x : 12$ $x = 9$
3. $3 : x :: x : 27$ $x = 9$
4. $x : 4 :: 36 : x$ $x = 12$
5. $x + 4 : x + 2 :: x + 8 : x + 5$ $x = 4$
6. $4x - 4 : 2x :: 3x + 7 : 2x + 6$ $x = 3$
7. $x^2 + x + 1 :: 62(x + 1) :: x^2 - x + 1 : 63(x - 1)$ $x = 5$
8. If $xy = mn$ and $yz = np$ then $x : m :: z : p$

8*a*. Find the 2*nd* proportional to the nos 32, 4, 1 Ans 8

8*b*. Find a fourth proportional to $2x$, $3x$ and $8x$ Ans $12x$

8*c*. Find the mean proportional to the nos 3 and 27 Ans 9

8*d*. Find the mean proportional between $\frac{x+y}{x-y}$ and $\frac{x^2-y^2}{x^2y^2}$

Ans $\frac{x+y}{xy}$

If $a : b :: c : d$ shew that

9*a*. $a + b : b :: c + d : d$ (Componendo)

9*b*. $a-b : b :: c-d : d$ (Dividendo)

9*c*. $a : a-b :: c : c-d$ (Convertendo)

9*d*. $a : c :: b : d$ (Alternendo)

9*e*. $b : a :: d : c$ (Invertendo)

9*f*. $a+b : a-b :: c+d : c-d$ (Componendo and Dividendo)

10. $a+c : c :: b+d : d$

11. $a+b : a :: c+d : c$

12. $a^2 : b^2 :: a^2+c^2 : b^2+d^2$

13. $md+nb : b :: mc+nd : d$

14. $a \pm \frac{c}{m} : a :: b \pm \frac{d}{m} : b$

15. $a^2+b^2 : a^2-b^2 :: c^2+d^2 : c^2-d^2$

16. $ma+nc : pa+qc :: mb+nd : pb+qd$

17. $a+mc : b+md :: a : b$

18. $ma+b : mb+a :: mc+d : md+c$

19. $a+b+c+d : b+d :: c+d : d$

20. $(a \pm b)^2 : ab :: (c \pm d)^2 : cd$

21. $a(a+c) : c^2 :: b(b+d) : d^2$

22. $a : c :: (a^3+b^3)^{\frac{1}{3}} : (c^3+d^3)^{\frac{1}{3}}$

23. $(a+b+c+d)(a+d-b-c)=(a+c-b-d)(a+b-c-d)$

If $x : y :: y : z$ Shew that

24. $xz=y^2$; $x^2-y^2 : x :: y^2-z^2 : z$

$x : z :: x^2 : y^2$

25. $7x+9y : 7y+9z :: 7x-9y : 7y-9z$

26. $x+2y+z : x^2 :: (y+z)^2 : z$

27. $x^{4p}+y^{4p}+z^{4p}=(x^{2p}-y^{2p}+z^{2p})(x^{2p}+y^{2p}+z^{2p})$

28. $y^4(x^{-2} - y^{-2} + z^{-2}) = x^2 - y^2 + z^2$; $y^{-2}(x + y + z) = x^{-1} + y^{-1} + z^{-1}$; $y^{-4}(x^2 + y^2 + z^2) = x^{-2} + y^{-2} + z^{-2}$; $y^{-2c}(x^c + y^c + z^c) = x^{-c} + y^{-c} + z^{-c}$

29. $\frac{x^2 + y^2}{xy + yz} = \frac{xy + yz}{y^2 + z^2}$

30. If $x : y :: z : w$ and $m : n = p : q$ shew that $mx : ny :: pz : qw$

31. $nx . my :: qz : pw$

32. If $wx = yz$ and $py = qw$, shew that $x : q :: z : p$

33. If $\frac{a}{b} = \frac{c}{d}$ prove that $(a - b)^{\frac{1}{2}} : (c - d)^{\frac{1}{2}} :: a^{\frac{1}{2}} - b^{\frac{1}{2}} : c^{\frac{1}{2}} - d^{\frac{1}{2}} :: a^{\frac{1}{2}} + b^{\frac{1}{2}} : c^{\frac{1}{2}} + d^{\frac{1}{2}}$

If $x : y = z : w = r : s$ then

34. $x^2 : y^2 :: rz : sw$

35. $z^2 : w^2 = x^2 + z^2 + r^2 : y^2 + w^2 + s^2$

36. $z^2 + w^2 : r^2 + s^2 :: wz : rs$

37. $x - mz + r : y - nw + s :: mx - nz + qr : mz + nw : x : z$

38. If $m : n :: a^{\frac{1}{2}} : b^{\frac{1}{2}}$ and $m^2 : n^2 = r^2 + x^2 : r^2 - y^2$ prove that $r^2(a - b) = ab(a + b)$

39. a_1, a_2, a_3, a_4, a_5, be in continued proportion then $a_1 : a_5 = a_1^4 : a_2^4$

40. If $2a + 3b : 4a + 5b :: 2x + 3y : 4x + 5y$ then $a : b = x : y$

41. There are 3 numbers in continued proportion, the middle number is 8 and the sum of the others is 20; find the numbers.

Ans. 4, 8, 16

42. If $x : y = z : w$ and x the greatest of the four, then $x + w > y + z$ and $x^p + w^p > y^p + z^p$

43. If $x : y = z : w$ shew that $(x - mz)^2 : (y - mw)^2 :: x^2 \pm z^2 : y^2 \pm w^2$

44. If $\frac{a+c}{b} = \frac{c}{a} = \frac{a}{c-b}$ determine the ratios $a : b : c$

Ans. $2 : 3 : 4$

If $x : y :: z : w$ prove that

45.✓ $x(x-z-y+w)=(x-y)(x-z)$; $(x+y)-(z+w)=\frac{(x+y)(y-w)}{y}$

46.✓ $\frac{x^3-5x^2y-5xy^2+y^3}{x^3+y^3}=\frac{z^3-5wz^2-5zw^2+w^3}{z^3+w^3}$

47.✓ $\frac{x+w}{xw}=\frac{1}{y}+\frac{1}{z}+\frac{(x-y)(x-z)}{xyz}$

48.✓ $mx \pm ny : px \pm qy :: mz \pm nw : pz \pm qw$

49.✓ $\frac{(x^2-z^2)(z^2-w^2)}{(x^2-y^2)(y^2-w^2)}=\frac{z^2}{y^2}$; $x(x+y+s+w)=(x+y)(x+z)$

50.✓ $\frac{x^2-xy+y^2}{x^2}=\frac{z^2-wz+w^2}{z^2}$

51.✓ If $4(x+y)(z+w)=yw\left(\frac{x+y}{y}+\frac{z+w}{w}\right)^2$

52. $\frac{1}{mx}+\frac{1}{ny}=\frac{1}{yz}\left(\frac{x}{q}+\frac{y}{p}+\frac{z}{n}+\frac{w}{m}\right)-\frac{1}{pz}-\frac{1}{qw}$

53. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ prove that each of these ratios $=\frac{x^2+y^2+z^2}{ax+by+cz}$

54. If $x : y :: z : w :: m : n$ shew that $(xz+mz+mx)(y^2+w^2+n^2)$
$=(yw+nw+ny)(x^2+z^2+m^2)$

If $x : y = m : n$ prove that.

55. $(x+m)(x^2+m^2)(y-n)(y^2-n^2)=(x-m)(x^2-m^2)(y+n)(y^2+n^2)$

56. $(px^2+qxy+ry^2)(lm^2+smn+tu^2)$
$=(pm^2+qmn+rn^2)(lx^2+sxy+ty^2)$

57. $nx\left(\frac{1}{x}-\frac{1}{2y}-\frac{1}{3m}+\frac{1}{4n}\right) \qquad =\frac{x}{4}-\frac{y}{3}-\frac{m}{2}+n$

58. $mx^2+m^2x : y^2n+yn^2 :: (x+m)^3 : (y+n)^3$

59. $\frac{x+y}{xy} : \frac{m+n}{mn} :: \frac{y-x}{xy} : \frac{n-m}{mn}$

60. If $\frac{b}{a+b}=\frac{a+c-b}{b+c-a}=\frac{a+b+c}{2a+b+2c}$ find the ratio between a, b and c. Ans 2, 3, 4

61. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ prove that $\frac{a^{3n}-c^{3n}}{b^{3n}-d^{3n}}=\frac{a^nc^ne^n-(a^n-c^n+e^n)^3}{b^nd^nf^n-(b^n-d^n+f^n)^3}$

62. Find 3 numbers in continued proportion such that their sum may be 14 and the sum of their squares 84 Ans 2, 4, 8

63. If $a : b=b : c$ shew that $a-2b+c=\frac{(a-b)^2}{a}=\frac{(b-c)^2}{c}$

64. If $a : b=b : c$ shew that $\frac{a+b+c}{a-b+c}=\frac{(a+b+c)^2}{a^2+b^2+c^2}$

65. $2a : b :: b : 2c$ shew $a : c :: 4a^2 : b^2$

66. If $a+x : a-x :: 11 : 7$ find the value of $a : x$ Ans $9 : 2$

67. Find two numbers in the ratio of $2 : 3$ that thier sum : their product : : 5 : 12 Ans 4 and 6

68. If $a : b : : b : c$ and $b : c :: c : d$ shew that $a+b : b+c : : b+c : c+d$

69. Shew that $a : b$ is duplicate of the ratio $a+c : b+c$ if c be a mean proportional between a and b

70. Divide the number n into 2 parts so that one shall be to the other in the ratio of $n : 1$.

Ans $\frac{n^2}{n+1}$, $\frac{n}{n+1}$

71. Find the number to which if one and three be successively added the resulting numbers are in the proportion of 2 : 7 Ans $-\frac{1}{5}$

72. If 4 quantities are proportionals and the 2nd is a mean proportional between the 3rd and 4th, the 3rd will be a mean proportional between the 1st and 2nd

If $a : b = c : d = e : g$ shew that

73. $a - e : b - g :: e : d$; $a : b = \sqrt[n]{(p^n a^n + q^n c^n + r^n e^n)} : \sqrt[n]{(p^n b^n + q^n d^n + r^n d^n)}$

74. $ma + nc + re : mb + nd + rg = pa + qc + le : pb + qd + lg$

75. $b^2 : d^2 :: (a + mb)^2 : (c + md)^2 :: a^2 - b^2 : c^2 - d^2$

76. If $a^2 + c^2 : ab + cd = ab + cd : b^2 + d^2$ prove that $a : b = c : d$

77. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ prove that $\frac{a}{d} = \frac{a^3}{b^3}$

78. If $m : n : p : q$: prove that $\frac{(m - n)(m - p)}{m} = m - n + q - p$

79. If the duplicate ratio of $a + c$ to $b + c$ be as a to b and if a and b are unequal prove that $c^2 = ab$.

80. If $x^2 : y^2 = m : n$ prove thet $(x^4 + y^4)\frac{m^3}{m + n} = \frac{x^6}{x^2 + y^2}(m^2 + n^2)$

81. If $\frac{7x - 6y}{8a + 9b} = \frac{7y - 6z}{8b + 9c} = \frac{7z - 6x}{8c + 9a}$ prove that $(172a + 337b + 239c)$

$(x + y + z) = 17(a + b + c)(95x + 59y - 110z)$

81. If $\frac{3x + 3y - z}{5a + b - c} = \frac{3z + 3x - y}{5c + a - b} = \frac{3y + 3z - x}{5b + c - a}$ prove that $\frac{x + y + z}{a + b + c}$

$= \frac{6x + 10y + 14z}{4a + 14b + 12c}$

83. If $x + z = 2y$; $x + y = 2z$; $y + z = 2x$ prove that $x = y = z$

84. If $\frac{2x - y}{2m + p} = \frac{2y - z}{2n + p} = \frac{2z - x}{2p + m}$ shew that $\frac{21(x + 2y + 3z)}{x + y + z}$

$= \frac{41m + 38n + 47p}{m + n + p}$

85. If $\frac{1+xy}{x-y}=\frac{1+wz}{w-z}$ prove that $\frac{x-w}{y-z}=\frac{1+wx}{1+yz}$

86. If $\frac{x+2y}{2a+b}=\frac{2y+z}{2b+c}=\frac{x+z}{2c+a}$ preve that $2(x+2y+z)(9a+16b+17c)$

$$=3(a+b+c)(7x+18y+12z)$$

If $a : b=c : d=e : f$ shew that

87. $a^2-ab+b^2 : ab+ad-bc :: c^2-cd+d^2 : cd-ad+bc$

88. $\frac{a}{b}=\frac{ma-ne}{mb-nf}=\frac{a-c-e}{b-d-f}=\frac{ma-nc-pe}{mb-nd-pf}=\frac{m(a+e)-nc}{m(b+f)-nd}$.

89. $a^{\frac{3}{2}}-\sqrt{(ace)}+e^{\frac{3}{2}} : b^{\frac{3}{2}}-\sqrt{(bdf)}+f^{\frac{3}{2}} :: (\sqrt{a}+\sqrt{c}+\sqrt{e})^3$

$: (\sqrt{b}+\sqrt{d}+\sqrt{f})^3$

90. $a+b : c+d :: a^2(c-d) : c^2(a-b)$

91. If $\frac{a}{b}=\frac{c}{d}=\frac{m}{n}$ prove that $\sqrt{(ab)}+\sqrt{(cd)}+\sqrt{(mn)}$

$=\sqrt{\{(a+c+m)(b+d+n)\}}$ and $(a^2+c^2+m^2)(b^2+d^2+n^2)$

$$=(ab+cd+mn)^2$$

92. If $\frac{ax+by}{cz}=\frac{cz+ax}{by}=\frac{by+cz}{ax}=x+y+z$ find x, y, z

$$x=\frac{2bc}{bc+ac+ab},\ y=\frac{2ac}{bc+ac+ab},\ z=\frac{2ab}{bc+ac+ab}$$

93. Before noon, a clock which is too fast, and points to afternoon time, is put back 5 hrs 40 min ; and it is observed that the time before shewn is to the true time as 29 : 105. Required the true time.

Ans 8 hrs 45 min.

94. A gentleman sitting in a railway saloon observes that another train running on a parallel line in the opposite direction occupies two seconds in passing him, but if the two trains had been proceeding in the same direction, it would have taken 30 seconds to pass him ; if the

rate of the faster train be 24 miles an hour, find the rate of the other and the length of the quicker train.

Ans 21 miles an hour and length 132 ft.

95. There are four towns situated in the order of the four letters A, B, C, D. The distance from A to D is 34 miles, the distance from A to B : distance from C to D : : 2 : 3, and $\frac{1}{4}$ of the distance from A to B added to half the distance from C to D is three times the distance from B to C; what are the respective distances.

Ans AB = 12, BC = 4, CD = 18

96. A was following B, and after a time B turned and without changing his pace walked in the oppostte direction and in consequence A approached B 3 times as fast as before. Compare the rates of A and B, Ans 2 : 1,

27. Two locomotive engines A and B set out to meet each other, A leaving the station Howrah at the same time that B left Raneegunge, and on meeting, it appeared that A had run 18 miles more than B; and that A could have run B's distance in $15\frac{3}{4}$ hours, but B would have been 28 hours in performing A's journey. What was the distance between Howrah and Raneegnuge and the distance run by each engine. Ans distance 126 miles; A 72, B 54 miles

98. Two pieces of long-cloth of equal goodness but of different lengths, were bought, the one for 5 Rs, the other for 6 Rs 8as; now if the lengths of both pieces were increased by 10 yds, the numbers resulting would be in the proportion of 5 to 6. How long was each piece and how much did they cost a yd.

Ans 20 and 26 yds; 4as

Exercise 28.

IDENTITIES.

Shew that

1. $(x^2+y^2)(a^2+b^2)=(ax+by)^2+(bx-ay)^2$

2. $(x+y)^2+(y+z)^2+(x+z)^2=(x+y+z)^2+x^2+y^2+z^2$

3 $x(1-y^2)(1-z^2)+y(1-z^2)(1-x^2)+z(1-x^2)(1-y^2)-4xyz$

$=(x+y+z-xyz)(1-xy-yz-xz)$

4. $(x+y+z)^3=x^3+y^3+z^3+3(x+y)(x+z)(y+z)=x^3+y^3+z^3$

$+3x^2(y+z)+3y^2(x+z)+3z^2(x+y)+6xyz$

5. $$\frac{x^4(y^2-z^2)+y^4(z^2-x^2)+z^4(x^2-y^2)}{x^2(y-z)+y^2(z-x)+z^2(x-y)}=(x+y)(y+z)(x+z)$$

(L. C. E. 1881)

6. $(a-b)(x-a)(x-b)+(b-c)(x-b)(x-c)+(c-a)(x-c)(x-a)$

$=(a-b)(a-c)(b-c)$

7. Shew that the difference of the squares of any two consecutive numbers is equal to the sum of the numbers.

8. $$(1+\frac{x}{y})(1+\frac{y}{z})(1+\frac{z}{x})+1=(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

9. $$\frac{x^2\left(\frac{1}{y}-\frac{1}{z}\right)+y^2\left(\frac{1}{z}-\frac{1}{x}\right)+z^2\left(\frac{1}{x}-\frac{1}{y}\right)}{x\left(\frac{1}{y}-\frac{1}{z}\right)+y\left(\frac{1}{z}-\frac{1}{x}\right)+z\left(\frac{1}{x}-\frac{1}{y}\right)}=x+y+z$$ (F. A. 1881)

10. $(a+b+c)^2+(a+b+c)(a-b-c)-2ab-2ac=2a^2$

11. $(a+b+c)^3-(b+c-a)^3-(a-b+c)^3-(a+b-c)^3=24abc$

12. $(x+y+z)^2+(x+y-z)^2+(x-y+z)^2+(y+z-x)^2$

$=4(x^2+y^2+z^2)$

13. $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$

14. $(m^2-np)^3+(n^2-mp)^3+(p^2-mn)^3=(m^3+n^3+p^3-3mnp)^2$
$+3(m^2-np)(n^2-mp)(p^2-mn)$

15. $(x+y)^2+(x+z)^2+(x+w)^2+(y+z)^2+(y+w)^2+(z+w)^2$
$=(x+y+z+w)^2+2(x^2+y^2+z^2+w^2)$

16. $\{(4x+3y)^2+(6x-2y)^2\}\{(4x+3y)^2-(6x-2y)^2\}$
$=13(4x^2+y^2)(5y^2-20x^2+48xy)$

17. $(x+y)(x-y)-\{x+y-z-(y-x-z)+(y+z-x)\}(x-y-z)$
$=z^2+2yz$

18. $(a+b)^2+(a+c)^2-(c+d)^2-(b+d)^2=2(a-d)(a-b+c+d)$

19. Shew that the continued product of any four consecutive numbers together with unity is a square number.

20. $(m+n+p)^4+m^4+n^4+p^4=12mnp(m+n+p)+(n+p)^4+(m+p)^4$
$+(m+n)^4$

21. $\left(\frac{a}{b}+\frac{b}{a}\right)^2+\left(\frac{a}{c}+\frac{c}{a}\right)^2+\left(\frac{b}{c}+\frac{c}{b}\right)^2=4+\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{b}{c}+\frac{c}{b}\right)$

22. $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}=(x+y)(y+z)(x+z)$

23. $(x-y)^4+(z-x)^4+(y-z)^4=2\{(x-y)^2(y-z)^2+(x-y)^2(z-x)^2$
$+(y-z)^2(z-x)^2\}$

24. $\{(m-n)^2+(n-p)^2+(p-m)^2\}^2=2\{(m-n)^4+(n-p)^4+(p-m)^4\}$

25. $(x+y)^7-(x^7+y^7)=7xy(x+y)(x^2+xy+y^2)^2$

26. $\left(1-\frac{z}{x}\right)^{-1}\left(1-\frac{y}{x}\right)^{-1}+\left(1-\frac{x}{y}\right)^{-1}.\left(1-\frac{z}{y}\right)^{-1}+\left(1-\frac{y}{z}\right)^{-1}\left(1-\frac{x}{z}\right)^{-1}=1$

27. Shew that the product of any four consecutive integers increased by 16 is a perfect square.

28. $\frac{2^{3x+1}+2\times 8^x-16^{\frac{3x}{4}}}{4^{\frac{3x}{2}}+32^{\frac{3x}{5}}+2^{3x}}=1$

29. $(x+y+z)(x^3+y^3+z^3+xyz)=x^4+y^4+z^4+(xy+yz+xz)$
$(x^2+y^2+z^2)$

30. $(my - nx)^2 + (nx - mz)^2 + (nz - py)^2 + (mx + ny + pz)^2$
$= (m^2 + n^2 + p^2)(x^2 + y^2 + z^2)$

31. $(y - z)^{-2} + (z - x)^{-2} + (x - y)^{-2} = \{(y - z)^{-1} + (z - x)^{-1} + (x - y)^{-1}\}^2$

32. $\frac{25}{21}\{(y - z)^7 + (z - x)^7 + (x - y)^7\}$
$= \frac{\{(y - z)^5 + (z - x)^5 + (x - y)^5\}^2}{(y - z)^3 + (z + x)^3 + (x - y)^3}$

33. $(1 - x)(1 + x)(1 + x^2)(1 + x^4)$ &c. to $n + 1$ factors $= 1 - x^{2^n}$

34. $\{(mp + nq)x + (mq - np)y\}^2 - \{(mp + nq)x - (mq - np)y\}^2$
$= (m^2 + n^2)(p^2 + q^2)(x^2 + y^2)$

35: $x^4 + y^4 + z^4 - \{(x^2 - z^2)^2 + (y^2 - z^2)^2 + (x^2 - y^2)^2\} = (x + y + z)$
$(x + y - z)(x - y + z)(y + z - x)$

Exercise 29.

CONDITIONAL IDENTITIES.

If $x + y + z = 0$ prove that

1. $x^3 + y^3 + z^3 = 3xyz$

2. $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy} = 3$

3. $x.\frac{y^3 - z^3}{y - z} + y.\frac{z^3 - x^3}{z - x} + z.\frac{x^3 - y^3}{x - y} = 0$

4. $x(x^2 - yz) + y(y^2 - xz) + z(z^2 - xy) = 0$

5. $x^2 - yz = y^2 - xz = z^2 - xy$

6. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$ prove that $(x + y + z)^3 = 27xyz$

7. If $x(y + z)^{-1} + z(x + y)^{-1} = 2y(x + z)^{-1}$ prove that $\frac{1}{2}(x^2 + z^2)$
$= y^2$ and $x + y = -z$

8. If $a=2c-b$ prove that $\frac{a^2(b+c)+b^2(a+c)+c^2(a+b)}{c^3}=6$

9. If $x+\frac{1}{x}=p+\frac{1}{p}$ prove that $x^3+\frac{1}{x^3}=p^3+\frac{1}{p^3}$

10 If $x(y^2-z^2)=\frac{y^{-2}}{x^{-2}}y(x^2-z^2)$, shew that $x^2=(y+z)(y-z)$

11. If $\frac{y^{-2}}{x-z}+\frac{x^{-2}}{y-z}=\frac{(xy)^{-1}}{(x-z)(y-z)}$ prove that $(x+y)z=xy$

$=x^2+y^2$

12. If $\frac{2x}{y-x}+\frac{2z}{y-z}=0$ prove that $x^{-1}+z^{-1}=2y^{-1}$

13. If $\frac{x+2a}{x-b}+\{2(a+b)-x\}(x-a)^{-1}=2$ prove that $x=2(a+b)$

14. If $x^2+z^2=2y^2$ prove that $(y+z)^{-1}+(x+y)^{-1}=2(x+z)^{-1}$

15. If $x^y\,y^x=1$ shew that $\frac{x}{y}=y^{-\left(\frac{x}{y}+1\right)}$

16. If $x^2+y^2=z^2+w^2=1$ shew that $\frac{xw+yz}{x+z}=\frac{x-z}{xw-yz}$

and $(xz-yw)^2+(xw+yz)^2=1$

17. If $2b=a+c$ prove that $\frac{2}{9}(a+b+c)^3=a^2(b+c)+b^2(a+c)$

$+c^2(a+b)$

18. If $2xy=\sqrt{(x+y+z)(x+y-z)(x+z-y)\,(y+z-x)}$ shew that

$z^2=x^2+y^2$

19. If $c=a(1-b^2)^{\frac{1}{2}}+b\,(1-a^2)^{\frac{1}{2}}$, shew that $(a+b+c)(a+b-c)$

$(a-b+c)(b+c-a)=4a^2b^2c^2$

20. If $a^2-bc=x^2$, $b^2-ac=y^2$, and $c^2-ab=z^2$; shew that

$ax^2+by^2+cz^2=(a+b+c)(x^2+y^2+z^2)$,

21. If $n^2p^2x=m^2(y-z)$, $m^2p^2y=n^2(z-x)$ and $m^2n^2z=p^2(x-y)$,

shew that $n^4p^4+m^4p^4+m^4n^4+m^4n^4p^4=0$

22. If $\frac{my+nx}{m}=\frac{ny-mx}{n}=1$ shew that $x^2+y^2=1$

23. If $x^2(x^2-2m^2)=(y^2+z^2)^2$, $y^2(y^2-2m^2)=(z^2+x^2)^2$ prove that

$$x^2+y^2+z^2=m^2$$

24. If $(m^2-np)(n^2-mp)(p^2-mn)=0$ prove that $m^{-3}+n^{-3}+p^{-3}$

$$=(m^3+n^3+p^3)(mnp)^{-2}$$

If $x=a+b+c$ prove that

25. $x(x-2b)(x-2c)+x(x-2c)(x-2a)+x(x-2a)(x-2b)$

$$=(x-2a)(x-2b)(x-2c)+8abc$$

26. $a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2=\frac{x(x-2c)(x-2b)(x-2a)}{4b^2}$

27. $x(x-a)(x-b)+c(x+a)(x+b)+x(x-a)(x-c)+x(x+a)(x-c)$

$$=(x+a)(x+b)(x+c)$$

28. $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$

$$=2(a^3+b^3+c^3-3abc)$$

29. $x^3=a^3+b^3+c^3+3a^2(x-a)+3b^2(x-b)+3c^2(x-c)+6abc$

If $2x=a+b+c$ shew that

30. $(x-a)^2+(x-b)^2+(x-c)^2+x^2=a^2+b^2+c^2$

31. $(x-a)(x-b)(x-c)=x^3-\frac{x}{2}(a^2+b^2+c^2)-abc$

32. $(x-a)^{-1}+(x-b)^{-1}+(x-c)^{-1}-x^{-1}$

$$=abcx^{-1}.(x-a)^{-1}.(x-b)^{-1}.(x-c)^{-1}$$

33. $2(x-a)(x-b)+2(x-b)(x-c)+2(x-c)(x-a)+a^2+b^2+c^2$

$$=2x^2$$

34. $2(x-a)(x-b)(x-c)+a(x-b)(x-c)+b(x-c)(x-a)$

$$+c(x-a)(x-b)-abc=0$$

35. $16x(x-a)(x-b)(x-c)=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$

36. $(2x-b)(x-c)+(2x-c)(x-a)+(2x-a)(x-b)=ab+ac+bc$

37. If $x=a+b+c+\dots\dots$to r terms shew that $\frac{x-a}{x}+\frac{x-b}{x}+\frac{x-c}{x}$
$+\dots\dots=r-1$

38. If $p_1+p_2+p_3+\dots\dots\dots p_r=\frac{rx}{3}$, $p_1^2+p_2^2+p_3^2+\dots p_r^2=\frac{r}{3}x$
and $p_1^3+p_2^3+p_3^3+\dots\dots p_r^3=0$
prove that $(x-p_1)^3+(x-p_2)^3+(x-p_3)^3+\dots(x-p_r)^3=r$

39. If $2x=a+b+c$ and $2s^2=a^2+b^2+c^2$ shew that,
$(s^2-a^2)(s^2-b^2)+(s^2-b^2)(s^2-c^2)+(s^2-c^2)(s^2-a^2)$
$=4x(x-a)(x-b)(x-c)$

40. If $x^{-1}+y^{-1}+z^{-1}=(x+y+z)^{-1}$ shew that $(x^{-1}+y^{-1}+z^{-1})^3$ $=(x^3+y^3+z^3)^{-1}$ and generally $(x^{-1}+y^{-1}+z^{-1})^{2r+1}=(x^{2r+1}+y^{2r+1}$ $+z^{2r+1})^{-1}$

41. If $x^2+y^2=1$ shew that $2(x^6+y^6)-3(x^4+y^4)+1=0$

42. If $m=ax+cy+bz$, $n=cx+by+az$, $p=bx+ay+cz$ shew that $m^3+n^3+p^3-3mnp=(a^3+b^3+c^3-3abc)\,(x^3+y^3+z^3-3xyz)$

43. If $\frac{a+b-c}{a+b}=\frac{b+c-a}{b+c}=\frac{c+a-b}{a+c}$ and that $a+b+c$ is not $=0$
shew that, $a=b=c$

44. If $x^m=m^0$, $x=y^n$, $y=z^p$ shew that $mnp=0$

45. If $\frac{a^2+b^2}{c^3}+\frac{a^2+c^2}{b^3}+\frac{b^2+c^2}{a^3}=\frac{a^3-1}{a}+\frac{b^3-1}{b}+\frac{c^3-1}{c}$ shew that $\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=1$, if $a^2+b^2+c^2$ is not$=0$

46. If $\frac{a+b+c-abc}{2b}=\frac{1-bc-ac-ab}{1-b^2}$ prove that $a+c=b$
$(1-ac)$

47. If $ab+ac+bc=1$ prove that $\frac{a+b}{1-ab}+\frac{b+c}{1-bc}+\frac{a+c}{1-ac}$
$$=\frac{(a+b)(b+c)(a+c)}{(1-ab)(1-bc)(1-ac)}$$

48. If $\frac{a-b}{c}+\frac{b-c}{a}+\frac{a+c}{b}=1$ and $a-b+c$ is not $=0$

then $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$

49 If $x+y+z=0$ shew that $\frac{x^2}{2x^2+yz}+\frac{y^2}{2y^2+xz}+\frac{z^2}{2z^2+xy}=1$

50. If $\frac{a}{b+c}=x, \frac{b}{a+c}=y, \frac{c}{a+b}=z$ shew that

$\frac{a^2}{x(1-yz)}=\frac{b^2}{y(1-xz)}=\frac{c^2}{z(1-xy)}$, and $xy+yz+xz+2xyz=1$

51. If $x^2=ab+ac+bc$ shew that $(x-a)(x-b)(x-c)+b(x-a)$
$(x-c)+c(x-a)(x-b)+a(x-b)(x-c)-2abc=0$

52. If $2x=a+b+c$ shew that $(x-c)(2x-c)+(x-b)(2x-b)$
$+(x-a)(2x-a)=a^2+b^2+c^2$

53. If $(x^2+yz)(y^2+xz\ (z^2+xy\ =0$ prove that $\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{z^3}$
$=\frac{x^3+y^3+z^3}{x^2y^2z^2}$

54. If $2s=a+b+c$ shew that $s^3-(s-a)^3-(s-b)^3-(s-c)^3=3abc$

55. If $x=cy+bz,\ y=az+cx,\ z=bx+ay$ shew that

$\frac{x^2}{1-a^2}=\frac{y^2}{1-b^2}=\frac{z^2}{1-c^2}$

56. If $x^2(y+z)=m^3,\ y^2(x+z)=n^3,\ z^2(x+y)=p^3,\ xyz=mnp$
shew that $m^3+n^3+p^3+mnp=0$

57. If $\left(\frac{x+yz}{y+xz}\right)^2=\frac{1-y^2}{1-x^2}$ prove that $x^2+y^2+z^2+2xyz=1$

(Cambridge examination papers.)

58. If $ab+ac+bc=1$ shew that $\frac{a}{1-a^2}+\frac{b}{1-b^2}+\frac{c}{1-c^2}$
$=\frac{4abc}{(1-a^2)(1-b^2)(1-c^2)}$

59. If $x=ny+pz+qu$, $y=mx+pz+qu$, $z=mx+ny+qu$, $u=mx+ny+pz$ shew that $\frac{m}{1+m}+\frac{n}{1+n}+\frac{p}{1+p}+\frac{q}{1+q}=1$

60. If $xy+yz+xz=1$ prove that $\frac{1+x^2}{(x+y)(x+z)}+\frac{1+y^2}{(y+x)(y+z)}$

$$+\frac{1+z^2}{(x+\ )(y+z)}=3$$

61. If $x+y+z=xyz$ prove that $\frac{3x-x^3}{1-3x^2}+\frac{3y-y^3}{1-3y^2}+\frac{3z-z^3}{1-3z^2}$

$$=\frac{(3x-x^3)(3y-y^3)(3z-z^3)}{(1-3x^2)(1-3y^2)(1-3z^2)}$$

62. If $c=\frac{2ab}{a+b}$ shew that $\frac{1}{c-a}+\frac{1}{c-b}=\frac{1}{a}+\frac{1}{b}$

63. If $(a^2+bc)^2(b^2+ac)^2(c^2+ab)^2=(a^2-bc)^2(b^2-ac)^2(c^2-ab)^2$ prove that either $a^3+b^3+c^3+abc=0$ or $a^{-3}+b^{-3}+c^{-3}+a^{-1}b^{-1}c^{-1}=0$

64. If $c^2=a^2+b^2$ shew that $(a+b+c)(a+b-c)(a+c-b)$

$$(b+c-a)=4a^2b^2$$

65. $2y+b^{-x}=b^x$ prove that $b^x=y+\sqrt{(1+y^2)}$

66. If $x+y+z=2a$, and $x^2+xy+y^2+a^2=2a(x+y)$ shew that $(x-a)^2+(y-a)^2+(z-a)^2=a^2$

67. $\sqrt[4]{x}+\sqrt[4]{y}+\sqrt[4]{z}=0$ Shew that

$$\{x^2+y^2+z^2-2(yz+xz+xy)\}^2=128\,xyz(x+y+z)$$

68. If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$ shew that

(1) $(b-c)x+(c-a)y+(a-b)z=0$

(2) $(a+b+c)\{x(y+z)+y(b+x)+z(x+y)\}$
$=2(x+y+z)(ax+by+cz)$

69. If $b^2=ac$ prove that $a^2b^2c^2(a^{-3}+b^{-3}+c^{-3})=a^3+b^3+c^3$

70. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ Shew that $\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}+\frac{z^2+c^2}{z+c}$

$$=\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}$$ (Cambridge papers.)

71. If $\frac{a+b}{a-b}=\frac{b+c}{2(b-c)}=\frac{a+c}{3(c-a)}$ shew that

$8a+9b+5c=0$

72. If $\frac{x+y}{3a-b}=\frac{y+z}{3b-c}=\frac{x+z}{3c-a}$ shew that

$(x+y+z)(a^2+b^2+c^2)=(ax+by+cz)(a+b+c)$

73. If $2x=a^2-b^2-c^2+d^2$ prove that $(ad+bc)^2-x^2$
$=\frac{1}{4}(a+b+c-d)(a+b+d-c)(a+c+d-b)(b+c+d-a)$

74. If $x+y+z=\frac{14x}{3}=\frac{7y}{2}$ prove that $x+y=z$

75. If $x-y=7z$ and $x-z=4y$ shew that $x=9(y-z)$

76. If $x^2+y^2=123z$ and $x^2-y^2=27z$ shew that $xy=60z$

77. If $ab=\frac{1}{2}(a+b)(p+q)-pq$ and $cd=\frac{1}{2}(c+d)(p+q)-pq$
shew that $\left(\frac{p-q}{2}\right)^2=\frac{(a-c)(a-d)(b-c)(b-d}{(a+b-c-d)^2}$

78. If $ax^3=by^3=cz^3$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{d}$ shew that $ax^2+by^2+cz^2$
$=(a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}})^3d^2$

79. If $2x=a+b+c$ shew that $(x-a)^3+(x-b)^3+(x-c)^3$
$-3(x-a)(x-b)(x-c)=\frac{1}{4}(a^3+b^3+c^3-3abc)$

80. If $x+y=2z$, $x+z=2y$ and $y+z=2x$ prove that $x=y=z$

81. If $x+y+z=xyz$ prove that $\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}$
$=\frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}$

82. If $\frac{x^2-yz}{x-xyz}=\frac{y^2-xz}{y-xyz}$ prove that each of these ratios

$$=\frac{z^2-xy}{z-xyz}=x+y+z=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$ (Cambridge papers.)

83. If $x^2+bc=a^2$, $y^2+ac=b^2$, $z^2+ab=c^2$ shew that

$$(a+b+c)(x^2+y^2+z^2)=ax^2+by^2+cz^2$$

84. If $\frac{cy+bz}{b-c}=\frac{az+cx}{c-a}=\frac{bx+ay}{a-b}$ shew that

$$(a+b+c)(x+y+z)=ax+by+cz$$

85. If $a+b+c=0$ shew that $c(a^2+b^2-c^2)=b(a^2+c^2-b^2)$

$$=a(b^2+c^2-a^2)$$

86. If $x=\frac{2ab}{b^2+1}$ shew that $\frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}=b$

87. If $a+b+c=0$ shew that $(a^2+b^2+c^2)^2=2(a^4+b^4+c^4)$

$$=4(a^2b^2+b^2c^2+c^2a^2)$$

88. If $a=b^x$, $b=c^y$ and $c=a^z$ shew that $xyz=1$

89. If $x=yz$, $y=xz$ and $z=xy$ prove that $xyz=1$

90. If $\frac{bx^2+a^2y}{a}=\frac{ay^2+b^2x}{b}$ shew that $bx+ay=ab$ or $ay=bx$

Exercise 30.

INEQUALITIES.

1. If x and y are any two positive integers prove that $x^2+y^2 > 2xy$.

2. If x, y, z be such that any two of them are greater than the third, prove that $2xy+2xz+2yz-x^2-y^2-z^2 > 0$.

3. Also $2x(y+z)+2y(x+z)+2z(x+y) > (x+y+z)^2$.

4. Prove that $(x+y+z)^3 > 27xyz$ unless $x=y=z$.

5. Shew that $a^2+b^2+c^2 > ab+ac+bc$ unless $a=b=c$.

6 Shew that $9(x^3+y^3+z^3) > (x+y+z)^3$.

7. Shew that $\sqrt[3]{x} > \sqrt[4]{(x+1)}$ for all values of x not less than 3.

8. Prove that $x^4+y^4+z^4 > xyz(x+y+z)$.

9 Prove that $x^3+y^3 > x^2y+y^2x$.

10. Given that $\frac{x+2}{4}+\frac{x}{3} < \frac{x+2}{2}$ and $> \frac{3x+5}{6}$, find a whole number for the value of x Ans $x=5$

11. Which is greater $2(y+1)^3$ or $y+2$

12. Shew that $abc > (a+b-c)\ (a+c-b)\ (b+c-a)$ unless $a=b=c$

13 What is the integral value of x when $6x-7 < 4x+3$, and $3x+1 > 13-x$ Ans $x=4$

14 If $x^2=a^2+b^2$, and $y^2=c^2+d^2$, shew that $xy > ac+bd$ unless $ad=bc$.

15. Prove that $n^3+1 > n^2+n$ unless $n=1$

16. Given that $\frac{x+2}{4}+\frac{x}{3} < \frac{x-4}{2}+3$, and $> \frac{x+1}{2}+\frac{1}{3}$, find x

$x=5$

17. Prove that $xy(x+y)+yz(y+z)+xz(x+z) > 6xyz$

18. Shew that $\frac{x^2-x+1}{x^2+x+1}$ lies between 3 and $\frac{1}{3}$ for all real values of x.

19. If x be greater than y prove that $x^n-y^n < nx^{n-1}(x-y)$, and $> ny^{n-1}(x-y)$

20. Shew that $(x^4+y^4+1)(z^4+w^4+1) > (x^2z^2+y^2w^2+1)^2$

21. Prove that $x^6+x^4y^2+x^2y^4+y^6 > (x^3+y^3)^2$

22. Which is greater $\left(\frac{1}{2}\right)^{\frac{1}{2}}$ or $\left(\frac{2}{3}\right)^{\frac{2}{3}}$ Ans $\left(\frac{2}{3}\right)^{\frac{2}{3}}$

23. Which is greater $a - b$ or $(\sqrt{a} - \sqrt{b})^2$ if $a > b$ Ans $a - b$

24. If $x > 3$ prove that $x^{\frac{1}{2}} > (x+1)^{\frac{1}{3}}$.

25. The double of a certain number increased by 7 is not greater than 19, and its triple diminished by 5 is not less than 13, What is the number ? Ans 6

26. Shew that $2(x^4 + x^2 + 1) > 3x(1 + x^2)$ unless $x = 1$

27. Shew that $a + b + c + d > 4\sqrt[4]{(abcd)}$ unless $a = b = c = d$.

Exercise 31.

ELIMINATIONS.

1. Eliminate x and y from the equations $x + y = m$, $x^2 + y^2 = n$, $x^3 + y^3 = p$. Ans $2p = 3mn - m^3$

2. Eliminate m and n from the equations $a + m + n = x$, $am + an + mn = y$, $amn = z$ Ans $a^3 - a^2x + ay = z$

3. Eliminate x and y from the equations $(ax + by)^2 = c^2x^2 + d^2y^2$ and $(ay - bx)^2 = c^2y^2 + d^2x^2$ Ans $a^2 + b^2 = c^2 + d^2$

4. Eliminate x and y from the equations $x - y = a$, $x^3 - y^3 = b$, $3xy = a^2$ Ans $b = 2a^3$

5. Eliminate x and y from the equations $x^3 + y^3 = \frac{m}{2}$, $(x + y)^3 = m$, $3xy = n$. Ans $m^2 = 8n^3$

6. Eliminate x and y from the equations $y^2 - ay = x^2 - bx$, $4xy = ax + by$, $x^2 + y^2 = 1$. Ans $(a+b)^{\frac{2}{3}} + (a-b)^{\frac{2}{3}} = 2$

7. Eliminate x, y and z from the equations $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = m$, $\frac{x}{z} + \frac{y}{x} + \frac{z}{y} = n$, $\left(\frac{x}{y} + \frac{y}{z}\right)\left(\frac{y}{z} + \frac{z}{x}\right)\left(\frac{z}{x} + \frac{x}{y}\right) = p$ Ans $mn = 1 + p$

8. Eliminate x and y from the equations $x^2+xy+y^2=a$, $x+y=b$, $xy=a^2$ Ans $a^2+a=b^2$

9. Eliminate a, b, c from the equations $a+b+c=m$, $a^2+b^2+c^2=n$, $ab+ac+bc=p$ Ans $n+2p=m^2$

10. Eliminate x and y from the equations $\frac{x^2-a^2}{y^2-b^2}=\frac{2a+3b}{3a+2b}$,

$x^2-y^2=(a-b)^2$, $x^{\frac{3}{2}}+y^{\frac{3}{2}}=c^{\frac{3}{2}}$ Ans $a^{\frac{1}{2}}+b^{\frac{1}{2}}=c^{\frac{1}{2}}$

11. Eliminate x, y, z from the equations

$$(x-y)(y-z)(z-x)=a^3$$
$$(x+y)(y+z)(z+x)=b^3$$
$$(x^2+y^2)(y^2+z^2)(z^2+x^2)=\frac{a^6}{2}$$
$$(x^4+y^4)(y^4+z^4)(z^4+x^4)=\frac{a^6b^6}{2}$$

Ans $2a^6=b^6$

12. Eliminate m, n, p, q from the equations

$$\frac{x-p}{m}+\frac{y+q}{n}=\frac{pm}{a}+\frac{qn}{b^2}=\frac{m^2}{a^2}-\frac{n^2}{b^2}=\frac{p^2}{a^2}+\frac{q^2}{b^2}-1=0$$

Ans $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$

13. Eliminate b and c from the equations

$a+b+c+p=0$, $a(b+c)=q-bc$, $abc+r=0$

Ans $a^3+pa^2+qa+r=0$

14. Eliminate m, n, p from the equations

$$\left(\frac{x}{m}\right)^a+\left(\frac{y}{n}\right)^a+\left(\frac{z}{p}\right)^a=1=\left(\frac{m}{q}\right)^b+\left(\frac{n}{q}\right)^b+\left(\frac{p}{q}\right)^b \text{ and}$$

$$\frac{x^a}{m^{a+b}}=\frac{y^a}{n^{a+b}}=\frac{z^a}{p^{a+b}}$$

Ans $x^{\frac{ab}{a+b}}+y^{\frac{ab}{a+b}}+z^{\frac{ab}{a+b}}=q^{\frac{ab}{a+b}}$

Exercise 32.

APPLICATION OF ALGEBRA TO GEOMETRY.

1. One side of a right angled triangle is 8 and the hypotenuse is 10 : find the other side. Ans 6

2. One side of a right angled triangle is 5 and the difference between the hypotenuse and the other side $=1$; find the hypotenuse and the other side. Ans 13. 12.

3. One side of a right angled triangle is 40, and the sum of the hypotenuse and the other side $=50$ find the hypotenuse and the other side. Ans 41, 9.

4. The hypotenuse $=5$ and the sum of the two sides $=7$: find the sides. Ans 3 and 4

5. The difference of the hypotenuse and one side $=3$; and the difference of the hypotenuse and the other side $=4$; find the sides.

Ans 12, 16 and 20

5 *a*. The area of a right angled triangle $=24$, and the hypotenuse $=10$; determine the sides. Ans 8, 6.

6. Two sides of a triangle are 50 and 41, and the perpendicular from the vertex to the base $=40$; find the base. Ans 39

7. The area of an equilateral triangle $=16\sqrt{3}$: find its perimeter

Ans 24

8. The difference between the two sides of a triangle $=2$, and the segments into which the base is divided by a perpendicular from the vertex are 5 and 9 ; determine the sides. Ans 15, 13.

9. The sides of a triangle are a, b, and c, find the diameter of the circumscribed circle , if $a=15$, $b=15$ and $c=18$ find the diameter, and the height of the segment on the chord 18.

Ans $\dfrac{abc}{2 \text{ area of the triangle}}$, $18\frac{3}{4}$, $6\frac{3}{4}$

10. In a certain lake the tip of a bud of lotus was seen 1 foot above the surface of the water, forced by the wind it gradually advanced

and submerged at a distance of 5ft ; calculate the depth of the water.

Ans 12 ft.

11. Find the length of a string which is tied in one corner on the floor of a room and stretched so as to reach the opposite corner under the ceiling; the length of the room $=32$, breadth $=24$ and the height $=9$.

Ans 41

12. Find the radius of the circle inscribed in a triangle of which the sides are a,b and c; if $a=8$, $b=6$ and $c=10$, find the radius,

Ans $\frac{\text{area}}{\text{Semiperimeter}}$, 2ft.

13. Find the side of a decagon inscribed in a circle of which the radius $=10$; and hence shew the method of finding its area.

Ans 6·3 nearly

14. A ladder whose foot rests in a given position, just reaches a a window on one side of a street, and when turned about its foot, just reaches a window on the other side. If the two positions of the ladder be at right angles to each other and the heights of the windows be 39 and 52 feet respectively, find the width of the street and the length of the ladder.

Ans 91 ft, 65ft.

15. What is the length of a diagonal of a square whose side $=10$

Ans $10\sqrt{2}$

16. If $s=$ semiperimeter of a triangle prove that its area

$=\sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the sides.

17. If A B C be an equilateral triangle and the length of AD, a perpendicular on BC, be a; find the length of AB

Ans $\frac{2a}{\sqrt{3}}$

18. In order to ascertain the height of a steeple I measured 150 feet from its base, and found the angle of elevation of its summit from that point to be 60° ; what is the height of the tower ?

Ans 259. 8ft nearly

19. From the top of a hill there are observed two consecutive milestones, on a horizontal road, running from the base. The angles of

depression are found to be 60° and 30°. Find the height of the hill

Ans $\frac{\sqrt{3}}{2}$ miles

20. From the top of a light house 200ft above the sea the angle of depression of a ship's hull is found to be 30°. How far is the ship distant. Ans 346ft nearly

21. The radii of two circles which intersect one another are 30 and 25ft. and the distance of their centres=25ft.; find the length of their common chord. Ans 48ft.

22. If from one of the angles of a rectangle, a perpendicular be drawn to its diagonal d, and from the point of their intersection, lines p, q be drawn perpendicular to the sides which contain the opposite angle ; shew that $p^{\frac{2}{3}} + q^{\frac{2}{3}} = d^{\frac{2}{3}}$.

23. The length of a kite string is 180 yds in length and the angle of elevation of the kite is 30°. Find the height of the kite.

Ans 90 yds

24. A coconut tree measuring 50ft. in height and standing upon level ground was broken in one place by a storm ; the broken part instantly inclined towards the ground so that its extremity reached a distance of 10ft. from the foot of the tree, at how many feet from the foot was the tree broken ? Ans 24ft.

25. A peacock perched on the top of a pillar 9 cubits in height. A snake's hole was at the foot of the pillar and at a distance equal to three times its height was seen a snake ; seeing the snake glide towards the hole, the peacock pounced upon it at a place which was equidistant between the top of the pillar and the place where the snake was first seen ; at how many cubits from the snake's hole did they meet. Ans 12 cubits

26. Two monkeys were sitting on the top of a tree 100 cubits high, and at the distance of 200 cubits from the foot of the tree there was a pool of water. One of the monkeys gradually descended

from the tree and went directly to the pool; the other vaulted to some height perpendicularly from the top of the tree and from thence leaped diagonally to the pool. Both monkeys went over the same space in these several ways. Required the height of the leap.

Ans 50 cubits

27. If p, h, d be the sides of a regular pentagon, hexagon and decagon respectively, inscribed in the same circle; shew that $p^2 = h^2 + d^2$

28. Show that the area of a dodecagon inscribed in a circle is equal to that of a square on the side of an equilateral triangle inscribed in the same circle.

29. ABC is a triangle AB = 16, AC = 20, BC = 18; D is the middle point of BC, find AD. Ans 15·7 nearly.

30. Two straight rods AB and CD each 12 ft. and 6ft. in length respectively are fixed perpendicularly in the ground at the points A and C; two strings are fastened one from A to D and the other from C to B; find the distance of the point where the two strings cross each other from the ground. Ans 4.

31. Find the area of a quadrilateral whose diagonals are 40 and 50 ft. and which are inclined to each other at an angle of 45°.

Ans $500\sqrt{2}$

32. A bar 14 ft. long is bent into a right angle, so that the lengths of the portions which meet at the angle are 8 ft. and 6 ft. respectively; find the distance of the middle of the hypotenuse from the point of the bar which was the middle when the bar was straight. Ans $3\sqrt{2}$

33. Find the area of a rhombus two of whose sides are inclined to each other at an angle of 30°; the side being 20 ft. Ans 200s ft.

34. A person stood on the top of a hill $\frac{1}{2}$ a mile high, from the sea level and observed the radius of his offing to be 63 miles; required the radius of the earth. Ans 3969 miles

35. Find the side of a pentagon inscribed in a circle the radius

of which is 16 feet; and hence shew the method of finding its area
Ans 18. 7 feet, nearly

36. Two roads AB, AC diverge from the same town, making with each other an angle of 45° ; the length of the road $AB = 40\sqrt{2}$ and of AC = 70 miles ; find the distance of C from B. Ans 50 miles

37. A crow wishing to quench its thirst came to a vessel which contained 28 cubic inches of water. The crow being unable to reach the water picked up several small stones each $\frac{3}{4}$ of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel. If the size of the vessel was such that it would exactly hold 73 cubic inches of water, find the number of stones dropped in by the crow. Ans 60 stones.

38. The light of a light house on the Alguada reef is 120 ft. above the sea level and a spectator's eye 6 ft. above the same datum ; find the greatest distance in miles at which the light is visible.
Ans 16 miles nearly

39. Find the height of a tree by the help of a mirror, by a bucket of water or by an artificial horizon.

40. The sides of a triangle are 13, 14 and 15 ; find the perpendicular on the side 14 ; in how many ways can you solve this problem ?
Ans 12

41. ABC is a triangle AB = 4, BC = 5, AC = 6, AD is the line which bisects the vertical angle ABC ; find BD and DC.
Ans 2 and 3.

42. If r be the radius of the inscribed circle of a triangle whose sides are a, b and c and R the radius of the circumscribed circle, prove that $2Rr = \frac{abc}{a+b+c}$

43. ABCD is a quadrilateral inscribed in a circle, $AB = a$, $BC = b$, $CD = c$, $AD = d$; find the two diagonals,

Ans $AC = \sqrt{\left(\frac{(ac+bd)(ad+bc)}{ab+cd}\right)}$, $BD = \sqrt{\left(\frac{(ac+bd)(ab+cd)}{ad+bc}\right)}$

44. From the preceding ex shew that the rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle is equal to both the rectangles contained by its opposite sides. (Ptolemy's theorem Euclid Prop. D Book VI)

45. If s = semiperimeter of a quadrilateral inscribed in a circle and if a, b, c, d are the four sides, prove that its area =

$$\sqrt{(s-a)(s-b)(s-c)(s-d)}$$

46 If a quadrilateral, whose sides are a, b, c, d, is capable of having both a circle inscribed in it and one circumscribed about it, prove that its area $=\sqrt{abcd}$

47. Having given that two points, each 10 feet above the surface, cease to be visible from each other over still water at a distance of 8 miles, find the Earth's diameter. Ans 8448 miles.

48. The sides of a triangle are a, b and c, find the magnitude of the escribed circle which touches the side c and the other two produced.

Ans radius $=\dfrac{\text{area of the triangle}}{s-c}$, where s = semiperimeter of the triangle.

Exercise 33.

ADFECTED QUADRATIC EQUATIONS.

Solve the following equations,

1.	$x^2-3x=-2$	$x=1$ or 2
2.	$x^2+x=12$	$x=-4$ or 3
3.	$x^2+4x=12$	$x=2$ or -6
4.	$x^2-(a+b)x=-ab$	$x=a$ or b
5.	$x^2-4ax=-4a^2$	$x=2a$
6.	$12x^2-7x+1=0$	$x=\frac{1}{3}$ or $\frac{1}{4}$
7.	$a^2b^2x^2-(2b^2+3a^2)x=-6$	$x=\dfrac{2}{a^2}$ or $\dfrac{3}{b^2}$

8. $6x^2-5x=-1$ $\qquad x=\frac{1}{2}$ or $\frac{1}{3}$

9. $5x^2-12x+3=12$ $\qquad x=3$ or $-\frac{3}{5}$

10. $x^2+2x+2\sqrt{(x^2+2x+1)}=47$ $\qquad x=5$

11 $3x^2-2x+\sqrt{(3x^2-4x-6)}=18+2x$ $\qquad x=3$ or $-\frac{5}{3}$

12. $x^4+1=0$ $\qquad x=\frac{\pm 1\pm\sqrt{(-1)}}{\sqrt{2}}$

13. $x^3-1=0$ $\qquad x=1$ or $\frac{-1+\sqrt{(-3)}}{2}$

14. $x^2+\frac{1}{x}=2$ $\qquad x=-\frac{1}{2}\pm\frac{\sqrt{5}}{2}$

15. Form an equation whose roots are 4 and 5, Ans $x^2-9x=-20$

16. Form an equation whose roots are a and b,

Ans $x^2-(a+b)x=-ab$

Exercise 34.

MAXIMA AND MINIMA*,

1. Divide a given number 8 into two such parts that their product may be the greatest possible. Ans 4 and 4

2. To find such a value of x as may make $\frac{x}{(x+8)(x+2)}$ a maximum. Ans $x=4$.

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* "The problems which relate to the maxima and minima, or the greatest or least values of variable quantities, are among the most interesting in the mathematics; they are connected with the highest attainments of wisdom and the greatest exertions of power; and seem like so many unmoveable columns erected in the infinity of space, to mark the eternal boundary which separates the regions of possibility and impossibility from one another."

2nd Diss Ency. Brit.

3. For what value of x the expression $p - (x - 6)^2$ a maximum.
Ans $x = 6$

4. To find such a value of x as will make $\frac{x}{1 + x^2}$ a maximum
Ans $x = 1$

5. Divide the number 16 into two such factors that the sum of their squares shall be a minimum. Ans 4 and 4

6. What fraction exceeds its square by the greatest possible number. Ans $\frac{1}{2}$.

7. For what value of x the expression $m + \sqrt[3]{(m^3 - 2m^2x + mx^2)}$ a minimum. Ans $x = m$

8. Determine the greatest rectangle that can be inscribed in a triangle. Ans The greatest rectangle is that whose altitude $= \frac{1}{2}$ the altitude of the triangle.

9. Of all right angled triangles of the same area, find that the sum of whose legs is the least possible.
Ans When the two legs are equal

10. Of all triangles upon the same base 8ft and having the same perimeter of 18ft, find that which has the greatest area.
Ans An isosceles triangle whose equal sides are each 5ft

11. Inscribe the greatest rectangle in a semicircle of which the radius $= 2$. Ans The rectangle whose sides are $2\sqrt{2}$ and $\sqrt{2}$

12 Of all squares inscribed in a square whose side $=$ 8ft. find that which is the least Ans The square found by joining the middle points of the sides of the given square.

13. Find the least triangle which can be described about a given quadrant Ans When the triangle is isosceles.

14. Find for what value of x the expression $m^4 + n^3x - p^2x^2$ becomes a maximum
Ans $x = \frac{n^3}{2p^2}$

15. In a given circle of which the radius$=4$, inscribe the greatest rectangle possible Ans A square whose sides are $4\sqrt{2}$

16, Find that number which being added to its reciprocal, the sum may be minimum. Ans 1

17. Find the value of x when $\frac{x(m-x)}{m^2}$ is a maximum

Ans $x=\frac{m}{2}$

18. Find the value of x when $\frac{(x+m)(x-n)}{x^2}$ is maximum

Ans $x=\frac{2mn}{m-n}$

19. Find the height at which a lamp should be placed so that the greatest quantity of light may be thrown on a book placed on the table, at a given horizontal distance of 4 feet. * Ans $2\sqrt{2}$ ft,

20. Find for what value of x the expression $\frac{m^2x^2+n^2}{(m^2-n^2)x}$ . . . mum. Ans $\frac{n}{m}$

21. Divide the number 16 into two such parts that if the square of one of these be subtracted from their product, the remainder is the greatest possible. Ans 4 and 12

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* This problem is for those who have gone through trigonometry, I have placed this here at the request of many F. A. students for its practical utility; students during their night studies and workmen generally will find this useful.

Exercise 35.

CALCUTTA UNIVERSITY ENTRANCE PAPERS.

1877 – Rev. G. H. Rouse and Mr. A. M. Nash, M. A.

1. Simplify $\frac{x+2}{1+x+x^2}+\frac{x-2}{1-x+x^2}-\frac{2x^2-4}{1-x^2+x^4}$; multiply together $a+b+c$, $b+c-a$, $c+a-b$, $a+b-c$; and divide $x^4+x^3-24x^2-35x+57$ by x^2+2x-3. Ans $\frac{4x^4+8}{x^8+x^4+1}$; $2b^2c^2+2a^2c^2+2a^2b^2-a^4-b^4-c^4$; x^2-x-19.

2. Solve the equations :——

(1) $\frac{2x-3}{6}+\frac{3x-8}{11}=\frac{4x+15}{33}+\frac{1}{2}$, $x=4\frac{1}{2}$

(2) $2(x+2)=1+\sqrt{(4x^2+9x+14)}$ $x=\frac{5}{3}$

(3 $3x+4y-11=0$, $5y-6z=-8$, $7z-8x-13=0$,

$x=1$, $y=2$, $z=3$

3. Find the G, C, M, of $x^4+x^3-11x^2-9x+18$ $x^4-10x^3+35x^2-50x+24$, Ans x^2-4x+3

4. Find the first four terms of the square root of a^2+x^2 and from the result deduce the square root of 101 correct to six places of decimals.

Ans $a+\frac{x^2}{2a}-\frac{x^4}{8a^3}+\frac{x^6}{16a^5}-$ &c. 10·0498756.

5 If $a:b=c:d$ prove that $a^2+c^2:b^2+d^2::ac:bd$.

6. A and B together can do a piece of work in 15 days; A can do it alone in 24 days; how long would B take to do it alone?

Ans 40

7. Two passengers have together 5 cwt. of luggage, and are charged for the excess above the weight allowed 5*s.* 2*d.* and 9*s.* 10*d.*

respectively ; but if the luggage had all belonged to one of them he would have been charged 19*s*. 2*d*. How much luggage is each passenger allowed to carry free of charge ? and how much luggage had each passenger ? Ans $\frac{25}{28}$ cwt, 2 cwt, 3 cwt.

1878 —Mr. W. Booth, B. A. and Mr. Mowat. M. A.

1. Divide $x(1+y^2)(1+z^2)+y(1+z^2)(1+x^2)+z(1+x^2)(1+y^2)$ $+4xyz$ by $1+xy+yz+xz$ Ans $x+y+z+xyz$

2. Extract the square root of $(a^2+b^2+c^2)(x^2+y^2+z^2)-(bz-cy)^2$ $-(cx-az)^2-(ay-bx)^2$ Ans $ax+by+cz$

3. If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$, find the value of $(b-c)x$ $+(c-a)y+(a-b)z$ Ans 0

4. Solve the equations

   (*a*) $\sqrt{4x^2+20x+17}-\sqrt{16x^2-11x+10}=2(x+2)$ $x=-3$

   (*b*) $\frac{4x+3}{9}+\frac{13x}{108}=\frac{8x+19}{18}$ $x=6$

5. See ex 43 page 64.

1879.

1. Rs 49 was divided amongst 150 children, each girl had 8 *as* and each boy 4 *as* ; how many boys were there ? Ans 104

2. Multiply $a^{2n}-a^nx^n+x^{2n}$ by a^n+x^n and find the greatest common measure of $x^2+\frac{7}{6}x+\frac{1}{3}$ and $x^2+\frac{2}{3}x+\frac{1}{12}$

   Ans $a^{3n}+x^{3n}$; $x+\frac{1}{2}$

3. Divide $x^{2^n}-y^{2^n}$ by $x^{2^{n-1}}+y^{2^{n-1}}$ and Simplify $\frac{x-y}{x-z}+\frac{x-z}{x-y}$ $-\frac{(y-z)^2}{(x-y)(x-z)}$ Ans $x^{2^{n-1}}-y^{2^{n-1}}$; 2.

4. Solve the equations

(a) $x - k + \sqrt{k^2 + x^2} = m$ $\qquad x = \dfrac{m(m+2k)}{2(m+k)}$

(b) $\left.\begin{aligned} a^x \cdot a^{y+1} &= a^7 \\ a^{2y} \cdot a^{3x+5} &= a^{20} \end{aligned}\right\}$ $\qquad \begin{aligned} x &= 3 \\ y &= 3 \end{aligned}$

(c) $\left.\begin{aligned} &\frac{4}{x} + \frac{}{y} \\ &\frac{3}{x} + \frac{2}{y} = \frac{19}{20} \end{aligned}\right\}$ $\qquad \begin{aligned} x &= 4 \\ y &= 10 \end{aligned}$

5. If $\dfrac{a}{b} = \dfrac{c}{d}$ prove that $\dfrac{a^2 + b^2}{a^2 - b^2} = \dfrac{ac + bd}{ac - bd}$

6. Two armies number 11,000 and 7000 men respectively, before they fight each is reinforced by 1000 men: in favor of which army is the increase. Ans In favor of the latter

7 From two towns 561 miles apart two men start, one from each at the same time: one goes 24 and the other 27 miles a day: in how many days will they meet. Ans 11

1880.

7. Simplify $\left\{\dfrac{x}{a} + \dfrac{2x^2}{a(b-x)}\right\} \left\{\dfrac{a}{x} - \dfrac{2ax}{x(b+a)}\right\}$ Ans 1

8. Find the highest common factor and the *l. c. m.* of $3x^2 - 10ax + 7a^2$ and $x^3 - 5ax^2 + 7a^2x - 3a^3$. Ans H. C. M. $= x - a$; L. C.M $= (x-a)(3x-7a)(x-3a)(x-a)$

9. Solve the equations

(a) $15 + \sqrt{x+7} = 19$ Ans 9

(b) $4x - \dfrac{x-1}{2} = x + \dfrac{2x-2}{5} + 24$ Ans 11

(c) $$\left.\begin{array}{l} \dfrac{7+x}{5} - \dfrac{2x-y}{4} = 3y-5 \\ \dfrac{5y-7}{2} + \dfrac{4x-3}{6} = 18-5x \end{array}\right\} \qquad \begin{array}{l} x=3 \\ y=2 \end{array}$$

10. If $a : b = c : d$ shew that $ma + nc : mb + nd :: (a^2 + c^2)^{\frac{1}{2}} : (b^2 + d^2)^{\frac{1}{2}}$

11. Extract the square root of $x^{\frac{8}{5}} - 2a^{-\frac{3}{5}}x^{\frac{11}{5}} + 2a^{\frac{4}{5}}x^{\frac{4}{5}} + a^{-\frac{6}{5}}x^{\frac{14}{5}} - 2a^{\frac{1}{5}}x^{\frac{7}{5}} + a^{\frac{8}{5}}$

Ans $a^{-\frac{3}{5}}x^{\frac{7}{5}} - x^{\frac{4}{5}} - a^{\frac{4}{5}}$

12. A boat goes up stream 30 miles and down stream 44 miles in 10 hours, it also goes up stream 40 miles and down stream 55 miles in 13 hours. Find the rate of the stream and of the boat

Ans Stream 3 miles and boat 8 miles.

1881.

1. What do you mean by a negative quantity ?
   Prove that $a - (b - c) = a - b + c$

2. Simplify $\dfrac{1}{abx} + \dfrac{1}{a(a-b)(x-a)} + \dfrac{1}{b(b-a)(x-b)}$ and resolve into elementary factors the expressions :—

   $x^2 - 5ax - 66a^2$ and $(1-c^2)(1+a)^2 - (1-a^2)(1+c)^2$

   Ans $\dfrac{1}{x(x-a)(x-b)}$; $(x+6a)(x-11a)$

3. A man receives $\dfrac{x}{y}$ ths of 10 Rs, and afterwards $\dfrac{y}{x}$ ths of 10 Rs. He then gives away 20 Rs. Show that he cannot lose by the transaction.

4. What is an equation ? Prove that *a* simple equation has only one root.

5. Solve the equations

(1) $\sqrt{x^2+11x+20} - \sqrt{x^2+5x-1} = 3$ $\qquad x=5$

(2) $\dfrac{4{\cdot}05}{9x} - \dfrac{{\cdot}3}{{\cdot}8-2x} = \dfrac{1{\cdot}8}{x} - \dfrac{3{\cdot}6}{2{\cdot}4-6x}$ $\qquad x={\cdot}3$

(3) $ax+by=c,\ a^2x+b^2y=c^2$ $\qquad x=\dfrac{c(c-b)}{a(a-b)},\ y=\dfrac{c(a-c)}{b(a-b)}$

6. A challenged B to ride a bicycle race of 1040 yds. He first gave B 120 yds. start, but lost by 5 seconds : he then gave B 5 seconds start and won by 120ft. How long does each take to ride the distance ? Ans A, 1 min $55\frac{9}{11}$ sec ; B, 2 min $5\frac{3}{11}$ sec.

Exercise 36.

MISCELLANEOUS EXAMPLES.

1. Multiply $1+x^{-1}+x^{-2}$, $1-x^{-1}+x^{-2}$ and $1+x^{-2}+x^{-4}$
Ans $1+x^{-4}+x^{-8}$.

2. If $\dfrac{b^2+c^2-a^2}{2bc}+\dfrac{c^2+a^2-b^2}{2ac}+\dfrac{a^2+b^2-c^2}{2ab}=1$ shew that two of the 3 fractions on the left hand side are each equal to 1 and the other to -1.

3. If n is a positive whole number shew that $13^{2n+1}+1$ is divisible by 14.

4. Solve $\dfrac{2x+3y-4z}{x+5}=\dfrac{3x+4y-2z}{5x}=\dfrac{4x+2y-3z}{4x-1}$
$=\dfrac{x+y+z}{6}$ $\qquad x=5,\ y=4,\ z=3$

5. If $\dfrac{bx+ay-cz}{a^2+b^2}=\dfrac{cy+bz-ax}{b^2+c^2}=\dfrac{az+cx-by}{c^2+a^2}$ shew that $\dfrac{x+y+z}{a+b+c}=\dfrac{ax+by+cz}{ab+bc+ac}$

7. If $x=\frac{1}{2}$, $x+y=x+y+z=0$ find the value of $(y^2-z^2)\{y^2+z^2-y(x-z)\}$ Ans $\frac{1}{8}$

8. Divide $a+b^2+c^3-3\sqrt[3]{(ab^2c^3)}$ by $a^{\frac{1}{3}}+b^{\frac{2}{3}}+c$

Ans $a^{\frac{2}{3}}+b^{\frac{4}{3}}+c^2-a^{\frac{1}{3}}b^{\frac{2}{3}}-a^{\frac{1}{3}}c-b^{\frac{2}{3}}c$

9. Eliminate x and y from the equations

$a=\frac{1}{x}-x,\ b=\frac{1}{y}-y,\ x^2+y^2=1.$ Ans $a^{\frac{2}{3}}b^{\frac{2}{3}}(a^{\frac{2}{3}}+b^{\frac{2}{3}})=1$

10. If $x^2+y^2+z^2=xy+yz+xz$ shew that $x^3+y^3+z^3=3xyz$

11. If $a^2+b^2=c^2+d^2=1$ shew that $(ad+bc)(ad-bc)=a^2-c^2$

12. If $x=\sqrt[3]{a+\sqrt{(a^2+b^3)}}+\sqrt[3]{a-\sqrt{(a^2+b^3)}}$ shew that $x^3+3bx=2a$

13. If $a=x^2-1$, $b=2x$ and $c=x^2+1$ shew that

$(a+b+c)(b+c-a)(a+c-b)(a+b-c)=4a^2b^2$

14. If $a+b+c=0$ shew that $2(a^2b^2+b^2c^2+a^2c^2)=a^4+b^4+c^4$

15. If $x=\frac{1}{2}\left\{\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right\}$ shew that $\frac{2a\sqrt{(1+x^2)}}{x+\sqrt{(1+x^2)}}=a+b$

16. If $x^2-yz=-8$, $y^2-xz=1$, $z^2-xy=10$, find x, y and z

$x=2,\ y=3,\ z=4$

17. Find a number of two digits such that its quotient by their sum exceeds the second digit by 1,and and is 4 times the other. Ans 72

18. Simplify $\left\{\frac{1+x}{1-x}+\frac{4x}{1+x^2}+\frac{8x}{1+x^4}-\frac{1-x}{1+x}\right\}$

$\div\left\{\frac{1+x^2}{1-x^2}+\frac{4x^2}{1+x^4}-\frac{1-x^2}{1+x^2}\right\}$ Ans $2x^{-1}$

19. A flagstaff $5\frac{1}{3}$ ft. high stands at the top of a wall, whose height is 3 ft. Find the distance from the bottom of the wall, at which the flagstaff subtends the greatest angle. Ans 5 ft.

20. $\frac{x+\sqrt{(x^2-25)}}{x-\sqrt{(x^2-25)}}=125\frac{\sqrt{(x+5)}-\sqrt{(x-5)}}{\sqrt{(x+5)}+\sqrt{(x-5)}}$ Ans $x=13$

21. Explain the following anomally.

$$1=\frac{x^2-x^2}{x^2-x^2}=\frac{(x+x)(x-x)}{x(x-x)}=\frac{x+x}{x}=2$$

22. Solve $3^x+2^y=17$ and $3^{x+1}+2^{y+2}=59$ Ans $x=2, y=3$

23. Two boys start from the right angle of a triangular field, and run along the sides with velocities in the ratio of 13 : 11. They meet first in the middle of the opposite side and again 30 yds from the starting point. Find the length round the field. Ans 360 yds

24 Solve the equation $\left(\frac{x+a}{b}\right)^{\frac{2}{3}}+\left(\frac{x+c}{d}\right)^{\frac{2}{3}}=2\left\{\frac{(x+a)(x+c)}{bd}\right\}^{\frac{1}{3}}$

$$x=\frac{bc-ad}{d-b}$$

25. Two persons started at the same time from A. One rode on an elephant at the rate of $7\frac{1}{2}$ miles an hour and arrived at B 30 minutes later than the other who trevelled the same distance by train at the rate of 30 miles an hour. Find the distance between A and B. Ans 5 miles.

26, Fnd the *l. c. m* of $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$

Ans x^4-y^4

27. Simplify $\frac{x}{x-a}-\frac{x}{x+a}-\frac{\frac{x+a}{x-a}-\frac{x-a}{x+a}}{\frac{x+a}{x-a}+\frac{x-a}{x+a}}$ Ans $\frac{4a^3x}{x^4-a^4}$

28. The third law of Kepler is that the squares of the periodic times of the planets are to each other as the cubes of their mean distances from the Sun, if the distance of the earth from the Sun be 95,000,000 miles and the periodic times of the Earth and Venus be respectively 365 and 224 days ; find the distance of Venus from the Sun. Ans 68,610,000miles

29. Find for what value of n the expression $2x^3-6nx^2-6m^2n$ is divisible by $x-m$ without a remainder. Ans $x=\frac{1}{6}m$

30. If $a+b+c=0$ prove that $\frac{a^5+b^5+c^5}{a^4+b^4+c^4} = \frac{5(a^3+b^3+c^3)}{3(a^2+b^2+c^2)}$

31. If $\frac{x-y}{y-z} = \frac{3x-y}{y-3z} = \frac{4x-z}{z-4y}$ prove that $\frac{19x-3y-3z}{-9y-4z}$

$$= \frac{13x-5y-z}{y-8z}$$

32. Mr. Leivin an aeronaut who made a baloon ascent in Calcutta on the 29th December 1877, wishing to know the altitude of the baloon when it was at its maximum height, threw down a bottle and observed that the time occupied for its descent to the surface of the earth was half a minute. Find the height of the baloon. (see Ex, 62, page 89)

Ans 14490 ft.

33. With a given perimeter of 32 feet, find what rectangle has the maximum area. Ans A square whose side is 8 feet

34. Disintegrate $4a^4 - 37a^2b^2 + 49b^4$ into its elementary factors,

Ans $(2a^2 - 7b^2 + 3ab)(2a^2 - 7b^2 - 3ab)$

35. A man is allowed the option of choosing 36 sq feet of ground for a room, find the length and breadth of the room that the cost of the walls may be minimum. Ans A square whose side is 6 ft.

36. Simplify $\frac{x^2-yz}{(x+y)(x+z)} + \frac{y^2-xz}{(x+y)(y+z)} + \frac{z^2-xy}{(x+z)(y+z)}$ Ans 0

37. Find the G. C. M. of $2x^3+x^2+x-4$ and $4x^4+6x^3+10x^2+3x+4$. Ans $2x^2+3x+4$

38. If $x^2+\frac{1}{x}=2$ shew that $x(x+1)=1$

39. Solve $8x^3-24x=65$ $x=8\frac{1}{8}; \frac{1}{2}$

40. Solve $\frac{\sqrt{(2+\sqrt{x})}}{\sqrt[3]{x}} + \frac{\sqrt{(2-\sqrt{x})}}{\sqrt[3]{x}} = \sqrt[6]{x}$ Ans $x=4$

41. Two men travel in the same direction in the circumference of a circular island, they both start at the same instant and it is known

that one can travel the circumference in 24 hours and the other in 27 days 7 hours; after what time they will be together again.

Ans 24 hours 55 min nearly

42. From the foregoing example deduce that if a man has the Moon in his meridian at a certain time of the day, it will be in his meridian again after 24 hours 55 minutes.

43. Investigate the rule for finding two square numbers whose sum is a square number.

44. Find the value of $\frac{a^3}{3(a^2-x)}+\frac{b^3}{3(b^2-a)}$ when $x=\frac{a^3+b^3}{2}$

45. Simplify $\frac{x^{\frac{1}{4}(3m-1)}\,y^{-1}}{x^{\frac{1}{4}(m-3)}\,y^{\frac{1}{2}(m-1)}}$ Ans $\left(\frac{x}{y}\right)^{\frac{1}{2}(m+1)}$

46. The trinomial mx^2+nx+p becomes 9, 18 and 31 if x is equal to 1, 2 and 3 respectively what will its value be if $x=4$ Ans 48

47. $x+y+z=0$

$x(b+c)+y(a+c)+z(a+b)=0$

$x^3+y^3+z^3=3(a-b)(b-c)(c-a)$ Ans $x=b-c, y=c-a, z=a-b$

48. $x(b-c)+y(c-a)+z(a-b)=0$ prove that

$$\frac{bz-cy}{b-c}=\frac{cx-az}{c-a}=\frac{ay-bx}{a-b}$$

49. If $\frac{x_0}{x_1}=\frac{x_1}{x_2}=\frac{x_2}{x_3}=\dots\dots\dots=\frac{x_{r-1}}{x_r}$ shew that each of the expressions

$$=\frac{x_0^{\frac{1}{r}}+x_1^{\frac{1}{r-1}}+x_2^{\frac{1}{r-2}}+\dots\dots x_{r-2}^{\frac{1}{2}}+x_{r-1}}{x_r^{\frac{1}{r}}+x_r^{\frac{1}{r-1}}+x_r^{\frac{1}{r-2}}+\dots\dots x_r^{\frac{1}{2}}+x_r}$$

50. If the sides of a triangle be a, b, c, prove that $(b+c-a)^{-1}+(c+a-b)^{-1}+(a+b-c)^{-1}>a^{-1}+b^{-1}+c^{-1}$

51. The area of an equilateral triangle is $16\sqrt{3}$; find the side

Ans 8

52. Eliminate m and n from the equations $y = mx + \sqrt{(a^2m^2 + b^2)}$ $= nx + \sqrt{(a^2n^2 + b^2)}$ and $mn + 1 = 0$ Ans $a^2 + b^2 = x^2 + y^2$

53. If $x = a^2 - bc$, $y = b^2 - ac$, $z = c^2 - ab$ shew that

$$\frac{x^2 - yz}{a} = \frac{y^2 - xz}{b} = \frac{z^2 - xy}{c} = (a + b + c)(x + y + z)$$

54. Divide $(a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3$ by $a^2 + 2a(b + c)$ $+ (b + c)^2$ Ans $a + b + c$

55. Eliminate x and y from the equations $ay = x^2 + 3y^2$, $bx = y^2$ $+ 3x^2$, $xy = c$ Ans $(a + b)^{\frac{2}{3}} - (a - b)^{\frac{2}{3}} = 4c^{\frac{1}{3}}$

56. Solve $\frac{1}{x + a + b} = \frac{1}{x} + \frac{1}{a} + \frac{1}{b}$ Ans $x = -b$ or $-a$

57. Multiply $x^{2^p} + x^{2^{p-1}}y^{2^{p-1}} + y^{2^p}$ by $x^{2^p} - x^{2^{p-1}}y^{2^{p-1}} + y^{2^p}$

Ans $x^{2^{p+1}} + x^{2^p}y^{2^p} + y^{2^{p+1}}$

58. Solve $xy + yz + xz = 30 - x^2 = 35 - y^2 = 42 - z^2$ $x = 2$, $y = 3$, $z = 4$

59. Solve $2^x + 2^{2-x} = 4$ $x = 1$

60. Eliminate $a = x^3 + 3y^2$, $b = y^2 + 3x^2$, $xy = c$

61. $\sqrt{3} + 1 = \frac{2}{3^x - 1}$ $x = \frac{1}{2}$

62. Simpliy $\frac{x - a}{x - b} + \frac{x - b}{x - a} - \frac{(a - b)^2}{(x - a)(x - b)}$ Ans 2

63. Simplify $\frac{x^3 + y^3 + z^3 - 3xyz}{(x - y)^2 + (y - z)^2 + (z - x)^2}$ Ans $\frac{x + y + z}{2}$

64. The sum of two numbers is 4225 and their G. C. M. is 845; find the numbers. Shew that there are two pairs of numbers satisfying the condition. Ans 845, 3380 ; 1690, 2535

65. A market woman bought eggs at 2 a penny and as many more at 3 a penny; and thinking to make her money again, she sold them

at 5 for two pence ; she lost however 4*d* by the business ; how much did she lay out ?

Ans $8^{s}.\,4^{d}$

66. Find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$, when $x=\frac{4ab}{a+b}$ Ans 2

67. Solve $\sqrt{(x^3-3ax+2a^2)}+\sqrt{(x^3-7ax+5a^2)}=\sqrt{(x^3-6ax-6a^2)}$
$+\sqrt{(x^3-10ax-3a^2)}$ $x=-\frac{8}{3}a$

68. A waterman rowed $3\frac{1}{2}$ miles down a river and up again in 100′ : supposing the stream to have a current of 2 miles an hour, find at what rate he would row in still water. Ans 5 miles

69. A gamester loses $\frac{1}{3}$ of his money, and then wins 10*s* ; he loses $\frac{1}{3}$ of this and then wins £1, when he leaves of as he began, what had he at first. Ans £2, 8*s*.

70. Divide $(x^3-1)a^3-(x^3+x^2-2)a^2+(4x^2+3x+2)a-3(x+1)$
by $(x-1)a^2-(x-1)a+3$ Ans $(x^2+x+1)a-(x+1)$

71. Reduce to its lowest term,

$$\frac{x^4+x^2+1}{x^{\frac{5}{2}}-x^2-x^{\frac{3}{2}}+x+x^{\frac{1}{2}}-1}\qquad \text{Ans } \frac{x^2+x+1}{x^{\frac{1}{2}}-1}$$

72. Solve $\left(\frac{2x+3}{2x-3}\right)^{\frac{1}{3}}+\left(\frac{2x-3}{2x+3}\right)^{\frac{1}{3}}=\frac{8}{13}\cdot\frac{4x^2+9}{4x^2-9}$, $x=\frac{27}{14}$

73. Solve $\frac{x^{(m-n)^2}+x^{-4mn}}{x^{(m-n)^2}-x^{-4mn}}=p^{\frac{a}{b}}$ $\qquad x=\left(\frac{p^{\frac{a}{b}}+1}{p^{\frac{a}{b}}-1}\right)^{\frac{1}{(m+n)^2}}$

74. Divide $x^{8^{x+\frac{1}{3}}}-y^{8^{x+\frac{1}{3}}}$ by $x^{8^{x}}+y^{8^{x}}$ Ans $x^{2^{3x}}-y^{2^{3x}}$

75. If x^2+ax+b and $x^2+a'x+b'$ have a common measure
prove that $(a-a')(ab'-a'b)+(b-b')^2=0$

76. Find the square root of $8\frac{1}{5}-\frac{12a}{5b}+\frac{9a^2}{25b^2}+\frac{49b^2}{4a^2}-\frac{28b}{2a}$

Ans $\frac{3a}{5b}-2+\frac{7b}{2a}$

77. Solve $\frac{243+324\sqrt{(3x)}}{16x-3}=(4\sqrt{x}-\sqrt{3})^2$ Ans $x=3$

78. Find the G. C. M. of $x^3+(y+z)x^2-(y-z)^2x-(y+z)(y-z)^2$ and $x^3+y^3+z^3-3xyz$ Ans $x+y+z$

79. Solve $\frac{\sqrt{(x+1)}-\sqrt{(x-1)}}{\sqrt{(x+1)}+\sqrt{(x-1)}}=\frac{x+\sqrt{(x^2-1)}}{x-\sqrt{(x^2-1)}}$ $x=-\frac{1}{2}$

80. Solve $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-a-b}{c}=\frac{2x}{a+b}$ $x=a+b$

81. Solve $5-x=2(\sqrt{(xy)}-\sqrt{6})+y$ Ans $x=3$, $y=2$

82. Find the value of x which will make $x^3+3cx^2+2c^2x+5c^3$ equal to the cube of $x+c$ Ans $x=4c$

83 An officer can form the men in his battalion into a solid square and also into a hollow square 12 deep: if the front in the latter formation exceed the front in the former by 3, find the number of men in the battalion. Ans 1296

84. Simplify $\frac{x^2+xy+y^2}{x\sqrt{x}+y\sqrt{y}}$ Ans $\frac{x+\sqrt{(xy)}+y}{\sqrt{x}+\sqrt{y}}$

85. If $x+m$ be the G. C. M of x^2+ax+b and x^2+cx+d shew that $m=\frac{b-d}{a-c}$

86. Two trains start at the same time from two towns and each proceeds at a uniform rate towards the other town. When they meet it is found that one train has run 108 miles more than the other and that if they continue to run at the same rates, they will finish the journey in 9 and 16 hours respectively. Find the distance between the towns and the rates of the trains.

Ans 756 miles, 36 and 27 miles

87. The height of a light house on the Cocos reef is 132 ft How far is it visible from a ship at sea, the radius of the earth = 4000 miles

Ans $10\sqrt{2}$ miles nearly

88. Simplify $\dfrac{x^2+x+1}{(x-y)(x-z)}+\dfrac{y^2+y+1}{(x-y)(z-y)}+\dfrac{z^2+z+1}{(z-x)(z-y)}$ Ans 1

89. Find the *g.c.m.* of $x^4+(m+n)x^3-(n+1)x^2-(m+n)x+n$ and $x^4-(p+q)x^3+(q-1)x^2+(p+q)x-q$ Ans x^2-1

90. Eliminate x and y from the equations $(mx+ny)^2=m^2+n^2$, $x^2+y^2=1$, $\dfrac{x^2}{p^2}+\dfrac{y^2}{q^2}=\dfrac{1}{m^2+n^2}$ Ans $\dfrac{m^2}{p^2}+\dfrac{n^2}{q^2}=1$

91. Find the condition that x^2+mx+n^2 may be a multiple of $x+p$ Ans $n^2+p^2=mp$

92. Find the *g. c. m* of $\dfrac{x^2}{3}+\dfrac{11x}{6}\sqrt{(x+1)}-x-1$ and $x^2-\frac{x}{4}-\frac{1}{4}$

Ans $x-\frac{1}{2}\sqrt{(x+1)}$

93 If $\dfrac{2b}{c-a}=\dfrac{a}{b-c}+\dfrac{c}{a-b}$ prove that $2(a^3+c^3-2b^3)=(a+b+c)$ $(a^2+c^2-2b^2)$

94. Find the length of a zig-zag road ascending by a gradient of 1 in 8, to the top of Parishnoth which is 4500 ft. high. Ans 36000 ft.

95. Divide $3+xy^{-1}+xz^{-1}+x^{-1}y+yz^{-1}+x^{-1}z+y^{-1}z$ by $x^{-1}+y^{-1}+z^{-1}$ Ans $x+y+z$

96. If $xy+x+y=2$ prove that $\dfrac{x^4-8x}{1+x^3}=\dfrac{y^4-8y}{1+y^3}=\dfrac{xy(4-xy)}{xy-1}$

97. If $yz+xz+xy=\dfrac{1}{yx}+\dfrac{1}{xz}+\dfrac{1}{zy}=3$ prove that

$(1+yz)(1+xz)(1+xy)=(1+x^2)(1+y^2)(1+z^2)$

98. If $x+y+z+w=$ prove that $6(x^5+y^5+z^5+w^5)$ $=5(x^3+y^3+z^3+w^3)(x^2+y^2+z^2+w^2)$

99. Shew that $(ab)^n-(bc)^n+(cd)^n-(ad)^n$ is divisible by $ab-bc+cd-ad$, if n be any positive integer.

100. If $3x=a+b+c$ prove that $(x-a)^4+(x-b)^4+(x-c)^4$
$=2\{x-b)^2(x-c)^2+(x-c)^2(x-a)^2+(x-a)^2(x-b)^2\}$

101. Simplify $\dfrac{2^{x+4}-2\cdot 2^x}{2^{x+2}\times 4}$ Ans $\frac{7}{8}$

102. Solve $\dfrac{a+b}{x+b}+\dfrac{a+c}{x+c}=\dfrac{2(a+b+c)}{x+b+c}$ $x=-\dfrac{b^2+c^2}{b+c}$

103. A and B run a mile. First A gives B a start of 44 yds and beats him by 51 seconds; at the second heat A gives B a start of 1 min. 15 sec. and is beaten by 88 yds; find the time in which A and B can run a mile respectively. Ans A in 5, and B in 6 min.

104. Add together $x^2-(x-y+z)(x+y-z)$, $y^2-(y-x+z)(y+x-z)$, $z^2-(z-x+y)(z+x-y)$, Ans $2(x^2+y^2+z^2-xy-xz-yz)$

105. Simplify $\dfrac{x^3}{(x-y)(x-z)}+\dfrac{y^3}{(y-x)(y-z)}+\dfrac{z^3}{(z-x)(z-y)}$
Ans $x+y+z$

106. AB is a railway 220 miles long, and three trains P. Q. R. travel upon it at the rate of 25, 20 and 30 miles per hour respectively, P and Q leave A at 7 a. m. and 8 15 a. m. respectively, and R leaves B at 10 30 a. m. When and where will P be equidistant from Q & R
Ans 12 O'clock; 125 miles from A.

107. Simplify $\dfrac{1+x+x^2}{1-x^3}+\dfrac{1-x+x^2}{1+x^3}-\left(\dfrac{x}{1+x}+\dfrac{1-x}{x}\div\dfrac{1+x}{x}\right)\times\dfrac{1}{1-x}$
Ans $\dfrac{1}{1-x^2}$

108. Simplify $\dfrac{\dfrac{a^3}{b^3}-\dfrac{b^3}{a^3}}{\left(\dfrac{a}{b}-\dfrac{b}{a}\right)\left(\dfrac{a}{b}+\dfrac{b}{a}-1\right)}\times\dfrac{\dfrac{1}{b}-\dfrac{1}{a}}{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{ab}}$ Ans $a-b$

109. If $\dfrac{x}{a^2+x^2}=\dfrac{2y}{a^2+y^2}=\dfrac{4z}{a^2+z^2}$ shew that $x(y^2-z^2)+2y(z^2-x^2)$
$+4z(x^2-y^2)=0$

110. If $x+y+z=0$ shew that $\left(\frac{y-z}{x}+\frac{z-x}{y}+\frac{x-y}{z}\right)$
$$\left(\frac{x}{y-z}+\frac{y}{z-x}+\frac{z}{x-y}\right)=9$$

111. If $x+y+z=x^2+y^2+z^2=2$ prove that
$x(1-x)^2=y(1-y)^2=z(1-z)^2$;

112. Simplify $\{x\sqrt{(1-y^2)}+y\sqrt{(1-x^2)}\}^2+\{xy-\sqrt{(1-x^2)(1-y^2)}\}^2$
Ans 1

113. If $\frac{y}{z}-\frac{z}{y}=m, \frac{z}{x}-\frac{x}{z}=n, \frac{x}{y}-\frac{y}{x}=p$
prove that $m^4+n^4+p^4=2n^2p^2+2m^2p^2+2m^2n^2+m^2n^2p^2$

114. Simplify $(4x^3-3x)^2+\{3\sqrt{1-x^2}-4(1-x^2)^{\frac{3}{2}}\}^2$ Ans 1

115. If $x^2+xy+y^2=c^2, x^2+xz+z^2=b^2, y^2+yz+z^2=a^2$ shew that $xy+yz+xz=\sqrt{\{\frac{1}{3}(2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4)\}}$

116. Divide $x^3-3ax^2+3a^2x-a^3+b^3$ by $x-a+b$
Ans $(x-a)^2-b(x-a)+b^2$

117. Eliminate $4(x^2+y^2)=ax+by, 2(x^2-y^2)=ax-by, xy=c^2$

118. Prove that $x^n-na^{n-1}x+(n-1)a^n$ is divisible by $(x-a)^2$ if n be a positive whole number,

119. Solve $\frac{x+4a+m}{x+a+m}+\frac{4x+a+2m}{x+a-m}=5$ $x=-m$

120. The slope of Baharinoth hill in Ranigunge is 1 in 8. A man whose usual pace on level ground is 4 miles an hour, ascends and descends in 5 hours. His pace uphill : pace on level : : 3 : 4 and pace down hill to pace on level : : 5 : 4. What is the height of the hill? Ans 2062½ yds.

121. If $\frac{x_1}{}=\frac{x_2}{}=\frac{x_3}{}=\&c, =\frac{x_n}{x_{n+1}}$ prove that
$$\left(\frac{x_1+x_2+x_3+\dots\dots\dots x_n}{x_2+x_3+x_4+\dots\dots x_{n+1}}\right)^n=\frac{x_1}{x_{n+}}$$

122. Prove that $(1+\sqrt{(-1)})^2 + (1-\sqrt{(-1)})^2 = 0$

123. Find the G. C, M, of $a^2+b^2+c^2-2ab-2ac+2bc$ and $a^2-b^2-c^2-2bc$ Ans $a-b-c$

124. If T = the time between two successive inferior or superior conjunctions of a planet, E = periodic time of the earth, P = periodic time of the planet prove that $P = \dfrac{T \times E}{T \pm E}$

125. If the interval between the inferior conjunctions of Venus be 584 days and the periodic time of the earth be 365 days find by ex 25 the periodic time of Venus. Ans 224 days nearly

126, Solve $\dfrac{2x+11}{x+5} - \dfrac{9x-9}{3x-4} = \dfrac{4x+13}{x+3} - \dfrac{15x-47}{3x-10}$ Ans $-\frac{5}{6}$

127, If $\dfrac{a}{x} = \dfrac{b}{y} = \dfrac{c}{z}$ and $\dfrac{x^2}{m^2} + \dfrac{y^2}{n^2} + \dfrac{z^2}{p^2} = 1$ prove that

$$\frac{a^2}{m^2} + \frac{b^2}{n^2} + \frac{c^2}{p^2} = \frac{a^2+b^2+c^2}{x^2+y^2+z^2}$$

128. Divide $x^{3n+1} + y^{n-3}$ by $x^{n+\frac{1}{3}} + y^{\frac{n}{3}-1}$

Ans $x^{\frac{2}{3}(3n+1)} - x^{n+\frac{1}{3}}y^{\frac{n}{3}-1} + y^{\frac{2}{3}(n-3)}$

129. Solve $x^3 + 24 = 3x^2 + 10x$ $x = 2,\ -3$ and 4

130. Divide $a^4(b-c) + b^4(c-a) + c^4(a-b)$ by $(a-b)(b-c)(c-a)$ Ans $a^2+b^2+c^2+ab+ac+bc$

131. Supposing that $(2x-a)(2x-b)(2x-c)$ is an exact cube shew that $(ab+ac+bc)\left\{\dfrac{1}{ab} + \dfrac{1}{bc} + \dfrac{1}{ac}\right\}$ is an exact square

132. Simplify $\dfrac{\dfrac{a+b+c-abc}{1-ab-ac-bc} - \dfrac{b+c}{1-bc}}{1 + \dfrac{a+b+c-abc}{1-ab-ac-bc} \cdot \dfrac{b+c}{1-bc}}$ Ans a

133. Solve $\dfrac{ax}{a+x} + \dfrac{by}{b+y} = \dfrac{c(a+b)}{a+b+c}$, $x+y=c$, $x = \dfrac{ac}{a+b}$, $y = \dfrac{bc}{a+b}$

RECREATIONS, AMUSING PROBLEMS.

134. A man has a wolf, a goat and a cabbage, to carry over a river , but being obliged to transport them one by one on account of the smallness of the boat, in what manner is this to be done, that the wolf may not be left with the goat, nor the goat with the cabbage ?

135. "Tell me, illustrious Pythagoras, how many pupils frequent thy school ? One half, replied the philosopher, study mathematics, one fourth natural philosophy, one seventh observe silence, and there are three females besides." Ans 28

136. A mule and an ass travelling together, the ass began to complain that her burden was too heavy, "Lazy animal" said the mule you have little reason to complain , for if I take one of your bags, I shall have twice as many as you, and if I give you one of mine we shall then have only an equal number," With how many bags was each loaded. Ans Mule 7 bags , ass 5 bags

137. Three gentlemen, with their servants, having to cross a river at a ferry, find a boat without a boatman ; but the boat is so small that it can contain no more than two of them at once. How can these six persons cross the river, two and two, so that none of the servants shall be left in company with any of the gentlemen, unless when his master is present.

SOLUTIONS.

NOTATION.

Ex. 1. 73. $(-\frac{1}{2})^4-(\frac{1}{2})(-\frac{1}{2})^3+\frac{1}{2}\cdot 0=\frac{1}{16}-(\frac{1}{2})(-\frac{1}{8})=\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$

74. $a-b=x$ ∴ $a=b+x=b+3$ also from the 2nd $a+(b+x)$ $=a+a=2$ ∴ $2a=2$ ∴ $a=1$ ∴ $b=-2$ ∴ the expression $=$ $(1+2)\{9-2\cdot1\cdot9+1\cdot3-(1-2)4\}=3\ \{9-18+3+4\}=-6$

ADDITION.

Ex. 2. 1.

$$\begin{array}{r} 4a-6b \\ 2a+3b \\ a-2b \\ a+b \\ \hline 8a-4b \end{array}$$

2.

$$\begin{array}{r} 5x^2+4y^2 \\ 4x^2-8y^2 \\ -2x^2+y^2 \\ 4x^2-y^2 \\ \hline 11x^2-4y^2 \end{array}$$

13.

$$\begin{array}{rrr} 2x^2 & +y^4 & +2z^3 \\ -4x^2 & & -5z^3 \\ 4x^2 & -3y^4 & +7z^3 \\ \hline 2x^2 & -2y^4 & +4z^3 \end{array}$$

17.

$$\begin{array}{l} 2\sqrt{(a+x)}-4\sqrt{(a-x)}+6\sqrt{(a^2-x^2)} \\ -3\sqrt{(a+x)}-6\sqrt{(a-x)}+2\sqrt{(a^2-x^2)} \\ -4\sqrt{(a+x)}+6\sqrt{(a-x)}+5\sqrt{(a^2-x^2)} \\ \hline -5\sqrt{(a+x)}-4\sqrt{(a-x)}+13\sqrt{(a^2-x^2)} \end{array}$$

19. Add the coefficients of x^4, x^3, x^2, x and y and arrange.

20. Proceed as in the foregoing example.

29.

$$\begin{array}{l} (a+b)(x+y)^2+(c+d)(x+y)+cx \\ (a-b)(x+y)^2+(c-d)(x+y)+cy \\ \quad -2a(x+y)^2 \\ \hline (a+b+a-b-2a)(x+y)^2+(c+d+c-d)(x+y)+c(x+y) \end{array}$$

or $2c(x+y)+c(x+y)=3c(x+y)$

SUBTRACTION.

Ex. 3. 1.
$$\begin{array}{r} 8a+16b \\ 3a+10b \\ \hline 5a+6b \end{array}$$

12. Sum of the 1st two $=2a+2c$,
and the sum of the last two is $2a$,
$\therefore$ diff$=2c$

18. $\{2a+b-c-(a+b-c)\}x^2+\{(a+b+c)-(a+c)\}x+\{a+b+c-(a+b)\}=ax^2+bx+c$

BRACKETS.

Ex. 4. 1. $3a-2b+2a-b=5a-3b$ 2. $a+b-c-a+b+c=2b$

3. $1-a+1+a^2-1-a^2-a+1=2-2a$

11. $10a-\{4a-5+6a-2a\}=10a-4a+5-6a+2a=2a+5$

12. $2-\{1-1+a\}-4=2-1+1-a-4=-a-2$

13. $x-\{-1-a-1+a\}=x+1+a+1-a=x+2$

14. $5a-[2a-\{1-2a+1\}]=5a-[2a-1+2a-1]$
$=5a-2a+1-2a+1=a+2$

20. $-[-1-\{1-(a-1+a)-a\}]=-[-1-\{1-a+1-2a\}]$
$=-[-1-1+a-1+2a]=1+1-a+1-2a=3-3a$

21. $-[-\{-(-a+1)\}]=-[-\{a-1\}]=-[-a+1]=a-1$

23. $a-[-a-1+\{-1-a+1-2\}-1]$
$=a-[-a-2-a+1-2-1]=a+a+2+a-1+2+1=3a+4$

24. $-[-1-\{-1+2-1\}]=-[-1+1-2+1]=1-1+2-1=1$

MULTIPLICATION.

Ex. 5. 21.
$$\begin{array}{l} a^2-ab+b^2 \\ a+b \\ \hline a^3-a^2b+ab^2 \\ \qquad a^2b-ab^2+b^3 \\ \hline a^3+b^3 \end{array}$$

54.
$$\begin{array}{r} x^{\frac{1}{2}}+y^{\frac{1}{2}} \\ x^{\frac{1}{2}}-y^{\frac{1}{2}} \\ \hline x+x^{\frac{1}{2}}y^{\frac{1}{2}} \\ -x^{\frac{1}{2}}y^{\frac{1}{2}}-y \\ \hline x-y \end{array}$$

64. $$\begin{array}{l} x^{m(n-1)} + y^{n(m-1)} \\ x^{-mn} - y^{-(n-1)} \\ \hline x^{-m} + x^{-mn}y^{n(m-1)} \\ \qquad\qquad - x^{m(n-1)}y^{-(n-1)} - y^{mn-2n+1} \\ \hline x^{-m} + x^{-mn}y^{n(m-1)} - x^{m(n-1)}y^{-(n-1)} - y^{mn-2n+1} \end{array}$$

66. $$\begin{array}{l} \frac{2}{3}x^2 - \frac{4}{5}x + \frac{1}{4} \\ \frac{3}{4}x - \frac{1}{2} \\ \hline \frac{1}{2}x^3 - \frac{3}{5}x^2 + \frac{3}{16}x \\ \qquad - \frac{1}{3}x^2 + \frac{2}{5}x - \frac{1}{8} \\ \hline \frac{1}{2}x^3 - \frac{14}{15}x^2 + \frac{47}{80}x - \frac{1}{8} \end{array}$$

68. $$\begin{array}{l} x^{2^n} + y^{2^n} \\ x^{2^n} - y^{2^n} \\ \hline x^{2^{n+1}} + x^{2^n}y^{2^n} \\ \qquad - x^{2^n}y^{2^n} - y^{2^{n+1}} \\ \hline x^{2^{n+1}} - y^{2^{n+1}} \end{array}$$

If we add 2^n to 2^n we get $2 \cdot 2^n = 2^{n+1}$

DIVISION.

Ex. 6. 14. $$x - 1 \,\big)\, x^2 - 2x + 1 \,\big(\, x - 1$$
$$\begin{array}{l} x^2 - x \\ \hline -x + 1 \\ -x + 1 \end{array}$$

19. $$2x + 6 \,\big)\, 6x^3 + 14x^2 - 4x + 24 \,\big(\, 3x^2 - 2x + 4$$
$$\begin{array}{l} 6x^3 + 18x^2 \\ \hline -4x^2 - 4x \\ -4x^2 - 12x \\ \hline 8x + 24 \\ 8x + 24 \end{array}$$

32. There will be altogether 81 terms in the quotient and the middle one will be the 41*st* ; now $\because$ the index of a in the first term is 80, in the 2nd term 79 and so on, the index of a in the 41*st* term is $81-41=40$ and $\because$ the sum of the indices of a and x in every term is 80 and the index of x in the middle term is 40 $\therefore$ the middle term is $a^{40}x^{40}$; all the terms are positive.

33. Proceed as in ex 32 remembering only that the odd terms in the quotient are positive.

35. $\because$ the index of a in the first term is $m-1$, in the 2nd term is $m-2$ and so on, the index of a in the $(m-5)$ term is $m-(m-5)=5$ and that of b is $m-6$ $\because$ the two indices will be together equal to $m-1$

60.
$$x+(ab-1)\,\Big)\,x^3+(ab-1)x^2-(ab+1)x-(a^2b^2-1)\,\Big(\,x^2-(ab+1)$$
$$\underline{x^3+(ab-1)x^2}$$
$$-(ab+1)x-(a^2b^2-1)$$
$$\underline{-(ab+1)x-(a^2b^2-1)}$$

67.
$$(a+b)x+1\,\Big)\,(a^3+b^3)x^3+(2a^2-ab)x^2+2ax+1\,\Big(\,\begin{array}{l}(a^2-ab+b^2)x^2\\ +(a-b)x+1\end{array}$$
$$\underline{(a^3+b^3)x^3+(a^2-ab+b^2)x^2}$$
$$(a^2-b^2)x^2+2ax+1$$
$$\underline{(a^2-b^2)x^2+(a-b)x}$$
$$(a+b)x+1$$
$$(a+b)x+1$$

70.
$$x^2-(x-1)\,\Big)\,\begin{array}{r}x^5-(1+m)x^4+(1+m+n)x^3-(m+n+p)x^2\\ +(p+n)x-p\end{array}\,\Big(\,\begin{array}{r}x^3-mx^2+nx\\ -p\end{array}$$
$$\underline{x^5-x^4+x^3}$$
$$-mx^4+(m+n)x^3$$
$$\underline{-mx^4+mx^3-mx^2}$$
$$nx^3-(n+p)x^2+(p+n)x$$
$$\underline{nx^3-nx^2+nx}$$
$$-px^2+px-p$$
$$-px^2+px-p$$

71.
$$x^2-(a+b)x+ab\,\Big)\,x^3-2ax^2+(a^2+ab-b^2)x-ab(a-b)\,\Big(\,x-(a-b)$$
$$\underline{x^3-(a+b)x^2+abx}$$
$$-ax^2+bx^2+(a^2-b^2)x-ab(a-b)$$
$$-ax^2+bx^2+(a^2-b^2)x-ab(a-b)$$

72. $$\frac{(2x+a)^2(5x+b)^2-(3x+a)^2(4x+b)^2}{(2x+a)\,(5x+b)+(3x+a)\,(4x+b)}=(2x+a)(5x+b)$$

$$-(3x+a)(4x+b)=(10x^2+2bx+5ax+ab)(12x^2+3bx+4ax+ab)$$

$$=10x^2+2bx+5ax+ab-12x^2-3bx-4ax-ab=-2x^2-bx+ax.$$

73 $$(x+3)+1\,\big)\,(x+3)^4-1\,\big(\,(x+3)^3-(x+3)^2+(x+3)-1$$

$$\begin{array}{r}
(x+3)^4+(x+3)^3 \\ \hline
-(x+3)^3-1 \\
-(x+3)^3-(x+3)^2 \\ \hline
(x+3)^2-1 \\
(x+3)^2+(x+3) \\ \hline
-(x+3)-1 \\
-(x+3)-1 \\ \hline
\end{array}$$

74. $$\begin{array}{l}(3x+2)\\+(7x+5)\end{array}\Big)\,(3x+2)^3+(7x+5)^3\,\Big(\begin{array}{l}(3x+2)^2-(3x+2)\\(7x+\cancel{2}5)+(7x+5)^2\end{array}$$

$$\begin{array}{r}
(3x+2)^3+(3x+2)^2(7x+5) \\ \hline
-(3x+2)^2(7x+5)+(7x+5)^3 \\
-(3x+2)^2(7x+5)-(3x+2)(7x+5)^2 \\ \hline
(3x+2)(7x+5)^2+(7x+5)^2 \\
(3x+2)(7x+5)^2+(7x+5)^2 \\ \hline
\end{array}$$

75. $$\begin{array}{l}(4x^2+6y)\\-(3x^2+2y)\end{array}\Big)\,(4x^2+6y)^3-(3x^2+2y)^3\,\Big(\begin{array}{l}(4x^2+6y)^2+(3x^2\\+2y)(4x^2+6y)\\+(3x^2+2y)^2\end{array}$$

$$\begin{array}{r}
(4x^2+6y)^3-(3x^2+2y)(4x^2+6y)^2 \\ \hline
(3x^2+2y)(4x^2+6y)^2-(3x^2+2y)^3 \\
(3x^2+2y)(4x^2+6y)^2-(3x^2+2y)^2(4x^2+6y) \\ \hline
(3x^2+2y)^2(4x^2+6y)-(3x^2+2y)^3 \\
(3x^2+2y)^2(4x^2+6y)-(3x^2+2y)^3 \\
\end{array}$$

77. $$x^p-1\,\big)\,x^{pq}-1\,\big(\,x^{(q-1)p}+x^{(q-2)p}+x^{(q-3)p}+\ldots\ldots x^{2p}+x^p+1$$

$$\begin{array}{l}
x^{pq}-x^{(q-1)p} \\ \hline
x^{(q-1)p}-1 \\
x^{(q-1)p}-x^{(q-2)p} \\ \hline
x^{(q-2)p}-1 \\
x^{(q-2)p}-x^{(q-3)p} \\ \hline
x^{(q-3)p}-1
\end{array}$$

We know that there will be $\frac{pq}{p}=q$ terms in [illegible] we see that the index of x in the 1*st* term is $(q-1)p$ in the 2*nd* term $(q-2)p$, in the 3*rd* term $(q-3)p$ $\therefore$ index of the last or the q*th* term is $(q-q)p=0$ $\therefore$ the last term is $x^0=1$, the last but one term is $x^{\{q-(q-1)\}p}=x^p$, the term immediately before it, is $x^{\{q-(q-2)\}p}$ $=x^{2p}$

78. $a^nb-ab^n-a^nc+ac^n+b^nc-bc^n=a^n(b-c)-a(b^n-c^n)+bc$ $(b^{n-1}-c^{n-1})$, and $(a-b\ (a-c)=a^2-a(b+c)+bc$

$$\begin{array}{r|l|l}
x^2-a(b+c) & a^n(b-c)-a(b^n-c^n)+bc(b^{n-1}-c^{n-1}) & a^{n-2}(b-c)+a^{n-3}(b^2 \\
+bc & a^n(b-c)-a^{n-1}(b^2-c^2)+a^{n-1}bc(b-c) & -c^2)+a^{n-4}(b^3-c^3) \\
\hline
 & a^{n-1}(b^2-c^2)-a(b^n-c^n)-a^{n-2}bc(b-c)+bc(b^{n-1}-c^{n-1}) & \\
 & a^{n-1}(b^2-c^2)-a^{n-2}(b-c)(b+c)^2+a^{n-3}bc\,b^2-c^2) & \\
\hline
 & a^{n-2}(b^3-c^3)-a^{n-3}bc(b^2-c^2)+a(b^n-c^n)+bc(b^{n-1}-c^{n-1}) &
\end{array}$$

By reversing the order of the dividend and divisor we may get the last two or more terms thus

$$\begin{array}{l|l|l}
bc-a(b+c)+a^2 & & \\
 & bc(b^{n-1}-c^{n-1})-a(b^n-c^n)+a^n(b-c) & (b^{n-1}-c^{n-1})+\&c. \\
 & bc(b^{n-1}-c^{n-1})-a(b+c)(b^{n-1}-c^{n-1})+a^2(b^{n-1}-c^{n-1}) &
\end{array}$$

In the 1st portion of this process if we assign any value to n say 4, 5, 6 &c, the remainder after each step of the division is divisible by $a^2-a(b+c)+bc$ thus let $n=4$ then the remainder $a^{n-2}(b^3-c^3)$ $-a^{n-3}bc(b^2-c^2)-a(b^n-c^n)+bc(b^{n-1}-c^{n-1})=a^2(b^{n-1}-c^{n-1})-a$ $(b+c)(b^{n-1}-c^{n-1})+bc(b^{n-1}-c^{n-1})=\{a^2-a(b+c)+bc\}(b^{n-1}-c^{n-1})$

89.
$$\begin{array}{r|l|l}
4x^{-\frac{1}{3}}+3y^{-\frac{2}{3}} & 64x^{-1}+27y^{-2} & \\
 & 64x^{-1}+48x^{-\frac{2}{3}}y^{-\frac{2}{3}} & 16x^{-\frac{2}{3}}-12x^{-\frac{1}{3}}y^{-\frac{2}{3}}+9y^{-\frac{4}{3}} \\
\cline{2-2}
 & -48x^{-\frac{2}{3}}y^{-\frac{2}{3}}+27y^{-2} & \\
 & -48x^{-\frac{2}{3}}y^{-\frac{2}{3}}-36x^{-\frac{1}{3}}y^{-\frac{4}{3}} & \\
\cline{2-2}
 & 36x^{-\frac{1}{3}}y^{-\frac{4}{3}}+27y^{-2} & \\
 & 36x^{-\frac{1}{3}}y^{-\frac{4}{3}}+27y^{-2} & \\
\cline{2-2}
\end{array}$$

FACTORS.

Ex. 8. 7. Resolve the last term 2 into its factors and see which factors being totalled will give a result equal to the coefficient of the 2nd term then split the coefficient in two parts according to the ratio of the factors; thus $2=2\times 1$

$\therefore\ x^2+3x+2=x^2+2x+x+2=x(x+2)+(x+2)=(x+1)(x+2)$

8. $x^2-3x-28=x^2-7x+4x-28=x(x-7)+4(x-7)=(x-4)(x-7)$

9. $x^2+x-30=x^2+6x-5x-30=x(x+6)-5(x+6)=(x-5)(x+6)$

19. $4x^2+8x+3=4x^2+6x+2x+3=2x(2x+3)+(2x+3)$
$=(2x+1)(2x+3)$

26. $12x^2-14x+2=2(6x^2-7x+1)=2\{(6x^2-x)-(6x-1)\}=2$
$\{x(6x-1)-(6x-1)\}=2(x-1)(6x-1)$

56. $a^2-b^2-c^2+2bc=a^2-(b^2-2bc+c^2)=a^2-(b-c)^2=(a+b-c)$
$(a-b+c)$ 57. $(a^2-2ab+b^2)-(c^2-2cd+d^2)=(a-b)^2-(c-d)^2$
$=(a-b+c-d)(a-b-c+d)$

70. Let $a+b=x$ then the expression $=4x^2-4x-3=(2x-3)(2x+1)$. Now substitute the value of x then the expression $=\{2(a+b)-3\}\{2(a+b)+1\}$ 73. $2a(a^2+ab+b^2)-(a-b)(a^2+ab+b^2)$
$=(a^2+ab+b^2)(2a-a+b)=(a^2+ab+b^2)(a+b)$

74. The given quantity $=(1+a^2+2a)(1+c^2)-(1+a^2)(1+c^2+2c)$
$=(1+a^2)(1+c^2)+2a(1+c^2)-(1+c^2)(1+a^2)-2c(1+a^2)=2a(1+c^2)-$
$2c(1+a^2)=2\{a+ac^2-c-a^2c\}=2\{(a-c)-ac(a-c)\}=2(a-c)(1-ac)$

79. $x^4+4=x^4+4x^2+4-(2x)^2=(x^2+2)^2-(2x)^2=(x^2+2+2x)$
(x^2+2-2x)

80. $x^4+a^2x^2+a^4=x^4+2a^2x^2+a^4-(ax)^2=(x^2+a^2)^2-(ax)^2$
$=(x^2-ax+a^2)(x^2+ax+a^2)$

90. $x^2-y^2-z^2+2yz+x+y-z=x^2-(y^2-2yz+z^2)+(x+y-z)=$
$x^2-(y-z)^2+(x+y-z)=(x+y-z)(x-y+z)+(x+y-z)=(x+y-z)$
$(x-y+z+1)$ 91. $35x^3+47x^2+13x+1=35x^3+7x^2+(40x^2$
$+8x)+(5x+1)=7x^2(5x+1)+8x(5x+1)+(5x+1)=(7x^2+8x+1)$
$(5x+1)=(7x+1)(x+1)(5x+1)$

94. The expression $= x^2(y+z) + (y^2z + yz^2) + (xy^2 + xyz) + (xyz + xz^2)$ $= x^2(y+z) + yz(y+z) + xy(y+z) + xz(y+z) = (x^2 + yz + xy + xz)(y+z)$ $= \{x(x+y) + z(x+y)\}(y+z) = (x+z)(x+y)(y+z)$

97 $\frac{(x+y)^3 + z^3}{(x+y) + z} = (x+y)^2 - (x+y)z + z^2$

$\therefore (x+y)^3 + z^3 = (x+y+z)(x^2 + 2xy + y^2 - xz - yz + z^2)$ $\therefore x^3 + y^3 + 3x^2y + 3xy^2 + z^3 = (x+y+z)(x^2 + 2xy + y^2 - xz - yz + z^2)$

Subtract $3x^2y + 3xy^2 + 3xyz = 3xy(x+y+z)$

$\therefore x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - xz - yz)$

98. Put. $x^{\frac{1}{3}}$ for x, $y^{\frac{1}{3}}$ for y, $-z^{\frac{1}{3}}$ for z

99 In ex 97 make $z = -1$ and the result will be the same as before.

100. Let $x + y + z = m$ and $a + b + c = n$ then the expression $= 2m^2 - 7mn + 3n^2 = (2m - n)(m - 3n) = (2x + 2y + 2z - a - b - c)$ $(x + y - z - 3a - 3b - 3c)$

101 The expression $= 1^3 - (x^2 + y^2 + z^2)1^2 + (x^2y^2 + x^2z^2 + y^2z^2)$

$\times 1 - x^2y^2z^2 - x\left(1 + \frac{xy}{z}\right) \times y\left(1 + \frac{xz}{y}\right) \times z\left(1 + \frac{yz}{x}\right)$

$= 1^3 - (x^2 + y^2 + z^2) + (x^2y^2 + x^2z^2 + y^2z^2) - x^2y^2z^2 - xyz$

$\left\{1^3 + \left(\frac{xy}{z} + \frac{xz}{y} + \frac{yz}{x}\right) \times 1^2 + \left(\frac{xy\,xz}{yz} + \frac{xy\,yz}{xz} + \frac{xz\,yz}{xy}\right) \times 1\right.$

$\left. + \frac{xy \times xz \times yz}{xyz}\right\} = 1 - (x^2 + y^2 + z^2) + x^2y^2 + x^2z^2 + y^2z^2$

$- x^2y^2z^2 - xyz\left\{1 + \frac{xy}{z} + \frac{xz}{y} + \frac{yz}{x} + x^2 + y^2 + z^2 + xyz\right\}$

$= 1 - x^2 - y^2 - z^2 + x^2y^2 + x^2z^2 + y^2z^2 - x^2y^2z^2 - xyz(1 + x^2 + y^2 + z^2)$

$- x^2y^2z^2 - x^2y^2 - x^2z^2 - y^2z^2 = 1 - x^2 - y^2 - z^2 - xyz - xyz$

$(x^2 + y^2 + z^2) - 2x^2y^2z^2 = 1 - x^2 - y^2 - z^2 - 2xyz + xyz - xyz$

$(x^2 + y^2 + z^2) - 2x^2y^2z^2 = (1 - x^2 - y^2 - z^2 - 2xyz) + xyz$

$(1 - x^2 - y^2 - z^2 - 2xyz) = (1 + xyz)(1 - x^2 - y^2 - z^2 - 2xyz)$

103. $(a^2+b^2-c^2)^2-(2ab)^2=\{(a^2+2ab+b^2)-c^2\}\{(a^2-2ab+b^2)-c^2\}=\{(a+b)^2-c^2\}\{(a-b)^2-c^2\}=(a+b+c)(a+b-c)(a-b+c)(a-b-c)$

104. $x^4-y^4-(x^2-y^2)^2+2y(x^3+y^3)=(x^2-y^2)\{x^2+y^2-x^2+y^2\}+2y(x^3+y^3)=(x+y)(x-y)2y^2+2y(x+y)(x^2-xy+y^2)=2y(x+y)\{(x-y)y+x^2-xy+y^2\}=2x^2y(x+y)$

GREATEST COMMON MEASURE.

7 The 1st quantity $=\left(x-\frac{1}{a}\right)(x-a)$ and the 2nd quantity $=\left(x-\frac{1}{a}\right)\left(x-\frac{1}{a}\right)$ ∴ G. C. M. is $x-\frac{1}{a}$

By division

$$\begin{array}{l|l|l}
x^2-ax- & x^2-2\frac{1}{a}x+\frac{1}{a^2} & \\
\frac{1}{a}x+1 & x^2-ax-\frac{1}{a}x+1 & 1 \\
\cline{2-2}
 & ax-\frac{1}{a}x+\frac{1}{a^2}-1 & \\
 & =a\left(x-\frac{1}{a}\right)-\frac{1}{a}\left(x-\frac{1}{a}\right) & \\
 & =\left(a-\frac{1}{a}\right)\left(x-\frac{1}{a}\right) &
\end{array}$$

Rejecting the factor in which no x is involved we have

$$\begin{array}{l|l|l}
x-\frac{1}{a} & x^2-ax-\frac{1}{a}x+1 & x-a \\
 & x^2-\frac{1}{a}x & \\
\cline{2-2}
 & -ax+1 & \\
 & -ax+1 & \\
\cline{2-2}
\end{array}$$

Hence $x-\frac{1}{a}$ is the G. C. M.

10. $x^2+11x+30=x^2+5x+6x+30=x(x+5)+6(x+5)=(x+6)(x+5)$, $x^2+13x+42=x^2+7x+6x+42=(x+7)(x+6)$ $\therefore$ $x+6$ is the G. C. M. By division. Make either of the 2 quantities which has the same highest power as the dividend and the other the divisor and as a general rule make that quantity the dividend which has the highest power.

18.
$$\begin{array}{r|r|l}
6x^2+x-2 & 9x^3+48x^2+52x+16 & \\
 & 2 & 3x \\
\hline
 & 18x^3+96x^2+104x+32 & \\
 & 18x^3+3x^2-6x & \\
\hline
 & 93x^2+110x+32 & \\
 & 2 & \\
 & 186x^2+220x+64 & 31 \\
 & 186x^2+31x-62 & \\
\hline
 & 189x+126 & \\
 & =63(3x+2) &
\end{array}$$

$$\begin{array}{r|l|l}
3x+2 & 6x^2+x-2 & 2x-1 \\
 & 6x^2+4x & \\
\hline
 & -3x-2 & \\
 & -3x-2 & \\
\hline
 & 0 &
\end{array}$$

thus $3x+2$ is the G. C. M.

5. The 1st quantitity $=m^{2x}(a^3+1)-(a^3+1)=(m^{2x}-1)(a^3+1)=(m^x+1)(m^x-1)(a+1)(a^2-a+1)$.

The 2nd quantity $=m^{2x}(a^2-1)+2m^x(a^2-1)+(a^2-1)=(m^{2x}+2m^x+1)(a^2-1)=(m^x+1)^2(a+1)(a-1)$ thus $(m^x+1)(a+1)$ or am^x+m^x+a+1 is the G. C. M.

36. Find the G. C. M. of the 1st and 2nd and then find the G. C. M. of this result and the 3rd,

43.
$$\begin{array}{r|l|l}
x^2-y^2+(x-y)\sqrt{xy} & x^2+2y^2+(x+2y)\sqrt{(xy)} & 1 \\
 & x^2-y^2+(x-y)\sqrt{(xy)} & \\
\hline
 & 3y^2+3y\sqrt{xy}=3y\sqrt{y}(\sqrt{x}+\sqrt{y}) &
\end{array}$$

$$\begin{array}{r|l|l}
\sqrt{x}+\sqrt{y} & x^2+x\sqrt{(xy)}-y\sqrt{(xy)}-y^2 & x\sqrt{x}-y\sqrt{y} \\
 & x^2+x\sqrt{(xy)} & \\
\hline
 & -y\sqrt{(xy)}-y^2 & \\
 & -y\sqrt{(xy)}-y^2 & \\
\hline
\end{array}$$

thus $\sqrt{x}+\sqrt{y}$ is the G.C.M

44. $$\begin{array}{r|l}
x^3+(a-1)x^2-(a-1)x+a & x^3+(a+1)x^2+(a+1)x+a \;(\;1 \\
 & \underline{x^3+(a-1)x^2-(a-1)x+a} \\
 & 2x^2+2ax=2x(x+a)
\end{array}$$

$$\begin{array}{r|l}
x+a & x^3+(a-1)x^2-(a-1)x+a \;(\;x^2-x+1 \\
 & \underline{x^3+ax^2} \\
 & -x^2-(a-1)x+a \\
 & \underline{-x^2-ax} \\
 & x+a \\
 & \underline{x+a}
\end{array}$$

hence $x+a$ is the G. C. M.

46. $x^4-2a(a-b)$

$$\begin{array}{r|l}
x^2+(a^2+b^2) & x^4-(a-b)x^3+(a-b)b^2x-b^4 \;(\;1 \\
(a-b)x-a^2b^2 & \underline{x^4-2a(a-b)x^2+(a^2+b^2)(a-b)x-a^2b^2} \\
 & -(a-b)x^3+2a(a-b)x^2-(a-b)a^2x+b^2(a^2-b^2)
\end{array}$$

reject $-(a-b)$ from all the terms.

$$\begin{array}{r|l}
x^3-2ax^2+a^2x & x^4-2a(a-b)x^2+(a^2+b^2)(a-b)x-a^2b^2 \;(\;x+2 \\
-b^2(a+b) & \underline{x^4-2ax^3+a^2x^2-b^2(a+b)x} \\
 & 2ax^3-(3a^2-2ab)x^2+(a^3+2ab^2-a^2b)x-a^2b^2 \\
 & \text{or } 2x^3-(3a-2b)x^2+(a^2+2b^2-ab)x-ab^2 \\
 & \underline{2x^3-4ax^2+2a^2x-2b^2(a+b)} \\
 & (a+2b)x^2+(2b^2-ab-a^2)x+b^2(a+2b) \\
 & =(a+2b)\{x^2+(b-a)x+b^2\}
\end{array}$$

$$\begin{array}{r|l}
x^2+(b-a)x+b^2 & x^3-2ax^2+a^2x-b^2(a+b) \;(\;x-(a+b) \\
 & \underline{x^3+(b-a)x^2+b^2x} \\
 & -(a+b)x^2+(a^2-b^2)x-b^2(a+b) \\
 & -(a+b)x^2+(a^2-b^2)x-b^2(a+b)
\end{array}$$

thus $x^2+(b-a)x+b^2$ is the G. C. M.

47. $$\begin{array}{r|l}
a^2-b^2+c^2-2ac & a^2+b^2+c^2-2ab-2ac+2bc \;(\;1 \\
 & \underline{a^2-b^2+c^2-2ac} \\
 & -2ab+2bc+2b^2=-2b(a-c-b)
\end{array}$$

$$\begin{array}{r|l}
a-c-b & a^2-b^2+c^2-2ac \;(\;a-c+b \\
 & \underline{a^2-ac-ab} \\
 & -ac+ab-b^2+c^2 \\
 & \underline{-ac+c^2+bc} \\
 & ab-b^2-bc \\
 & \underline{ab-bc-b^2} \\
 & 0
\end{array}$$

hence $a-c-b$ is the G. C. M.

LEAST COMMON MULTIPLE.

Ex. 10. 11. $\begin{array}{r|lll} a-x & a-x, & a^2-x^2, & a^2+ax \\ \hline a+x & 1, & a+x, & a^2+ax \\ \hline & 1, & 1\;, & a \end{array}$ $\quad\therefore a(a-x)(a+x)$ is the L. C. M.

32. $x^2+5x+6=(x+3)(x+2)$, $x^2+6x+8=(x+4)(x+2)$

$$\therefore \text{L. C. M.}=\frac{\text{Product of the quantities}}{\text{G. C. M.}}=\frac{(x+3)(x+2)(x+4)(x+2)}{x+2}$$
$$=(x+3)(x+4)(x+2)$$

48. $x^3-3x^2+3x-1=(x-1)(x^2-2x+1)$; $x^3-x^2-x+1=(x+1)(x-1)^2$; $x^4-2x^3+2x-1=(x+1)(x-1)(x^2-2x+1)$; $x^4-2x^3+2x^2-2x+1=(x^2+1)(x^2-2x\quad)$ $\therefore$ $(x^4-1)(x^2-2x+1)$ is the L. C. M.

50. $\text{L. C. M}=(x^4-x^2+1)(x^2-x+1)(x^2+x+1)=(x^4-x^2+1)$
$\{(x^2+1)^2-x^2\}=(x^4-x^2+1)(x^4+x^2+1)=x^8+x^4+1$

EASY IDENTITIES.

Ex. 11. 1. $(xy+xz)(y^2+z^2-x^2)+(xy+yz)(z^2+x^2-y^2)+(xz+zy)$
$(x^2+y^2-z^2)=xy^3+xyz^2-x^3y+xy^2z+xz^3-x^3z+xyz^2+x^3y-xy^3$
$+yz^3+x^2yz-y^3z+x^3z+xy^2z-xz^3+x^2yz+y^3z-yz^3$
$$=2xyz^2+2xy^2z+2x^2yz=2xyz(x+y+z)$$

2.
$$\begin{array}{l}(x+y+z)^3=x^3+y^3+z^3+x^2(y+z)+y^2(x+z)+z^2(x+y)+6xyz \\ (x-y-z)^3=x^3-y^3-z^3-x^2(y+z)+y^2(x-z)+z^2(x-y)+6xyz \\ (y-z-x)^3=y^3-z^3-x^3+x^2(y-z)-y^2(x+z)-z^2(x-y)+6xyz \\ (z-y-x)^3=z^3-y^3-x^3-x^2(y-z)-y^2(x-z)-z^2(x+y)+6xyz \\ \hline \therefore \text{Sum}=24xyz\end{array}$$

3. $a^2(b^2+2bc+c^2)+b^2(a^2+2ac+c^2)+c^2(a^2+2ab+b^2)+2a^2bc$
$2ab^2c+2abc^2=a^2b^2+2a^2bc+a^2c^2+a^2b^2+2ab^2c+b^2c^2+a^2c^2$
$+2ac^2b+b^2c^2+2a^2bc+2ab^2c+2abc^2=2(a^2b^2+a^2c^2+b^2c^2$
$+2a^2bc+2ab^2c+2abc^2)=2(ab+ac+bc)^2$

4. $x(y^2+2yz+z^2)+y(z^2+2xz+x^2)+z(x^2+2xy+y^2)$
$=xy^2+2xyz+xz^2+yz^2+2xyz+x^2y+x^2z+2xyz+y^2z$
$=(xy^2+xz^2+yz^2+x^2y+x^2z+y^2z+2xyz)+4xyz$
$=\{(xy^2+y^2z)+(xz^2+x^2z)+(xyz+yz^2)+(x^2y+xyz)\}+4xyz$
$=\{y^2(x+z)+xz(x+z)+yz(x+z)+xy(x+z)\}+4xyz$
$=(y^2+xz+yz+xy)(x+z)+4xyz=(x+y)(y+z)(x+z)+4xyz$

5. The first expression $=(x+y+z)^2-(x+y)^2=(x+y)^2+2z(x+y)+z^2-(x+y)^2=2xz+2yz+z^2$ and the second expression $=y^2+2yz+z^2+x^2+2xz+z^2-x^2-y^2-z^2=2xz+2yz+z^2$

6. $(a-b)^3+b^3=a^3-b^3-3ab(a-b)+b^3=a^3-3ab(a-b)$

7. Let $x-y=a$, $y-z=b$, $z-x=c$ then $a+b+c=0$
then $a+b=-c$ or $(a+b)^3=-c^3$ or $a^3+b^3+3ab(a+b)=-c^3$
or $a^3+b^3-3abc=-c^3$ (for $a+b=-c$) $\therefore$ $a^3+b^3+c^3=3abc$
or $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$

9. The expression $=(x+y+z)\{x(y+z)+yz\}-xyz=x^2(y+z)+xyz+x(y+z)^2+yz(y+z)-xyz=(y+z)(x^2+xy+xz+yz)$
$=(y+z)\{x(x+y)+z(x+y)\}=(y+z)(x+z)(x+y)$

10. $(a+b+c)^3=a^3+3a^2(b+c)+3a(b+c)^2+(b+c)^3=a^3+3a^2(b+c)+3a(b^2+2bc+c^2)+(b^3+3b^2c+3bc^2+c^3)=a^3+b^3+c^3+3a^2(b+c)+3b^2(a+c)+3c^2(a+b)+6abc$

11. $\{x^3+y^3+z^3+3x^2(y+z)+3y^2(x+z)+3z^2(x+y)+6xyz\}$
$-\{y^3+z^3-x^3+3y^2(z-x)+3z^2(y-x)+3x^2(y+z)-6xyz\}-\{x^3$
$+z^3-y^3+3x^2(z-y)+3z^2(x-y)+3y^2(x+z)-6xyz\}-\{x^3+y^3$
$-z^3+3x^2(y-z)+3y^2(x-z)+3z^2(x+y)-6xyz\}=24xyz$

12. The expression $=4\{(a+b+c)^2+(a+b-c)^2+(a-b+c)^2+(b+c-a)^2\}=4\{a+b^2+c^2+2ab+2ac+2bc+a^2+b^2+c^2+2ab$
$-2ac-2bc+a^2+b^2+c^2-2ab+2ac-2bc+b^2+c^2+a^2+2bc$
$-2ab-2ac\}=4\{4(a^2+b^2+c^2)\}=16(a^2+b^2+c^2)$

13. $(a+b)^2+(a-b)^2=2a^2+2b^2$ and $(a-c)^2+(a+c)^2=2a^2+2c^2$
$\therefore$ Sum $=4a^2+2b^2+2c^2$

14. The expression $=(x+y)^2+2(x^2-y^2)+(x-y)^2=(\overline{x+y}+\overline{x-y})^2$
$=(2x)^2=4x^2$

REDUCTION OF FRACTIONS.

Ex. 12. 1. $$\begin{array}{r|l} 11\,) & 61x\,(\,5x \\ & \underline{55x} \\ & 6x \end{array} \qquad \therefore \text{ result} = 5x + \frac{6x}{11}$$

2. $$\begin{array}{r|l} 7\,) & 21ax + 4x\,(\,3ax \\ & \underline{21ax} \\ & 4x \end{array} \qquad \therefore \text{ result} = 3ax + \frac{4x}{7}$$

3. $$\begin{array}{r|l} x+4\,) & 10x^2 + 40x + 6\,(\,10x \\ & \underline{10x^2 + 40x} \\ & 6 \end{array} \qquad \therefore \text{ result} = 10x + \frac{6}{x+4}$$

16. The expression $= \dfrac{(a^2+b^2)(a-b)(a+b)}{(a+b)(a+b)} = \dfrac{(a^2+b^2)(a-b)}{a+b}$

17. The expression $= \dfrac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)xy} = \dfrac{1}{(x-y)xy}$

18. The fraction $= \dfrac{(x+7)(x+6)}{(x+7)(x+10)} = \dfrac{x+6}{x+10}$

19. The fraction $= \dfrac{x^2(x-11)-(x-11)}{x^2(x-11)+2(x-11)} = \dfrac{(x^2-1)(x-11)}{(x^2+2)(x-11)} = \dfrac{x^2-1}{x^2+2}$

25. The fraction $= \dfrac{(x+a)(x-c)}{(x+a)(x-b)} = \dfrac{x-c}{x-b}$

26. The fraction $= \dfrac{(x+b+a+c)(x+b-a-c)}{(x+c+a+b(x-a-b+c)} = \dfrac{x+b-a-c}{x-a-b+c}$

40. The fraction $= \dfrac{(x^4+a^4)(x^2+a^2)(x+a)(x-a)}{(x^3+a^3)(x^3-a^3)}$

$$= \frac{(x^4+a^4)(x^2+a^2)(x+a)(x-a)}{(x+a)(x^2-ax+a^2)(x-a)(x^2+ax+a^2)}$$

$$= \frac{(x^4+a^4)(x^2+a^2)}{(x^2-ax+a^2)(x^2+ax+a^2)} = \frac{(x^4+a^4)(x^2+a^2)}{x^4+a^2x^2+a^4}$$

41. Here l. c. m. of the denominators $= 42x^3 \therefore \dfrac{6}{7x} = \dfrac{6\times 6x^2}{42x^3}$

$$= \frac{36x^2}{42x^3},\ \frac{5}{21x^2} = \frac{5\times 2x}{42x^3} = \frac{10x}{42x^3},\ \frac{2}{14x^3} = \frac{2\times 3}{42x^3} = \frac{6}{42x^3}$$

48. The L. C. M. of the denominators $=(1+x^n)(1-x^n)(1+x^n+x^{2n})$
$=(1+x^n)(1-x^{3n})$

ADDITION AND SUBTRACTION OF FRACTIONS.

Ex. 13. 1. $\dfrac{18x+35x+30x}{45}=\dfrac{83x}{45}$

2. $\dfrac{20x+6x+40+x}{10}=\dfrac{40+27x}{10}$

19. $$x\left(\frac{1}{(4a)^x+1}-\frac{1}{(4a)^x-1}\right)=x\left(\frac{(4a)^x-1-(4a)^x-1}{(4a)^{2x}-1}\right)$$
$$=x\left(\frac{-2}{(4a)^{2x}-1}\right)=\frac{2x}{1-(4a)^{2x}}$$

30. $$\frac{\{\sqrt{(a^2+1)}+\sqrt{(a^2-1)}\}^2+\{\sqrt{(a^2+1)}-\sqrt{(a^2-1)}\}^2}{(a^2+1)-(a^2-1)}=\frac{4a^2}{2}=2a^2$$

33. The given expression $=\dfrac{1}{x^2+5x+6}-\dfrac{1}{x^2+4x+4}+\dfrac{1}{(x+3)^2}$
$$=\frac{1}{(x+2)(x+3)}-\frac{1}{(x+2)^2}+\frac{1}{(x+3)^2}=\frac{(x+2)(x+3)-(x+3)+(x+2)^2}{(x+2)^2(x+3)^2}$$
$$=\frac{x^2+3x+1}{(x+2)^2(x+3)^2}$$

36. The given expression $=\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(a-c)(b-c)}$
$$=\frac{b-c+a-b}{(a-b)(a-c)(b-c)}=\frac{a-c}{(a-b)(a-c)(b-c)}=\frac{1}{(a-b)(b-c)}$$

59. $$\frac{bc(x-a)^2}{(a-b)(a-c)}-\frac{ac(x-b)^2}{(b-c)(a-b)}+\frac{ab(x-c)^2}{(a-c)(b-c)}$$
$$=\frac{(b-c)bc(x-a)^2-ac(a-c)(x-b)^2+ab(a-b)(x-c)^2}{(a-b)(b-c)(a-c)}$$

$$=\frac{bc(b-c)(x^2-2ax+a^2)-ac(a-c)(x^2-2bx+b^2)+ab(a-b)(x^2-2cx+c^2)}{(a-b)(b-c)(a-c)}$$

$$=\frac{\{bc(b-c)-ac(a-c)+ab(a-b)\}x^2+2x\{abc(a-c)-abc(b-c)-abc}{(a-b)(b-c)(a-c)}$$

$$\frac{(a-b)\}+a^2bc(b-c)-ab^2c(a-c)+abc^2(a-b)}{(a-b)(b-c)(a-c)}$$

$$=\frac{(b^2c-bc^2-a^2c+ac^2+a^2b-ab^2)x^2+(a^2bc-abc^2-ab^2c+abc^2-a^2bc}{(a-b)(a-c)(b-c)}$$

$$\frac{+ab^2c)2x+a^2b^2c-a^2bc^2-a^2b^2c+ab^2c^2+a^2bc^2-ab^2c^2}{(a-b)(b-c)(a-c)}$$

$$=\frac{(b^2c-bc^2-a^2c+ac^2+a^2b-ab^2)x^2}{(a-b)(a-c)(b-c)}=\frac{(a-b)(a-c)(b-c)x^2}{(a-b)(a-c)(b-c)}=x^2$$

65. $$\frac{1}{2(3-\sqrt{x})}+\frac{1}{2(3+\sqrt{x})}-\frac{3}{9+x}=\frac{3+\sqrt{x}+3-\sqrt{x}}{2(9-x)}-\frac{3}{9-x}$$

$$=\frac{3}{9-x}-\frac{3}{9+x}=3\left(\frac{9+x-9+x}{81-x^2}\right)=\frac{6x}{81-x^2}$$

67. $y-x=-x+y=-(x-y)$ and similarly $z-x=-(x-z)$,

$z-y=-(y-z)$ $\therefore$ the expression $=\dfrac{x^2}{(x-y)(x-z)}$

$$-\frac{y^2}{(x-y)(y-z)}+\frac{z^2}{(x-z)(y-z)}=\frac{x^2(y-z)-y^2(x-z)+z^2(x-y)}{(x-y)(x-z)(y-z)}$$

$$=\frac{x^2y-x^2z-xy^2+y^2z+xz^2-yz^2}{(x-y)(x-z)(y-z)}=\frac{xy(x-y)-z(x^2-y^2)-z^2(x-y)}{(x-y)(x-z)(y-z)}$$

$$=\frac{\{xy-z(x+y)-z^2\}(x-y)}{(x-y)(x-z)(y-z)}=\frac{x(y-z)-z(y-z)}{(x-z)(y-z)}=\frac{(x-z)(y-z)}{(x-z)(y-z)}=1,$$

58. $$\frac{m}{a^m-1}+\frac{m}{\frac{1}{a^m}-1}=m\left\{\frac{1}{a^m-1}+\frac{1}{\frac{1-a^m}{a^m}}\right\}$$

$$=m\left\{\frac{1}{a^m-1}+\frac{a^m}{1-a^m}\right\}=m\left\{\frac{1}{a^m-1}-\frac{a^m}{a^m-1}\right\}$$

$$=m\left\{\frac{1-a^m}{a^m-1}\right\}=-m\left\{\frac{1-a^m}{1-a^m}\right\}=-m$$

71 $$\frac{1}{1+x^{m-n}+x^{m-p}}+\frac{1}{1+x^{-(m-n)}+x^{n-p}}+\frac{1}{1+x^{-(m-p)}+x^{p-n}}$$

$$=\frac{1}{1+x^{m-n}+x^{m-p}}+\frac{1}{1+\frac{1}{x^{m-n}}+x^{n-p}}+\frac{1}{1+\frac{1}{x^{m-p}}+x^{p-n}}$$

$$=\frac{1}{1+x^{m-n}+x^{m-p}}+\frac{x^{m-n}}{x^{m-n}+1+x^{m-p}}+\frac{x^{m-p}}{x^{m-p}+1+x^{m-n}}$$

$=\frac{1+x^{m-n}+x^{m-p}}{1+x^{m-n}+x^{m-p}}=1$ If 4, 5, 6 &c. terms be taken in the denominators as in the following ex. the result $=1$ $\frac{1}{1+a^{x-y}+a^{x-z}+a^{x-w}}$

$$+\frac{1}{1+a^{y-x}+a^{y-z}+a^{y-w}}+\frac{1}{1+a^{z-x}+a^{z-y}+a^{z-w}}$$

$$+\frac{1}{1+a^{w-x}+a^{w-y}+a^{w-z}}=1.$$

72. The expression $=\left(\frac{x^{3n}}{x^n-1}-\frac{1}{x^n-1}\right)-\left(\frac{x^{2n}}{x^n+1}-\frac{1}{x^n+1}\right)$

$$=\frac{x^{3n}-1}{x^n-1}-\frac{x^{2n}-1}{x^n+1}=(x^{2n}+x^n+1)-(x^n-1)=x^{2n}+2$$

77. Divide the numerator by the denominator of the fraction the result is $x+y-1$ $\therefore$ $(x+y-1)-y+1=x$

82 $$\frac{(x+y-z)(x-y+z)}{(x+y+z)(x-y+z)}+\frac{(y+x-z)(y-x+z)}{(x+y+z)(x+y-z)}$$

$$+\frac{(z+x-y)(z-x+y)}{(y+z+x)(y+z-x)}=\frac{x+y-z}{x+y+z}+\frac{y-x+z}{x+y+z}+\frac{z+x-y}{x+y+z}$$

$$=\frac{x+y-z+y-x+z+z+x-y}{x+y+z}=\frac{x+y+z}{x+y+z}=1$$

83. The expression $=\frac{1}{x(x-y)(x-z)}-\frac{1}{y(y-z)(x-y)}+\frac{1}{z(x-z)(y-z)}$

$$=\frac{yz(y-z)-xz(x-z)+xy(x-y)}{xyz(x-y)(x-z)(y-z)}=\frac{yz(y-z)-x^2z+xz^2+x^2y-xy^2}{xyz(x-y)(x-z)(y-z)}$$

$$=\frac{yz(y-z)+x^2(y-z)-x(y^2-z^2)}{xyz(x-y)(x-z)(y-z)}=\frac{yz+x^2-xy-xz}{xyz(x-y)(x-z)}$$

$$=\frac{x(x-y)-z(x-y)}{xyz(x-y)(x-z)}=\frac{1}{xyz}$$

84 $$\frac{1}{(a-b)(a-c)(x+a)}-\frac{1}{(a-b)(b-c)(x+b)}+\frac{1}{(a-c)(b-c)(x+c)}$$

$$=\frac{(b-c)(x+b)(x+c)-(a-c)(x+a)(x+c)+(a-b)(x+a)(x+b)}{(a-b)(a-c)(b-c)(x+a)(x+b)(x+c)}$$

of which the numerater $=(b-c)\{x^2+(b+c)x+bc\}-(a-c)\{x^2+(a+c)x+ac\}+(a-b)\{x^2+(a+b)x+ab\}=\{b-c-(a-c)+(a-b)\}x^2+(b^2-c^2-a^2+c^2+a^2-b^2)x+(b-c)bc-(a-c)ac+(a-b)ab,=(b-c)bc-(a-c)ac+(a-b)ab=(a-b)(ab-ac-ac+c^2)=(a-b)(a-c)(b-c)$ $\therefore$ the given fraction $=\dfrac{1}{(x+a)(x+b)(x+c)}$

88 The expression $=\dfrac{(1+xy)(1+xz)}{(x-y)(z-x)}-\dfrac{(1+yz)(1+xy)}{(x-y)(z-y)}-\dfrac{(1+xz)(1+yz)}{(z-x)(z-y)}$

$$=\frac{(1+xy)(1+xz)(z-y)-(1+yz)(1+xy)(z-x)-(1+xz)(1+yz)(x-y)}{(x-y)(z-x)(z-y)}$$

$=\dfrac{(x-y)(z-x)(z-y)}{(x-y)(z-x)(z-y)}$ (when the numerator is simplified) $=1$

89. The expression $=$

$$\frac{2x^3y^3+2xy-2xy(x^2+y^2)+4x^2y^2-(x^2+y^2)(1+x^2y^2)}{2x^3y^3+2xy+2xy(x^2+y^2)-4x^2y^2-(x^2+y^2)(1+x^2y^2)}$$

$$=\frac{2x^3y^3+4x^2y^2+2xy-2xy(x^2+y^2)-(x^2+y^2)(1+x^2y^2)}{2x^3y^3-4x^2y^2+2xy+2xy(x^2+y^2)-(x^2+y^2)(1+x^2y^2)}$$

$$=\frac{2xy(x^2y^2+2xy+1)-(x^2+y^2)(x^2y^2+2xy+1)}{2xy(x^2y^2-2xy+1)-(x^2+y^2)(x^2y^2-2xy+1)}$$

$$=\frac{(2xy-x^2-y^2)(xy+1)^2}{(2xy-x^2-y^2)(xy-1)^2}=\left(\frac{xy+1}{xy-1}\right)^2$$

90. When the fractions are all added the numerator becomes $2(b-c)(c-a)+2(a-b)(c-a)+2(a-b)(b-c)+(a-b)^2+(b-c)^2+(c-a)^2$ and this is evidently the square of $(b-c)+(c-a)+(a-b)$ *ie.* of zero ∴ the numerator is zero ∴ the result is zero.

MULTIPLICATION OF FRACTIONS.

Ex. 14. 1. $\frac{2x}{3}\times\frac{6}{8x}=\frac{1}{2}$

2. $\frac{2x}{a}\times\frac{a^2}{mx}\times\frac{m^2}{x}=\frac{2am}{x}$

3. $\frac{ax}{m+1}\times\frac{m^2-1}{a^2x^2}\times\frac{a}{x}=\frac{a^2x(m^2-1)}{a^2x^3(m+1)}=\frac{m-1}{x^2}$

4. $\frac{4x^2}{5y^2}\times\frac{20y^2}{6z^2}\times\frac{6z^2}{4x^2}=\frac{20y^2}{5y^2}=4$

5. $\frac{x^2+xy}{x^3-y^3}\times\frac{x^2-y^2}{(x+y)^2}\times\frac{x^2+xy+y^2}{x}$

$$=\frac{x(x+y)\times(x+y)(x-y)\times(x^2+xy+y^2)}{(x-y)(x^2+xy+y^2)(x+y)^2\times x}=1$$

7. $\frac{a^2}{a-x}\times\frac{a-x}{a-2x}\times\frac{a^2-4x^2}{a^3-x^3}\times\frac{a^2-x^2}{a^2}$

$$\frac{a^2(a-x)(a-2x)(a+2x)(a-x)(a+x)}{(a-x)(a-2x)(a-x)(a^2+ax+x^2)a^2}=\frac{(a+2x)(a+x)}{a^2+ax+x^2}$$

8 $\frac{a^{3m}-x^{3m}}{a^m-x^m}\times\frac{a^m x^{m+2}}{a^{2m}+a^m x^m+x^{2m}}\times\frac{1}{x^m x^2}$

$$\frac{(a^m-x^m)(a^{2m}+a^m x^m+x^{2m})(a^m x^{m+2})}{(a^m-x^m)(a^{2m}+a^m x^m+x^{2m})(x^{m+2})}=a^m$$

9 $\frac{x^2+5x+6}{x+2}\times\frac{x+5}{x^2+10x+21}\times\frac{x+7}{x^2}$

$$=\frac{(x+3)(x+2)(x+5)(x+7)}{(x+2)(x+7)(x+3)x^2}=\frac{x+5}{x^2}$$

10 $$\frac{\sqrt{x}-\sqrt{y}}{x-y} \times \frac{x+2\sqrt{(xy)}+y}{\sqrt{(xy)}} \times 4$$

$$= \frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})^2 \times 4}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})\sqrt{(xy)}} = \frac{4(\sqrt{x}+\sqrt{y})}{\sqrt{(xy)}}.$$

11. $$\frac{x+y}{(x-y)^3} \times \frac{x^3-y^3}{x^3+y^3} \times \frac{(x-y)^2+xy}{(x+y)^2-xy} \times (x-y)^2$$

$$= \frac{(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)(x-y)^2}{(x-y)^3 \times (x+y)(x^2-xy+y^2)(x^2+xy+y^2)} = 1$$

12. $$\frac{\frac{a+x}{a-x}+\frac{a-x}{a+x}}{a^2+x^2} \times \frac{a+x}{2} \times 6x$$

$$= \frac{2(a^2+x^2) \times (a+x) \times 6x}{(a^4-x^4) \times (a^2+a^2) \times 2} = \frac{6x}{a-x}$$

13 $$\left(\frac{(\sqrt{a}+a)}{(\sqrt{a}-x)} - \frac{(\sqrt{a}-x)}{(\sqrt{a}+x)}\right) \times (a^2-x^2) \times \sqrt{x}$$

$$= \frac{2x(a^2-x^2)\sqrt{x}}{(\sqrt{a^2}-x^2)} = 2 \times x^{\frac{3}{2}}\sqrt{(a^2-x^2)}$$

14 $$\frac{x^2-3x-10}{x^2+6x-7} \times \frac{x^2+10x+21}{x^2-3x-10} \times \frac{x^2+3x-4}{x^2+7x+12}$$

$$= \frac{x^2-3x-10}{(x+7)(x-1} \times \frac{(x+3)\ x+7)}{(x^2-3x-10)} \times \frac{(x+4)(x-1)}{(x+4)(x+3)} = 1$$

15. $$\frac{a^4+a^2+1}{a^2} \times \left(a-\frac{1}{a}\right) = \frac{a^4+a^2+1}{a^2} \times (a-a^{-1})$$

$$= \frac{a^5-a^{-1}}{a^2} = a^3-a^{-3}$$

16 $$x^{m+n+p} \times (2x)^{4n-m-p} \times 2^{m+p-4n} \times 4 = x^{m+n+p+4n-m-p}$$

$$\times\, 2^{4n-m-p+m+p-4n} \times 4 = x^{5n} \times 1 \times 4 = 4x^{5n}$$

17 $$(x^{m+n})^2(x^{n-p})^2(x^{-m+p})^2 = x^{2(m+n)+2(n-p)+2(-m+p)} = x^{4n}$$

18. $$\left(\frac{2x^2}{7} - \frac{99}{49}x + \frac{1}{7}\right) \times \frac{7}{x-7} = \frac{14x^2-99x+7}{49} \times \frac{7}{(x-7)}$$

$$= \frac{14x^2-99x+7}{7x-49} = 2x-\frac{1}{7}$$

19. $$\frac{(a-x)^2}{x(a+x)} \times \frac{x^2}{a^2-x^2} = \frac{x(a-x)}{(a+x)^2}$$

20. $$\left(x^3 - x + \frac{1}{x} - \frac{1}{x^3}\right) \times \left(x + \frac{1}{x}\right)$$

$$= (x^3 - x + x^{-1} - x^{-3})(x + x^{-1}) = x^4 - x^{-4}$$

21. $$\frac{a+b+c}{a+b-c} \times (a^2 - b^2 + c^2 - 2ac) = \frac{a+b+c}{a+b-c} + (a-c+b)(a-c-b)$$

$$= (a+b+c)(a-b-c) = a^2 - b^2 - c^2 - 2bc$$

22. $$\left(x + \frac{1}{x} - 1\right)\left(\frac{x^2+x+1}{x}\right)$$

$$= (x + x^{-1} - 1)(x + x^{-1} + 1) = (x + x^{-1})^2 - 1 = x^2 + x^{-2} + 1$$

23. $$\frac{(a+x)^2}{x^2+xy+y^2} \times \frac{a+x}{x-y} = \frac{(a+x)^3}{x^3-y^3}$$

24. $$\left(\frac{a+b}{c-d} + \frac{a-b}{c+d}\right) \times \frac{ac-bd}{ac+bd} = \frac{2(ac+bd) \times (ac-bd)}{(c^2-d^2)\,(ac+bd)} = \frac{2(ac-bd)}{c^2-d^2}$$

25. $$\frac{a^3-b^3}{x^3+y^3} \times \frac{x+y}{a-b} \times 9\left(\frac{x^2-xy+y^2}{a^2+ab+b^2}\right)$$

$$= \frac{(a-b)(a^2+ab+b^2)(x+y)9(x^2-xy+y^2)}{(x+y)(x^2-xy+y^2)(a-b)(a^2+ab+b^2)} = 9$$

DIVISION OF FRACTIONS.

Ex. 15 1. $\frac{5x}{7} \div \frac{25x}{21} = \frac{5x}{7} \times \frac{21}{25x} = \frac{3}{5}$ 2. $\frac{4x^2}{5y^4} \div \frac{2x}{6y^2} = \frac{4x^2}{5y^2} \times \frac{6y^2}{2x} = \frac{12x}{5}$

3. $$\frac{4x^2-4}{18} \div \frac{6(x+1)}{36} = \frac{4(x^2-1)}{18} \times \frac{6}{x+1} = \frac{4(x-1)}{3}$$

4. $$\frac{x^2+xy}{xy-y^2} \div \frac{x^2(x+y)^2}{(x-y)^2} = \frac{x(x+y) \times (x-y)^2}{y(x-y)(x+y)^2x^2} = \frac{x-y}{xy(x+y)}$$

5. $\frac{a-b}{ab+b^2} \div \frac{5(a^2-b^2)}{a^2+ab} = \frac{a-b}{b(a+b)} \times \frac{a(a+b)}{5(a^2-b^2)} = \frac{a}{5b(a+b)}$

6. $\frac{x^3-y^3}{x^2-y^2} \div \frac{(x-y)x}{x+y} = \frac{x^3-y^3}{x^2-y^2} \times \frac{x+y}{x(x-y)} = \frac{x^2+xy+y^2}{x(x-y)}$

7. $\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+1\right) \div \left(\frac{x}{y}+\frac{y}{x}+1\right) = \left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+2-1\right) \div \left(\frac{x}{y}+\frac{y}{x}+1\right)$

$$= \frac{\left(\frac{x}{y}+\frac{y}{x}\right)^2 - 1}{\left(\frac{x}{y}+\frac{y}{x}+1\right)} = \frac{x}{y}+\frac{y}{x}-1$$

8. $\frac{(a-c)^2-b^2}{abc} \div \left(\frac{a}{bc}-\frac{b}{ac}-\frac{c}{ab}-\frac{2}{a}\right) = \frac{(a-c+b)(a-c-b)}{abc}$

$$\div \left(\frac{a^2-b^2-c^2-2bc}{abc}\right) = \frac{(a-c+b)(a-c-b)}{abc}$$

$$\times \frac{abc}{(a-b-c)(a+b+c)} = \frac{a+b-c}{a+b+c}$$

9. The expression $= \frac{(x+a)(x+b)}{(x+a)(x-c)} \times \frac{x^2-c^2}{x^2-b^2} = \frac{x+c}{x-b}$

10. $\frac{a^4-b^4}{(a+b)^2} \div \frac{a-b}{a(a+b)} = \frac{a^4-b^4}{(a+b)^2} \times \frac{a(a+b)}{a-b} = a(a^2+b^2)$

11. $1-\frac{1}{1+x}$ by $1+\frac{x^2}{1-x^2} = \frac{x}{1+x} \times \frac{1-x^2}{1} = x(1-x)$

12. $(23x^2-3y+\frac{y}{8}) \div (6x-\frac{x}{4}) = (23x^2-\frac{23}{8}y) \div \frac{23x}{4} = 4x-\frac{y}{2x}$.

13. $\left(x+\frac{2x}{x-3}\right) \div \left(x-\frac{2x}{x-3}\right) = \frac{x^2-x}{x-3} \times \frac{x-3}{x^2-5x} = \frac{x-1}{x-5}$

14. $\frac{7a(a^2-x^2)}{3b(c^2-x^2)} \div \frac{a^2-ax}{bc+bx} = \frac{7a(a^2-x^2)}{3b(c^2-x^2)} \times \frac{b(c+x)}{a(a-x)} = \frac{7(a+x)}{3(c-x)}$

15. $(x^6+\frac{1}{x^6}+x^4+\frac{1}{x^4}+x^2+\frac{1}{x^2}+2) \div (x^3+\frac{1}{x^3}+x+\frac{1}{x})$

$$= \frac{x^6+x^4+x^2+2+x^{-6}+x^{-4}+x^{-2}}{x^3+x+x^{-1}+x^{-3}} = x^3+x^{-3}$$

16 $$\frac{b-3a}{2ab}\div\frac{6a-2b}{b^2-2ab}=\frac{b-3a}{2ab}\times\frac{b^2-2ab}{6a-2b}=\frac{2a-b}{4a}$$

17. $$\left(\frac{2a+x}{2a-x}+\frac{2a-x}{2a+x}\right)\div\left(\frac{2a+x}{2a-x}-\frac{2a-x}{2a+x}\right)$$

$$\frac{2(4a^2+x^2)}{4a^2-x^2}\times\frac{4a^2-x^2}{8ax}=\frac{4a^2+x^2}{4ax}$$

18 $$\left(\frac{a-1}{a}+\frac{b-1}{b}+\frac{c-1}{c}\right)\div\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$=\frac{abc-bc+abc-ac+abc-ab}{abc}\div\left(\frac{bc+ac+ab}{abc}\right)$$

$$=\frac{3abc-bc-ac-ab}{abc}\times\frac{abc}{bc+ac+ab}=\frac{3abc}{bc+ac+ab}-1$$

19 $$\left(\frac{x+y}{x-y}+\frac{x^2+y^2}{x^2-y^2}\right)\div\left(\frac{x-y}{x+y}-\frac{x^3-y^3}{x^3+y^3}\right)$$

$$=\frac{x^3-xy^2+yx^2-y^3+x^3+y^2x-yx^2-y^4}{(x-y)(x^2-y^2)}$$

$$\div\frac{x^4+y^3x-yx^3-y^4-x^4+xy^3-yx^3+y^4}{(x+y)(x^3+y^3)}=\frac{2(x^3-y^3)\times(x+y)(x^3+y^3)}{(x-y)(x^2-y^2)2(xy^3-yx^3)}$$

$$=\frac{2(x-y)(x^2+xy+y^2)(x+y)(x+y)(x^2-xy+y^2)}{2(x-y)(x+y)(x-y)xy(y-x)(x+y)}=\frac{x^4+x^2y^2+y^4}{xy(2xy-x^2-y^2)}$$

20 $$\left(\frac{x^2+y}{x^2-y}-\frac{x^2-y}{x^2+y}\right)\div\left(\frac{x+\sqrt{y}}{x-\sqrt{y}}-\frac{x-\sqrt{y}}{x+\sqrt{y}}\right)$$

$$\frac{4x^2y}{(x^2-y)(x^2+y)}\times\frac{x^2-y}{4x\sqrt{y}}=\frac{x\sqrt{y}}{x^2+y}$$

21 $$\left(\frac{2x+3y}{2x-3y}+\frac{2x-3y}{2x+3y}\right)\div\left(\frac{4x^2+9y^2}{(2x)^2-(3y)^2}-\frac{(2x)^2-(3y)^2}{4x^2+9y^2}\right)$$

$$=\frac{2(4x^2+9y^2)(4x^2-9y^2)(4x^2+9y^2)}{(4x^2-9y^2)\times2\times72x^2y^2}=\frac{(4x^2+9y^2)^2}{72x^2y^2}$$

22. $$\frac{\left(\frac{x-a}{x+a}-\frac{x+a}{x-a}\right)}{\left(\frac{x+a}{x-a}+\frac{x-a}{x+a}\right)}+\frac{x}{x-a}-\frac{x}{x+a}$$

$$=\left\{\frac{-4ax}{x^2-a^2}\times\frac{x^2-a^2}{2(x^2+a^2)}\right\}+\frac{x^2+ax-x^2+ax}{x^2-a^2}=\frac{-2ax}{x^2+a^2}+\frac{2ax}{x^2-a^2}=\frac{4a^3x}{a^3-ab}$$

23. $$\frac{1}{a+\frac{1}{b+\frac{1}{c}}}=\frac{1}{a+\frac{1}{\frac{bc+1}{c}}}=\frac{1}{a+\frac{c}{bc+1}}=\frac{1}{\frac{abc+a+c}{bc+1}}=\frac{bc+1}{abc+a+c}$$

24. $$\frac{1}{x-4+\frac{2}{x-\frac{2}{x}}}=\frac{1}{x-4+\frac{2}{\frac{x^2-2}{x}}}=\frac{1}{x-4+\frac{2x}{x^2-2}}$$

$$=\frac{1}{\frac{x^3-2x-4x^2+8+2x}{x^2-2}}=\frac{x^2-2}{x^3-4x^2+8}$$

INVOLUTION.

Ex. 16, 9. $(x+2)^3(x-2)^3=(x^2-4)^3=(x^2)^3-3(x^2)^2\cdot 4+3\cdot x^2\cdot 4^2$
$-4^3=x^6-12x^4+48x^2-64$

14. $(x-2)^5=x^5+5\cdot x^4\cdot(-2)+\frac{5\cdot 4}{1\cdot 2}x^3\cdot(-2)^2+\frac{5\times 4}{1\cdot 2}x^2(-2)^3$
$+5\cdot x(-2)^4+(-2)^5=x^5-10x^4+40x^3-80x^2+80x-32$

15. $(4x+1)^3=(4x)^3+3\cdot(4x)^2\cdot 1+3\cdot 4x\cdot 1^2+1^3=64x^3+48x^2+12x+1$

20. $(\overline{a+b}+c)^2=(a+b)^2+2c(a+b)+c^2=a^2+2ab+b^2+2ac+2bc+c^2$

24. $(\overline{a+b}+\overline{c+d})^2=(a+b)^2+2(a+b)(c+d)+(c+d)^2$
$=a^2+2ab+b^2+2ac+2ad+2bc+2bd+c^2+2cd+d^2$
$=a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$

44. $(x+1)^8 = x^8 + 8x^7 + \frac{8\cdot7}{1\cdot2}x^6 + \frac{8\cdot7\cdot6}{1\cdot2\cdot3}x^5 + \frac{8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4}x^4 + \frac{8\cdot7\cdot6}{1\cdot2\cdot3}x^3$

$+ \frac{8\cdot7}{1\cdot2}x^2 + 8x + 1 = x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1.$

49. $(x^{-2} - x^{-1})^2 = (x^{-2})^2 - 2\cdot x^{-2}\cdot x^{-1} + (x^{-1})^2 = x^{-4} - 2x^{-3} + x^{-2}$

50. $\left(\frac{a}{b} - \frac{b}{a}\right)^2 = \left(\frac{a}{b}\right)^2 - 2\frac{a\,b}{b\cdot a} + \left(\frac{b}{a}\right)^2 = \frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

EVOLUTION.

Ex. 17 28.

$$\begin{array}{r|l|l}
 & 4x^4 - 4x^2y + y^4 - 2y^3 + 4x^2y^2 + y^2 & 2x^2 - y + y^2 \\
 & 4x^4 & \\ \hline
 & -4x^2y + 4x^2y^2 + y^2 & \\
4x^2 - y & -4x^2y + y^2 & \\ \hline
 & 4x^2y^2 - 2y^3 + y^4 & \\
4x^2 - 2y + y^2 & 4x^2y^2 - 2y^3 + y^4 & \\ \hline
 & 0 &
\end{array}$$

29.

$$\begin{array}{r|l|l}
 & (a-b)^4 - 2(a^2+b^2)(a-b)^2 + (a^2+b^2)^2 & (a-b)^2 - (a^2 + b^2) \\
 & (a-b)^4 & \\ \hline
 & -2(a^2+b^2)(a-b)^2 + (a^2+b^2)^2 & \\
2(a-b)^2 - (a^2+b^2) & -2(a^2+b^2)(a-b)^2 + (a^2+b^2)^2 & \\ \hline
 & 0 &
\end{array}$$

31. $(x+\frac{1}{x})^2 - 4(x-\frac{1}{x}) = x^2 + 2 + \frac{1}{x^2} - 4x + \frac{4}{x}$

arrange thus

$$\begin{array}{r|l|l}
 & x^2 - 4x + 2 + \frac{4}{x} + \frac{1}{x^2} & x - 2 - \frac{1}{x} \\
2x - 2 & x^2 & \\ \hline
 & -4x + 2 & \\
2x - 4 - \frac{1}{x} & -4x + 4 & \\ \hline
 & -2 + \frac{4}{x} + \frac{1}{x^2} & \\
 & -2 + \frac{4}{x} + \frac{1}{x^2} & \\ \hline
\end{array}$$

Otherwise $x^2 + 2 + \frac{1}{x^2} - 4x + \frac{4}{x} = \left(x^2 - 2 + \frac{1}{x^2}\right) - 4(x - \frac{1}{x}) + 4$

$= (x - \frac{1}{x})^2 - 4(x - \frac{1}{x}) + 2^2 = (x - \frac{1}{x} - 2)^2$

32.

$$\begin{array}{r|l|l}
 & a^4+2(2b-c)a^3+(4b^2-4bc+3c^2)a^2 & a^2+a(2b-c) \\
 & \qquad +2c^2(2b-c)a+c^4 & \quad +c^2 \\
 & a^4 & \\
\cline{2-2}
 & 2(2b-c)a^3+(4b^2-4bc+3c^2)a^2 & \\
2a^2+a(2b-c) & 2(2b-c)a^3+(4b^2-4bc+c^2)a^2 & \\
\cline{2-2}
 & 2a^2c^2+2c^2(2b-c)a+c^4 & \\
2a^2+2a(2b-c)+c^2 & 2a^2c^2+2c^2(2b-c)a+c^4 & \\
\cline{2-2}
 & 0 &
\end{array}$$

33.

$$\begin{array}{r|l|l}
 & (a-2b)^2x^4-2a(a-2b)x^3+(a^2+4ab-6a & (a-2b)x^2 \\
 & -8b^2+12b)x^2-(4ab-6a)x+4b^2-12b+9 & -ax+(2b \\
 & & -3) \\
2(a-2b)x^2 & (a-2b)^2x^4 & \\
\cline{2-2}
-ax & -2a(a-2b)x^3+(a^2+4ab-6a-8b^2+12b)x^2 & \\
 & -2a(a-2b)x^3+a^2x^2 & \\
\cline{2-2}
2(a-2b)x^2 & (4ab-6a-8b^2+12b)x^2-(4ab-6a)x & \\
-2ax+(2b & (4ab-6a-8b^2+12b)x^2-(4ab-6a)x+4b^2 & \\
-3) & \qquad -12b+9 & \\
\cline{2-2}
 & 0 &
\end{array}$$

34.

$$\begin{array}{r|l|l}
 & 4a-12a^{\frac{1}{2}}b^{\frac{1}{3}}+9b^{\frac{2}{3}}+16a^{\frac{1}{2}}c^{\frac{1}{4}}-24b^{\frac{1}{3}}c^{\frac{1}{4}}+16c^{\frac{1}{2}} & 2a^{\frac{1}{2}}-3b^{\frac{1}{3}}+4c^{\frac{1}{4}} \\
 & 4a & \\
\cline{2-2}
4a^{\frac{1}{2}}-3b^{\frac{1}{3}} & -12a^{\frac{1}{2}}b^{\frac{1}{3}}+9b^{\frac{2}{3}} & \\
 & -12a^{\frac{1}{2}}b^{\frac{1}{3}}+9b^{\frac{2}{3}} & \\
\cline{2-2}
4a^{\frac{1}{2}}-6b^{\frac{1}{3}} & 16a^{\frac{1}{2}}c^{\frac{1}{4}}-24b^{\frac{1}{3}}c^{\frac{1}{4}}+16c^{\frac{1}{2}} & \\
+4c^{\frac{1}{4}} & 16a^{\frac{1}{2}}c^{\frac{1}{4}}-24b^{\frac{1}{3}}c^{\frac{1}{4}}+16c^{\frac{1}{2}} & \\
\cline{2-2}
 & 0 &
\end{array}$$

35.

$$\begin{array}{r|l|l}
 & 256x^{\frac{4}{3}}-512x+640x^{\frac{2}{3}}-512x^{\frac{1}{3}}+304-128x^{-\frac{1}{3}} & 16x^{\frac{2}{3}} \\
 & \qquad +40x^{-\frac{2}{3}}-8x^{-1}+x^{-\frac{4}{3}} & -16x^{\frac{1}{3}} \\
 & 256x^{\frac{4}{3}} & +12 \\
\cline{2-2}
32x^{\frac{2}{3}}-16x^{\frac{1}{3}} & -512x+640x^{\frac{2}{3}} & -4x^{-\frac{1}{3}} \\
 & -512x+256x^{\frac{2}{3}} & +x^{-\frac{2}{3}} \\
\cline{2-2}
32x^{\frac{2}{3}}-32x^{\frac{1}{3}}+12 & 384x^{\frac{2}{3}}-512x^{\frac{1}{3}}+304 & \\
 & 384x^{\frac{2}{3}}-384x^{\frac{1}{3}}+144 & \\
\cline{2-2}
32x^{\frac{2}{3}}-32x^{\frac{1}{3}} & -128x^{\frac{1}{3}}+160 & \\
+24-4x^{-\frac{1}{3}} & -128x^{\frac{1}{3}}+128-96x^{-\frac{1}{3}}+16x^{-\frac{2}{3}} & \\
\cline{2-2}
32x^{\frac{2}{3}}-32x^{\frac{1}{3}}+24 & 32-32x^{-\frac{1}{3}}+24x^{-\frac{2}{3}}-8x^{-1}+x^{-\frac{4}{3}} & \\
-8x^{-\frac{1}{3}}+x^{-\frac{2}{3}} & 32-32x^{-\frac{1}{3}}+24x^{-\frac{2}{3}}+8x^{-1}+x^{-\frac{4}{3}} & \\
\cline{2-2}
\end{array}$$

37. Arrange thus

$$\begin{array}{l|l|l}
 & x-2x^{\frac{1}{2}}-1+2x^{-\frac{1}{2}}+x^{-1} & x^{\frac{1}{2}}-1-x^{-\frac{1}{2}} \\
 & x & \\ \hline
2x^{\frac{1}{2}}-1 & -2x^{\frac{1}{2}}-1+2x^{-\frac{1}{2}}+x^{-1} & \\
 & -2x^{\frac{1}{2}}+1 & \\ \hline
2x^{\frac{1}{2}}-2-x^{-\frac{1}{2}} & -2+2x^{-\frac{1}{2}}+x^{-1} & \\
 & -2+2x^{-\frac{1}{2}}+x^{-1} & \\ \hline
\end{array}$$

42. See Ex. 31.

43. The expression can be put in the following form $x+1-2x^{\frac{1}{4}}(1+x^{\frac{1}{2}})+3x^{\frac{1}{2}}=x+1-2x^{\frac{1}{4}}-2x^{\frac{3}{4}}+3x^{\frac{1}{2}}=x-2x^{\frac{3}{4}}+3x^{\frac{1}{2}}-2x^{\frac{1}{4}}+1.$

Now find the root in the usual way.

$$\begin{array}{l|l|l}
 & x-2x^{\frac{3}{4}}+3x^{\frac{1}{2}}-2x^{\frac{1}{4}}+1 & x^{\frac{1}{2}}-x^{\frac{1}{4}}+1 \\
2x^{\frac{1}{2}}-x^{\frac{1}{4}} & x-2x^{\frac{3}{4}}+x^{\frac{1}{2}} & \\ \hline
 & 2x^{\frac{1}{2}}-2x^{\frac{1}{4}}+1 & \\
2x^{\frac{1}{2}}-2x^{\frac{1}{4}}+1 & 2x^{\frac{1}{2}}-2x^{\frac{1}{4}}+1 & \\ \hline
 & 0 & \\
\end{array}$$

44. The expression $=a^{2\sqrt{2}}+2+a^{-2\sqrt{2}}=\left(a^{\sqrt{2}}+a^{-\sqrt{2}}\right)^2$

49.

$$\begin{array}{l|l|l}
 & x^4+6x^3+11x^2+3x+31 & x^2+3x+1 \\
 & x^4 & \\ \hline
 & 6x^3+11x^2 & \\
2x^2+3x & 6x^3+9x^2 & \\ \hline
 & 2x^2+3x+31 & \\
2x^2+6x+1 & 2x^2+6x+1 & \\ \hline
 & -3x+30 & \\
\end{array}$$

Now $\because$ the given quantity is a perfect square the remainder $-3x+30=0$ $\therefore$ $3x=30$ $\therefore$ $x=10$.

54. Proceed as in ex. 31.

56.

$$\begin{array}{l|l|l}
 & \frac{x^2}{a^2}-\frac{2x}{a}+\frac{b^2}{x^2}-\frac{2b}{x}+\frac{2b}{a}+1 & \frac{x}{a}-1+\frac{b}{x} \\
 & \frac{x^2}{a^2} & \\
\cline{2-2}
\frac{2x}{a}-1 & -\frac{2x}{a}+1 & \\
 & -\frac{2x}{a}+1 & \\
\cline{2-2}
\frac{2x}{a}-2+\frac{b}{x} & \frac{2b}{a}-\frac{2b}{x}+\frac{b^2}{x^2} & \\
 & \frac{2b}{a}-\frac{2b}{x}+\frac{b^2}{x^2} &
\end{array}$$

57. The expression $=4(a^2-b^2)^2c^2d^2+4(c^2-d^2)a^2b^2+8(a^2-b^2)(c^2-d^2)abcd+(a^2-b^2)^2(c^2-d^2)^2-8(a^2-b^2)(c^2-d^2)abcd+16a^2b^2c^2d^2$

$=(a^2-b^2)^2\{(c^2-d^2)^2+4c^2d^2\}+4a^2b^2\{(c^2-d^2)^2+4c^2d^2$

$=\{(a^2-b^2)^2+4a^2b^2\}\{(c^2-d^2)^2+4c^2d^2\}$

$=\{a^4+2a^2b^2+b^4\}\{c^4+2c^2d^2+d^4\}=(a^2+b^2)^2(c^2+d^2)^2$

$\therefore$ Square root $=(a^2+b^2)(c^2+d^2)$

65. The expression can be put thus

$$9\left(\frac{x}{y}+\frac{y}{x}\right)^2-24\left(\frac{x}{y}+\frac{y}{x}\right)+16$$

and which is $=\left\{3\left(\frac{x}{y}+\frac{y}{x}\right)-4\right\}^2$

INDICES.

Ex. 18. 1. $36^{\frac{1}{2}}$ means the square root of 36 ; $\frac{1}{4^{\frac{1}{2}}}=\frac{1}{2}$; $8^{\frac{1}{3}}$ means the cube root of 8 ; $27^{-\frac{2}{3}}=\frac{1}{(27^{\frac{1}{3}})^2}=\frac{1}{(3)^2}=\frac{1}{9}$; $100^{\frac{3}{2}}=(100^{\frac{1}{2}})^3=10^3=1000$; $81^{\frac{5}{4}}=\left((81)^{\frac{1}{4}}\right)^5=3^5=243$

2. $x^{2\times4}=x^8$; $x^{-2\times6}=x^{-12}$; $(2x)^{-1\times5}=(2x)^{-5}=\frac{1}{(2x)^5}=\frac{1}{32x^5}$;

$(3x^2)^{-2\times-3}=(3x^2)^6=3^6x^{12}$

3. $(a^{-6})^{\frac12}=a^{-6\times\frac12}=a^{-3}$; $(a^{-10})^{\frac15}=a^{-10\times\frac15}=a^{-2}$;

$(a^{2m}b^m)^{\frac1m}=a^2b$; $(a^nb^2c^{2n})^{\frac1{2n}}=a^{\frac{n}{2n}}b^{\frac{2}{2n}}c^{\frac{2n}{2n}}=a^{\frac12}b^{\frac1n}c$

4. $(-x)^{3\times-5\times2}=(-x)^{-30}=\frac{1}{(-x)^{30}}=\frac{1}{x^{30}}=x^{-30}$; $(2x^{\frac12})^{\frac25}$

$=\left\{(2x^{\frac12})^2\right\}^{\frac15}=(4x)^{\frac15}$; $\left(x^{m\times n\times2p}\right)^{\frac{1}{2mnp}}=x$

5. $-(x^3)^{-\frac18}=-x^{-\frac38}$ and $\left\{-(-x^2)^{-3}\right\}^{\frac13}=-(-x^2)^{-1}$

$=-(-x^{-2})=-x^{-2}$ $\therefore$ product$=-x^{-\frac38-2}=-x^{-\frac{19}{8}}$

6. $\left(\frac{x^m}{y^n}\right)^{\frac1m}=\frac{x}{y^{\frac{n}{m}}}$

7. $(x^{2m-n+n}.\,y^{5m+1+m-1}.\,z^{3p+m-3p})^{\frac1m}=(x^{2m}.\,y^{6m}.\,z^m)^{\frac1m}=x^2y^6z$

8. $\left(\frac{a^{-2m}}{b^{-2n}}\right)^{\frac{1}{mn}}=\left(\frac{\frac{1}{a^{2m}}}{\frac{1}{b^{2n}}}\right)^{\frac{1}{mn}}=\left(\frac{b^{2n}}{a^{2m}}\right)^{\frac{1}{mn}}=\frac{b^{\frac2m}}{a^{\frac2n}}$

9. Dividend$=(x^{m+n-n})^{2m}=(x^m)^{2m}=x^{2m^2}$ and divisor

$=(x^{n-n-m})^{2(n-m)}=(x^{-m})^{2(n-m)}=x^{-2m(n-m)}$ $\therefore$ quotient

$=x^{2m^2+2m(n-m)}=x^{2mn}$

10. $\frac{2^n.\,2^{n^2-n}}{2^{n+1}.\,2^{n-1}}=\frac{2^{n^2}}{2^{2n}}=2^{n^2-2n}$

11. $\dfrac{a^x+b^y}{\frac{a^x+b^y}{a^xb^y}} \times \dfrac{a^y-b^x}{\frac{b^x-a^y}{b^xa^y}} = a^xb^y \times -(b^xa^y) = -(ab)^{x+y}$

12. The expression $= \left(\dfrac{a-b}{a+b}\right)^{-m\times-\frac{q}{m}} \times \left(\dfrac{(a+b)^2}{a^2-b^2}\right)^{-n\times-\frac{q}{n}}$

$$= \left(\frac{a-b}{a+b}\right)^q \times \left(\frac{a+b}{a-b}\right)^q = 1.$$

13. The expression $= \dfrac{4}{x} \times \dfrac{y^2}{x} \times \dfrac{x^4}{y^2} = x^2.$

14. The expression $= \left(\dfrac{y^{2b}}{x^{2a}}\right)^{\frac{cd}{2ab}} = \dfrac{y^{\frac{cd}{a}}}{x^{\frac{cd}{b}}} = \left(\dfrac{y^{\frac{1}{a}}}{x^{\frac{1}{b}}}\right)^{cd}.$

15. The expression $= x^{m+n+p} \times 2^{m+n-p} \times x^{m+n-p} \times x^{-n+p-m}$

$$= 2^{m+n-p}x^{m+n+p}$$

16. The expression $= \dfrac{3^{27^2}}{(3^3)^{9^2}} = \dfrac{3^{27^2}}{3^{3\cdot 9^2}} = 3^{27^2-3\cdot 9^2} = 3^{(3^3)^2-3(3^2)^2}$

$$= 3^{3^6-3^5} = 3^{3^5(3-1)} = 3^{3^5\times 2}$$

17. The expression $= \dfrac{x+\sqrt{(x^2-y)}}{2} + \dfrac{x-\sqrt{(x^2-y)}}{2}$

$$+2\left\{\left(\frac{x^2-(x^2-y)}{4}\right)\right\}^{\frac{1}{2}} = x+2\left\{\frac{x^2-(x^2-y)}{4}\right\}^{\frac{1}{2}} = x+2\frac{y^{\frac{1}{2}}}{2} = x+y^{\frac{1}{2}}$$

19. $x^{-\frac{1}{2}} = \dfrac{1}{x^{\frac{1}{2}}} = \frac{1}{2}$; $(2x)^{-\frac{1}{3}} = \dfrac{1}{(2x)^{\frac{1}{3}}} = \dfrac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$; $(4x)^{-\frac{1}{4}} = 16^{-\frac{1}{4}}\dfrac{1}{16^{\frac{1}{4}}} = \frac{1}{2}$;

$(x^3)^{-\frac{1}{6}} = x^{-\frac{1}{2}} = \dfrac{1}{x^{\frac{1}{2}}} = \frac{1}{2}$ ∴ expression $= 0$

20. The expression $= x^{\frac{a+b}{b} \times \frac{b}{a+b}} + \left(\frac{x^{2a}}{x^a}\right)^{\frac{1}{a}} = x + (x^a)^{\frac{1}{a}} = x + x = 2x$

21. The expression $= \frac{1}{1+x^{m-n}} + \frac{1}{1+x^{n-m}} = \frac{1}{1+x^{m-n}} + \frac{1}{1+x^{-(m-n)}}$

$$\frac{1}{1+x^{m-n}} + \frac{1}{1+\frac{1}{x^{m-n}}} = \frac{1}{1+x^{m-n}} + \frac{x^{m-n}}{1+x^{m-n}} = \frac{1+x^{m-n}}{1+x^{m-n}} = 1$$

22. The expression $= \frac{x^{2b+2c+2c+2d+2b+2d}}{x^{4(c+b+d)}} = \frac{x^{4b+4c+4d}}{x^{4(b+c+d)}} = 1$

SURDS.

Ex. 19. 1. $\sqrt{18} + 4\sqrt{8} + \sqrt{32} = 3\sqrt{2} + 8\sqrt{2} + 4\sqrt{2} = 15\sqrt{2}$

2 $\sqrt{12} + 2\sqrt{48} - 2\sqrt{27} = 2\sqrt{3} + 8\sqrt{3} - 6\sqrt{3} = 4\sqrt{3}$

3. $8 \times 2 - 4 \times 3 + 2 = 6$

4. $4\sqrt{5} + 3 \times \frac{3}{2} - 5 \times \frac{7}{3} = 4\sqrt{5} + 4\frac{1}{2} - 11\frac{2}{3} = 4\sqrt{5} - 7\frac{1}{6}$

5. $\frac{4+\sqrt{2}}{2+\sqrt{3}} = \frac{(4+\sqrt{2})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = 8 + 2\sqrt{2} - 4\sqrt{3} - \sqrt{6}$

6. $\frac{(\sqrt{7}+\sqrt{3})^2}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{7+3+2\sqrt{21}}{4} = 10 + 2\sqrt{21} \quad 5 + \sqrt{21}$

7. $\frac{(2\sqrt{3}-\sqrt{5})(3\sqrt{2}-\sqrt{3})}{(3\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})} = \frac{6\sqrt{6} - 3\sqrt{10} - 6 + \sqrt{15}}{15}$

8. $\frac{2\sqrt{3}-3\sqrt{2}}{3\sqrt{2}+2\sqrt{5}} = \frac{(2\sqrt{3}-3\sqrt{2})(3\sqrt{2}-2\sqrt{5})}{(3\sqrt{2}+2\sqrt{5})(3\sqrt{2}-2\sqrt{5})} = \frac{6\sqrt{6} - 18 - 4\sqrt{15} + 6\sqrt{10}}{-2}$

$= -3\sqrt{6} + 9 + 2\sqrt{15} - 3\sqrt{10}$

9. Let $\sqrt{3+2\sqrt{2}} = \sqrt{x} + \sqrt{y}$ $\therefore 3 + 2\sqrt{2} = x + y + 2\sqrt{xy}$
$\therefore x + y = 3$, $2\sqrt{xy} = 2\sqrt{2}$ $\therefore x^2 + 2xy + y^2 = 9$. Subtract this $4xy = 8$
then $x^2 - 2xy + y^2 = 1$ $\therefore x - y = 1$ and $x + y = 3$ $\therefore 2x = 4$ $\therefore x = 2$
and $2y = 2$ $\therefore y = 1$ $\therefore$ the root required is $1 + \sqrt{2}$

10. Assume $\sqrt{7-4\sqrt{3}}=\sqrt{x}-\sqrt{y}$ and then proceed as in the preceding ex. 11. Proceed as in ex. 9. 12. Preceed as in ex 10

13. Proceed as in ex 9 14. Proceed as in ex 10

15. $\dfrac{1}{\sqrt{(7+2\sqrt{10})}}=\dfrac{1}{\sqrt{5}+\sqrt{2}}=\dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\dfrac{\sqrt{5}-\sqrt{2}}{3}$

16. Proceed as in ex 15

17. The numerator $=4+2\sqrt{3}$ $\therefore$ quotient $=2$

18. Extract the root of the two quantities and add

19. Rationalize the denominator.

20. Let $\sqrt[3]{(7+5\sqrt{2})}=x+\sqrt{y}$ then $\sqrt[3]{(7-5\sqrt{2})}=x-\sqrt{y}$ $\therefore$ $\sqrt[3]{(49-50)}=x^2-y$ or $-1=x^3-y$ $\therefore$ $y=x^2+1$; again cubing the 1*st* equ we have $7+5\sqrt{2}=x^3+3x^2\sqrt{y}+3xy+y^{\frac{3}{2}}$ now equating the rational quantities from both sides $x^3+3xy=7$ or $x^3+3x(x^2+1)=7$ or $4x^3+3x=7$ whence by trial $x=1$ and $\therefore$ $y=x^2+1=2$ $\therefore$ cube root required is $1+\sqrt{2}$

21. Proceed as in ex 20.

22. $\sqrt{1+x}=\sqrt{1+\dfrac{\sqrt{3}}{2}}=\dfrac{\sqrt{3}+1}{2}$, and $\sqrt{1-x}=\sqrt{1-\dfrac{\sqrt{3}}{2}}$

$$=\frac{\sqrt{3}-1}{2}\qquad\therefore\ \frac{1+\dfrac{\sqrt{3}}{2}}{1+\dfrac{\sqrt{3}+1}{2}}+\frac{1-\dfrac{\sqrt{3}}{2}}{1+\dfrac{\sqrt{3}-1}{2}}=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2+\sqrt{3}}{1+\sqrt{3}}$$

$$=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2\sqrt{3}-3}{\sqrt{3}+3}=\frac{3\sqrt{3}-1}{3+\sqrt{3}}=\frac{(3\sqrt{3}-1)(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$=\frac{-12+10\sqrt{3}}{6}=\tfrac{5}{3}\sqrt{3}-2$$

23. Proceed as in ex 20. 24. Proceed as in ex 20.

25. The expression $=\dfrac{1+\dfrac{\sqrt{3}}{2}}{1+\dfrac{\sqrt{3}+1}{\ }}+\dfrac{1-\dfrac{\sqrt{3}}{2}}{1-\dfrac{\sqrt{3}-1}{2}}=\dfrac{2+\sqrt{3}}{3+\sqrt{3}}+\dfrac{2-\sqrt{3}}{3-\sqrt{3}}$

$$=\frac{(2+\sqrt{3})(3-\sqrt{3})+(2-\sqrt{3})(3+\sqrt{3})}{9-3}=\frac{6}{6}=1$$

27. Let $\frac{-1+\sqrt{(-3)}}{2}=r$ and $\frac{-1-\sqrt{-3}}{2}=s$ and suppose x a multiple of 3 so that $x=3p$ $\therefore$ $r^3=1$ $\therefore$ $r^{3p}=1$, $s^3=1$ $\therefore$ $s^{3p}=1$ $\therefore$ sum $=2$. Next suppose that when x is divided by 3 there is a remainder 1 so that $x=3p+1$. Now $r^{3p+1}=r^{3p}\times r=r$; $s^{3p+1}=s^{3p}\times s=s$ $\therefore$ Sum $=r+s=\frac{-1+\sqrt{(-3)}}{2}+\frac{-1-\sqrt{(3)}}{2}$ $=-1$. Lastly suppose that when is divided by 3 there is a remainder 2 so that x is of the form $3p+2$ $\therefore$ r^{3p+2} $=r^{3p}\times r^2=r^2$ and $s^{3p+2}=s^{3p}\times s^2=s^2$ $\therefore$ Sum $=r^2+s^2$

$$=\left(\frac{-1+\sqrt{(-3)}}{2}\right)^2+\left(\frac{-1-\sqrt{3}}{2}\right)^2=-1$$

29. $\sqrt{2+\sqrt{3}}=\left(\frac{4+2\sqrt{3}}{2}\right)^{\frac{1}{2}}=\frac{\sqrt{3}+1}{\sqrt{2}}$, $\sqrt{2-\sqrt{3}}=\left(\frac{4-2\sqrt{2}}{2}\right)^{\frac{1}{2}}$ $=\frac{\sqrt{3}-1}{\sqrt{2}}$, $\therefore$ the expression $=\frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\frac{\sqrt{3}-1}{\sqrt{2}}}$

$$=\sqrt{2}\left(\frac{2+\sqrt{3}}{3+\sqrt{3}}\right)+\sqrt{2}\left(\frac{2-\sqrt{3}}{3-\sqrt{3}}\right)=\sqrt{2}\left\{\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}\right\}$$

$$=\sqrt{2}\left(\frac{6+\sqrt{3}-3+6-\sqrt{3}-3}{6}\right)=\sqrt{2}$$

30. $$\frac{(\sqrt{(a+x)}-\sqrt{(a-x)})^2-(\sqrt{(a+x)}+\sqrt{(a-x)})^2}{(\sqrt{(a+x)}+\sqrt{(a-x)})(\sqrt{(a+x)}-\sqrt{(a-x)})}$$

$$=\frac{4\sqrt{(a^2-x^2)}}{2x}=\frac{2\sqrt{(a^2-x^2)}}{x}$$

31. $$\frac{a-\sqrt{(a^2-x^2)}+a+\sqrt{(a^2-x^2)}}{a^2-(a^2-x^2)}=\frac{2a}{x^2}$$

32. $$\frac{(a-\sqrt{b})(c-\sqrt{d})+(a+\sqrt{b})(c+\sqrt{d})}{c^2-d}=\frac{2ac+2\sqrt{(bd)}}{c^2-d}$$

33. $$\frac{(x+\sqrt{(x^2-b)})^2-(x-\sqrt{(x^2-b^2)})^2}{x^2-(x^2-b^2)}=\frac{4x\sqrt{(x^2-b^2)}}{b^2}$$

34. $$\frac{(\sqrt{(6x)})^2+(\sqrt{\{2x+1\}})^2}{\sqrt{(6x(2x+1))}}=\frac{8x+1}{\sqrt{(12x^2+6x)}}$$

35. Proceed as in Ex. 33 36. Proceed as in Ex. 31.

37. $$\frac{\dfrac{\sqrt{(1-a^2)}+1}{\sqrt{(1+a)}}}{\dfrac{\sqrt{(1-a^2)}+1}{\sqrt{(1-a^2)}}}=\frac{\sqrt{(1-a^2)}}{\sqrt{(1+a)}}=\sqrt{(1-a)}$$

38. The L. C. M. of the denominators $=(1-a^2)^{\frac{3}{2}}$ $\therefore$ the expression

$$=\frac{2a^2+(1-a^2)}{(1-a^2)^{\frac{3}{2}}}=\frac{a^2+1}{(1-a^2)^{\frac{3}{2}}}$$

39. $$(a+\sqrt{-1})^2+(a-\sqrt{-1})^2=a^2+2a\sqrt{-1}+(-1)+a^4-2a\sqrt{-1}$$
$$+(-1)=2a^2-2=2(a^2-1)$$

41. $$\frac{(x+y\sqrt{-1})^2+(x-y\sqrt{-1})^2}{x^2+y^2}$$

$$=\frac{x^2+2xy\sqrt{(-1)}-y^2+x^2-2xy\sqrt{-1}-y^2}{x^2+y^2}=\frac{2(x^2-y^2)}{x^2+y^2}$$

42. The product $=\sqrt[3]{(x-1)(x^2+x+1)(x+1)(x^2-x+1)}\cdot(x^6-1)^{\frac{2}{3}}$

$$=\sqrt[3]{(x^3-1)(x^3+1)}\,(x^6-1)^{\frac{2}{3}}=\sqrt[3]{a^6-1}.\ (x^6-1)^{\frac{2}{3}}$$

$$=(x^6-1)^{\frac{1}{3}+\frac{2}{3}}=x^6-1$$

43. The expression $$=\frac{1}{(4x^3-3x)^2}-\left\{\frac{\dfrac{(1-x^2)^{\frac{1}{2}}}{x}\left(3-\dfrac{1-x^2}{x^2}\right)}{1-3\left(\dfrac{1-x^2}{x^2}\right)}\right\}^2$$

$$= \frac{1}{(4x^3-3x)^2} - \left\{ \frac{\frac{(1-x^2)^{\frac{1}{2}}}{x}\left(\frac{4x^2-1}{x^2}\right)}{\frac{4x^2-3}{x^2}} \right\}^2 = \frac{1}{(4x^3-3x)^2}$$

$$- \left\{ \frac{(1-x^2)^{\frac{1}{2}}(4x^2-1)}{x(4x^2-3)} \right\}^2 = \frac{1}{(4x^3-3x)^2} - \frac{(1-x^2)(4x^2-1)}{(4x^3-3x)^2}$$

$$= \frac{1-(-16x^6+24x^4-9x^2+1)}{(4x^3-3x)^2} = \frac{16x^6-24x^4+9x^2}{(4x^3-3x)^2} = \frac{(4x^3-3x)^2}{(4x^3-3x)^2} = 1$$

SIMPLE EQUATIONS.

Ex. 20. 1. $4x-3x=12-8$ or $x=4$

2. $8x-5x=6+3$ or $3x=9$ $\therefore$ $x=3$

3. $3x-4x=-1-4$ or $-x=-5$ $\therefore$ $x=5$

14. $2ax+nx=2m+b$ or $(2a+n)x=2m+b$ $\therefore$ $x=\frac{2m+b}{2a+n}$

17. $3\cdot75x-2\cdot25x=8-\cdot5$ or $1\cdot5x=7\cdot5$ $\therefore$ $x=\frac{7\cdot5}{1\cdot5}=5$

28. $3x-3p-4p+4x=0$ $\therefore$ $7x=7p$ $\therefore$ $x=p$.

49. $(x-a)^2-(x+a)^2=5a^2$ or $(x-a+x+a)(x-a-x-a)=5a^2$

or $2x\times-2a=5a^2$ $\therefore$ $x=-\frac{5a}{4}$

63. From $A^3+B^3+C^3-3ABC=(A+B+C)(A^2+B^2+C^2-AB-AC-BC)$ we have $(x-1)^3+(x-2)^3+(x-3)^3-3(x-1)(x-2)(x-3)=(x-1+x-2+x-3)\{(x-1)^2+(x-2)^2+(x-3)^2-(x-1)(x-2)-(x-1)(x-3)-(x-2)(x-3)\}=0$

$\therefore$ 1st factor $3x-(1+2+3)=0$ $\therefore$ $x=\frac{1+2+3}{3}=2$

64. The left side expression $=(x+a+b)^2-c^2=x^2+a^2+b^2+2ax+2bx+2ab-c^2$ and this $=x^2+a^2+b^2+c^2$

$$\therefore\ 2(a+b)x=2c^2-2ab\quad\therefore\ x=\frac{c^2-ab}{a+b}$$

SIMPLE EQUATIONS INVOLVING FRACTIONS.

65. Multiply the whole eqn by the *l. c. m* of the denominators we have $15x+5x+3x=23\times15$ or $23x=23\times15$ ∴ $x=\frac{23\times15}{23}=15$

76. $6(x-1)+4(x-2)+3(x-3)=2(5x-1)$ or $6x-6+4x-8+3x-9=10x-2$ or $13x-10x=21$ ∴ $x=7$

112. $\frac{42}{x-2}=\frac{35\times2}{2(x-3)}=\frac{35}{x-3}$ ∴ $42x-126=35x-70$ ∴ $42x-35x=126-70=56$ ∴ $7x=56$ ∴ $x=8$

113. Multiply the whole eqn. by $12x$ then $240+4=61x$ ∴ $61x=244$ ∴ $x=4$

114. $\frac{10x+17}{18}-\frac{12x+2}{11x-8}=\frac{5x-4}{9}$ ∴ $(11x-8)(10x+17)-18(12x+2)=2(11x-8)(5x-4)$ or $110x^2+107x-136-216x-36=110x^2-168x+64$ or $-109x+168x=64+136+36$ or $59x=236$ ∴ $x=4$

115. $4(x-5)=10(x-8)$ or $4x-20=10x-80$ ∴ $4x-10x=20-80$ or $-6x=-60$ ∴ $x=10$.

116. $(2x-6)(9x-21)=(6x-15)(3x-8)$ or $18x^2-96x+126=18x^2-93x+120$ or $-3x=-6$ ∴ $x=2$

117. Multiply the whole eqn by $3x(x+2)$ then $3x(x-2)+3(x+2)=3x(x+2)$ or $3x^2-6x+3x+6=3x^2+6x$ ∴ $-9x=-6$ ∴ $x=\frac{6}{9}=\frac{2}{3}$

118. $6(3x-2)(3x-1)-6(2x-1)(4x-2)=(2x-1)(3x-2)$ ∴ $x=2$

119. Multiply the whole eqn by $7(x-1)(x+7)$ then $7(x+7)-14(x-1)=x+7$ or $6(x+7)-14x+14=0$ ∴ $-8x=-56$ ∴ $x=7$

120. Multiply the whole eqn by $(x-4)(x-6)(x-2)$ then $2(x-6)(x-2)+3(x-4)(x-2)=5(x-4)(x-6)$ or $2(x^2-8x+12)+3(x^2-6x+8)=5(x^2-10x+24)$ or $2x^2-16x+24+3x^2-18x+24=5x^2-50x+120$ $\therefore\ 50x-34x=120-48$ or $16x=72$ $\therefore\ x=4\frac{1}{2}$

121. Multiply the whole eqn by 7 then we have $x+1+7x(x-2)=7(x-1)^2$ or $x+1+7x^2-14x=7x^2-14x+7$ $\therefore\ x=6$

122. $6(3x-2)(3x-1)=(2x-1)(3x-2)+6(2x-1)(4x-2)$ or $54x^2-54x+12=6x^2-7x+2+6(8x^2-8x+2)$ or $54x^2-54x+12=54x^2-55x+14$ $\therefore\ x=2$

123. Multiply the equ by $x+2$ then $x^2+3x+2-x^2-3=2x+4$ $\therefore\ x=5$

124. Multiply the equ by $(x-2)(7x-26)$ then $7x^2-33x+26=7x^2-35x+42$ $\therefore\ x=8$

125. By cross multiplication $ac+afx=bd+cdx$ $\therefore\ (af-cd)x=bd-ac$ $\therefore\ x=\frac{bd-ac}{af-cd}$

126. $\frac{1}{x+3}(1-\frac{1}{6})=\frac{1}{x+5}$ or $\frac{5}{6}\cdot\frac{1}{x+3}=\frac{1}{x+5}$ $\therefore\ 5x+25=6x+18$ $\therefore\ x=7$

127. Multiply the whole eqn by $(2x+1)(x+12)$ then $6x^2+80x+96=2x^2+25x+12+4x^2+78x+38$ $\therefore\ 23x=46$ $\therefore\ x=2$

128. $7(3x^2-2x-8)=21x^2-48x+12$ or $21x^2-14x-56=21x^2-48x+12$ or $34x=68$ $\therefore\ x=2$.

129. $2(x-2)(3x+2)+(2x-3)(3x+2)=6(2x-3)(x-2)$ or $6x^2-8x-8+6x^2-5x-6=12x^2-42x+6$ $\therefore\ 29x=50$ $\therefore\ x=\frac{50}{29}$.

130. $$\frac{x^2-4x+3-x^2+4x-4}{(x-2)(x-3)}=\frac{x^2-12x+35-x^2+12x-36}{(x-6)(x-7)}$$ or $\frac{-1}{(x-2)(x-3)}=\frac{-1}{(x-6)(x-7)}$ $\therefore\ (x-2)(x-3)=(x-6)(x-7)$ or $x^2-5x+6=x^2-13x+42$ $\therefore\ 8x=36$ $\therefore\ x=4\frac{1}{2}$

131. $(64x^2-48x+9)(x-1)=(16x^2-8x+1)(4x-5)$ or $64x^3-112x^2+57x-9=64x^3-112x^2+44x-5$ $\therefore\ 13x=4$ $\therefore\ x=\frac{4}{13}$

132. $$\frac{(x-4)(x-6)-(x-5)^2}{(x-5)(x-6)}=\frac{(x-7)(x-9)-(x-8)^2}{(x-8)(x-9)}$$

or $$\frac{x^2-10x+24-x^2+10x-25}{(x-5)(x-6)}=\frac{x^2-16x+63-x^2+16x-64}{(x-8)(x-9)}$$

or $$\frac{-1}{(x-5)(x-6)}=\frac{-1}{(x-8)(x-9)}$$ or $(x-5)(x-6)=(x-8)(x-9)$

or $x^2-11x+30=x^2-17x+72$ $\therefore\ 6x=42$ $\therefore\ x=7$

133. $(2x+3)(3x-13)=(2x-8)(3x+9)$

$\therefore\ 6x^2-17x-39=6x^2-6x-72$ $\therefore\ 11x=33$ $\therefore\ x=3$

134. $(5x-4)(9x+20)=36(4x-12)+9x(5x-4)$ or $45x^2+64x-80=45x^2+108x-432$ $\therefore\ 44x=352$ $\therefore\ x=8$.

135. $(6x+7)(x+7)=(3x+1)(2x+19)$ or $6x^2+49x+49=6x^2+59x+19$ $\therefore\ 10x=30$ $\therefore\ x=3$

136. $(10x+17)(13x-16)-18(12x+2)=2(5x-4)(13x-16)$

$\therefore\ 130x^2+61x-272-216x-36=130x^2-264x+128$

$\therefore\ 109x=436$ $\therefore\ x=4$

137. Multiply the whole equation by the L. C. M. of the denominators which is $2(x-3)(x+7)$ $\therefore\ 2(x-7)(x-3)-(2x-15)(x+7)=-(x-3)$ or $2(x^2-10x+21)-(2x^2-x-105)=-(x-3)$ or $2x^2-20x+42-2x^2+x+105=-x+3$ $\therefore\ -18x=-144$ $\therefore\ x=8$

138. The left hand side $=\dfrac{x+3}{2(x^2-1)}-\dfrac{x+3}{2(x^2+1)}$

$$=\frac{(x^2+1)(x+3)-(x+3)(x^2-1)}{2(x^4-1)}=\frac{x^3+3x^2+x+3-x^3+x-3x^2+3}{2(x^4-1)}$$

$$=\frac{2(x+3)}{2(x^4-1)}=\frac{x+3}{x^4-1}\quad\therefore\ \frac{x+3}{x^4-1}=\frac{6}{x^4-1}\quad\therefore\ x+3=6\quad\therefore\ x=3$$

139. Clearing the equations of fractions we have $4(x^2+1)-4x(2x-1)+4x^2-1=0$ or $4x+3=0$ $\therefore\ x=-\frac{3}{4}$.

140. Clearing the eqns of fractions we have $(2x+1)^2-8=(2x-1)^2$ or $4x^2+4x+1-8=4x^2-4x+1$ or $8x=8$ ∴ $x=1$

141. $\frac{x^3+1}{x+1}-(x-1)=\frac{9-(x^2-1)}{x+1}=\frac{9}{x+1}-(x-1)$ ∴ $\frac{x^3+1}{x+1}=\frac{9}{x+1}$ ∴ $x^3+1=9$ ∴ $x^3=8$ ∴ $x=2$

142. $x^2+2x+2=\frac{20}{x^2-2x+2}$ ∴ $(x^2+2x+2)(x^2-2x+2)=20$ or $x^4+4=20$ ∴ $x^4=16$ ∴ $x=2$

143. $\frac{x^2+1}{x(x^2+1)}-\frac{1}{x+1}=\frac{3}{(x+1)^2}$ or $\frac{1}{x}-\frac{1}{x+1}=\frac{3}{(x+1)^2}$ or $\frac{x+1-x}{x(x+1)}=\frac{3}{(x+1)^2}$ or $\frac{1}{x}=\frac{3}{x+1}$ ∴ $x+1=3x$ or $2x=1$ ∴ $x=\frac{1}{2}$

144. $\frac{(3-2x)(2x-7)-(2x-5)(1-2x)}{(1-2x)(2x-7)}=\frac{7-16x+4x^2-4x^2+1}{7-16x+4x^2}$ or $\frac{20x-4x^2-21-12x+4x^2+5}{16x-7-4x^2}=\frac{8-16x}{7-16x+4x^2}$ or $\frac{16-8x}{7-16x+4x^2}=\frac{8-16x}{7-16x+4x^2}$ ∴ $16-8x=8-16x$ ∴ $8x=-8$ ∴ $x=-1$

145. $\frac{a}{x+a}+\frac{b}{x+b}=\frac{a}{x+c}+\frac{b}{x+c}$ ∴ $\frac{a}{x+a}-\frac{a}{x+c}=\frac{b}{x+c}-\frac{b}{x+b}$ or $\frac{ax+ac-ax-a^2}{(x+a)(x+c)}=\frac{bx+b^2-bx-bc}{(x+b)(x+c)}$ or $\frac{a(c-a)}{(x+a)(x+c)}=\frac{b(b-c)}{(x+b)(x+c)}$ ∴ $b(b-c)(x+a)=a(c-a)(x+b)$ ∴ $b(b-c)x+ab(b-c)=a(c-a)x+abc(c-a)$ ∴ $(b^2-bc-ac+a^2)x$

$$=ab(c-a)-ab(b-c)=ab(c-a-b+c)$$

$$\therefore x=\frac{ab(c-a-b+c)}{a^2+b^2-c(a+b)}=\frac{ab(a+b-2c)}{c(a+b)-a^2-b^2}$$

146. Clearing the equation of fractions we have $b(c-x)+a(b-x)=b(b-x)$ or $bc-bx+ab-ax=b^2-bx$

$$\therefore x=\frac{b}{a}(a-b+c)$$

147. Let $(x-1)^2=m$ then the given equation is $\frac{1}{5}\frac{m-4}{m-9}+\frac{1}{5}\frac{m-16}{m-25}-\frac{2}{13}\left(\frac{m-36}{m-49}\right)=\frac{92}{585}$ or $\frac{\frac{1}{5}(m-4)}{m-9}+\frac{\frac{1}{5}(m-16)}{m-25}-\frac{\frac{2}{13}(m-36)}{m-49}=\frac{92}{585}$ but $\frac{1}{5}m+\frac{1}{5}m-\frac{2}{3}m=\frac{92}{585}m$ ∴ by subtraction we have $\frac{1}{m-9}+\frac{1}{m-25}-\frac{2}{m-49}=0$ or $\frac{1}{m-9}-\frac{1}{m-49}+\frac{1}{m-25}-\frac{1}{m-49}=0$ or $\frac{40}{m-9}+\frac{24}{m-25}=0$

or $5m-125+3m-27=0$ ∴ $8m=152$ or $m=19$

∴ $(x-1)^2=19$ ∴ $x-1=\pm\sqrt{19}$ ∴ $x=1\pm\sqrt{19}$

148. The left hand side $=\frac{a^2+x^2}{ax}+\frac{a^2(x-a)^2-x^2(x+a)^2}{ax(x^2-a^2)}$

$$=\frac{x^4-a^4+a^2x^2-2a^3x+a^4-x^4-2ax^3-a^2x^2}{ax(x^2-a^2)}=\frac{-2ax(a^2+x^2)}{ax(x^2-a^2)}$$

$$=\frac{2(a^2+x^2)}{a^2-x^2} \quad \therefore \quad \frac{2(a^2+x^2)}{a^2-x^2}=\frac{ax}{a^2-x^2}-2=\frac{ax-2(a^2-x^2)}{a^2-x^2}$$

or $2a^2+2x^2=ax-2a^2+2x^2$ ∴ $ax=4a^2$ ∴ $x=4a$.

149. $(2x^3+4x^2+8x+16)(3x-6)=1440$ or $6x^4-96=1440$

∴ $6x^4=1536$ ∴ $x^4=256$ ∴ $x=4$

150. Simplifying the fractions on the left hand side of the eqn we have $\frac{4x^3}{x^8+x^4+1}=\frac{4x^3}{x^8+17}$ ∴ $x^8+x^4+1=x^8+17$ ∴ $x^4+1=17$

∴ $x^4=16$ ∴ $x=2$.

151. Clearing the eqn of fractions we have $x^2-2x+3+x^2-x-2-x^2+x-1=2x^2-5x$ or $-x^2-2x+5x=0$ or $x^2+2x-5x=0$

∴ $x^2-3x=0$ ∴ $x^2=3x$ ∴ $x=3$

152. Simplifying the fractions on the left hand side we have the result $=\frac{1}{(x-a)(x-b)(x-c)}$ ∴ $\frac{1}{(x-a)(x-b)(x-c)}$

$=\frac{1}{(x-a)(x-b)(2x-d)}$ ∴ $x-c=2x-d$ ∴ $2x-x=d-c$ or $x=d-c$

153 $$\frac{a}{x-a}-\frac{b}{x-b}=\frac{a-b}{x-c}=\frac{a}{x-c}-\frac{b}{x-c}\;\therefore\;\frac{a}{x-a}-\frac{a}{x-c}=\frac{b}{x-b}$$

$$-\frac{b}{x-c}\;\therefore\;\frac{ax-ac-ax+a^2}{(x-a)(x-c)}=\frac{bx-bc-bx+b^2}{(x-b)(x-c)}\text{ or }\frac{a(a-c)}{x-a}$$

$$=\frac{b(b-c)}{x-b}\text{ or }a(a-c)(x-b)=b(b-c)(x-a)\text{ or }(a^2-ac)x$$

$$-ab(a-c)=(b^2-bc)x-ab(b-c)\text{ or }(a^2-ac-b^2+bc)x=ab(a-b)\text{ or}$$

$$x=\frac{ab(a-b)}{a^2-ac-b^2+bc}=\frac{ab(a-b)}{(a^2-b^2)-c(a-b)}=\frac{ab(a-b)}{(a-b)(a+b-c)}=\frac{ab}{a+b-c}$$

154. $$\frac{x-a+c-(x-a)}{(x-a)(x-a+c)}=\frac{x-b-(x-b-c)}{(x-b)(x-b-c)}\text{ or }\frac{c}{(x-a)(x-a+c)}$$

$$=\frac{c}{(x-b)(x-b-c)}\;\therefore\;(x-a)(x-a+c)=(x-b)(x-b-c)\text{ or}$$

$$(x-a)^2+(x-a)c=(x-b)^2-(x-b)c\text{ or }x^2-2ax+a^2+cx-ac$$
$$=x^2-2bx+b^2-cx+bc\text{ or }2x(b-a)+2cx=b^2-a^2+c(a+b)$$
$$=(b-a+c)(a+b)\text{ or }2x(b-a+c)=(b-a+c)(a+b)$$

$$\therefore\;2x=a+b\;\therefore\;x=\frac{a+b}{2}.$$

155. $$\left(1+\frac{1}{x-a-1}\right)-\left(1+\frac{1}{x-a-2}\right)=\left(1+\frac{1}{x-b-1}\right)$$

$$-\left(1+\frac{1}{x-b-2}\right)\;\therefore\;\frac{1}{x-a-1}-\frac{1}{x-a-2}=\frac{1}{x-b-1}-\frac{1}{x-b-2}$$

$$\text{or }\frac{x-a-2-x+a+1}{(x-a+1)(x-a+2)}=\frac{x-b-2-x+b+1}{(x-b-1)(x-b-2)}$$

$$\text{or }\frac{-1}{(x-a+1)(x-a+2)}=\frac{-1}{(x-b+1)(x-b+2)}\text{ or }(x-a+1)(x-a+2)$$

$$=(x-b+1)(x-b+2)\quad\text{or }x^2-(2a+3)x+(a+1)(a+2)=x^2-(2b+3)$$
$$x+(b+1)(b+2)\text{ or }2(b-a)x=b^2+3b+2-a^2-3a-2=(b^2-a^2)$$
$$+3(b-a)=(b-a)(b+a+3)\;\therefore\;x=\tfrac{1}{2}(b+a+3)$$

156. $\dfrac{x^4-8x^3+21x^2-1}{x^4+x^2+1}=1-\dfrac{4(2x-3)}{x^2+x+1}=\dfrac{x^2+x+1-8x+12}{x^2+x+1}$
$=\dfrac{x^2-7x+13}{x^2+x+1}$ $\therefore$ $x^4-8x^3+21x^2-1=(x^2-x+1)(x^2-7x+13)=x^4$
$-8x^3+21x^2-20x+13$ $\therefore$ $-20x+13=-1$ $\therefore$ $-20x=-14$
$\therefore$ $x=\frac{14}{20}=\frac{7}{10}$.

157. Simplify the fractions on the left hand side and we get

$\dfrac{59}{(x-10)(x+1)(x+3)}$ for the result $\therefore \dfrac{59}{(x-10)(x+1)(x+3)}$
$=\dfrac{118}{(x-10)(x+1)(3x+4)}$ or $\dfrac{1}{x+3}=\dfrac{2}{3x+4}$ $\therefore$ $3x+4=2x+6$ $\therefore$ $x=2$

158. Divide the numerator of each fraction by the denominator and we get $x+2-x+3=\frac{5x}{2}$ or $5=\frac{5x}{2}$ $\therefore$ $5x=10$ $\therefore$ $x=2$

159. Multiply the whole equation by $6x$ then $6+3-2=14x$
$\therefore$ $7=14x$ $\therefore$ $x=\frac{1}{2}$.

160. $\dfrac{(x-2a)^2}{(x+2b)^2}-1=\dfrac{x-2a-2b}{x+2a+2b}-1$ $\therefore$ $\dfrac{(x-2a)^2-(x+2b)^2}{(x+2b)^2}$

$$=\frac{x-2a-2b-(x+2a+2b)}{x+2a+2b} \text{ or } \frac{(x-2a+x+2b)(x-2a-x-2b)}{(x+2b)^2}$$

$$=-\frac{4(a+b)}{x+2a+2b} \text{ or } \frac{2(x-a+b)x-2(a+b)}{(x+2b)^2}=-\frac{4(a+b)}{x+2a+2b}$$

$$\therefore \frac{x-a+b}{(x+2b)^2}=\frac{1}{x+2a+2b}$$

By cross multiplication we have $x^2+4bx+4b^2=x^2+2ax+2bx+bx$
$2ab+2b^2-ax-2a^2-2ab$ $\therefore$ $bx-ax=-2a^2-2b^2$ or $ax-bx$
$=2(a^2+b^2)$ $\therefore$ $x=\dfrac{2(a^2+b^2)}{a-b}$

161. Clearing the eqn of fractions we have $(x+a)(2x+b)^2=(x+b)(2x+a)^2$ i. e. $(x+a)(4x^2+4bx+b^2)=(x+b)(4x^2+4ax+a^2)$; multiplying we obtain $4x^3+4x^2(a+b)+x(4ab+b^2)+ab^2=4x^3+4x^2(a+b)$

$+x(4ab+a^2)+a^2b \;\therefore\; xb^2+ab^2=xa^2+a^2b \;\therefore\; x(a^2-b^2)=ab^2-a^2b$
$=ab(b-a)=-ab(a-b) \;\therefore x=-\dfrac{ab}{a+b}.$

162. Multiply the eqn by $3x(2x+1)$ then $3x^2(2x+3)+2x+1=3x$
$(x+1)(2x+1)$ or $6x^3+9x^2+2x+1=3x(2x^2+3x+1)=6x^3$
$+9x^2+3x \;\therefore\; 3x-2x=1 \;\therefore\; x=1.$

163. $\dfrac{2(66x+1)}{3x+2}+\dfrac{2(4x+5)}{x-2}=52 \;\therefore\; 2(66x+1)(x-2)+2(4x+5)$
$(3x+2)=52(x-2)(3x+2)$ or $2(66x^2-131x-2)+2(12x^2+23x$
$+10)=52(3x^2-4x-4)$ or $132x^2-262x-4+24x^2+46x+20$
$=156x^2-208x-208$ or $-8x=-224 \;\therefore x=28.$

164. The L. C. M. of the denominators is $210(x-1)$; multiply the whole equation by this; then $14(x-1)(6-5x)-15(7-2x^2)=10(x-1)$ $(3x+1)-7(x-1)(10x-11)+2(x-1)$ or $154x-70x^2-84-105+30x^2$ $=30x^2-20x-10-70x^2+147x-77+2x-2 \;\therefore\; 25x=100 \;\therefore\; x=4.$

165. The left hand side of the equation can be simplified thus

$$\frac{x}{2}-\frac{2}{3}\,\frac{2x-3}{x-1}+\tfrac{1}{2}\,\frac{3x-1}{x-1}=\frac{3x(x-1)-4(2x-3)+3(3x-1)}{6(x-1)}$$

$$=\frac{3x^2-3x-8x+12+9x-3}{6(x-1)}=\frac{3x^2-2x+9}{6(x-1)} \;\therefore\; \frac{3x^2-2x+9}{6(x-1)}=\tfrac{3}{2}\,\frac{x^2+2}{3x-2}$$

$$\therefore\; \frac{3x^2-2x+9}{6(x-1)}\times\tfrac{2}{3}=\frac{x^2+2}{3x-2} \text{ or } \frac{3x^2-2x+9}{9(x-1)}=\frac{x^2+2}{3x-2} \;\therefore\; 9x^3-12x^2+31x$$

$-18=9x^3+18x-9x^2-18 \;\therefore\; -3x^2=-13x$ or $3x=13 \;\therefore\; x=4\tfrac{1}{3}.$

166. Simplify both sides of the equation thus,

$$\frac{(3+2x)(7+2x)-(5+2x)(1+2x)}{(7+2x)(1+2x)=7+16x+4x^2}=\frac{7+16x+4x^2-4x^2+2}{7+16x+4x^2} \text{ or } 21+20x$$

$+4x^2-5-12x-4x^2=16+9$ or $16+8x=9+16x \;\therefore\; x=\tfrac{7}{8}.$

167. Divide the numerators by the denominators thus

$$x+\frac{1}{x-1}+x+\frac{1}{x+1}=2x \;\therefore\; \frac{1}{x-1}+\frac{1}{x+1}=0 \;\therefore\; \frac{2x}{x^2-1}=0$$

$$\therefore\; 2x=0 \;\therefore\; x=0,$$

168. Divide the numerator of each fraction by the denominator thus $\left(x+1+\frac{1}{x+1}\right)+\left(x+4+\frac{4}{x+4}\right)=\left(x+2+\frac{2}{x+2}+x+3+\frac{3}{x+3}\right)$ ∴ $\frac{1}{x+1}+\frac{4}{x+4}=\frac{2}{x+2}+\frac{3}{x+3}$ ∴ $\frac{1}{x+1}-\frac{2}{x+2}=\frac{3}{x+3}-\frac{4}{x+4}$ ∴ $\frac{x+2-2x-2}{(x+1)(x+2)}=\frac{3x+12-4x-12}{(x+3)(x+4)}$ or $\frac{-x}{(x+1)(x+2)}=\frac{-x}{(x+3\ (x+4)}$ or $\frac{1}{(x+1)(x+2)}=\frac{1}{(x+3)(x+4)}$ ∴ $(x+1)(x+2)=(x+3)(x+4)$ or $x^2+3x+2=x^2+7+12$ ∴ $7x-3x=-10$

or $4x=-10$ ∴ $x=-2\frac{1}{2}$

169. $\frac{(x-10)(x+3)}{2(x-10)}+\frac{(x+4)(x-3)}{x-3}+\frac{(x-4)(x+5)}{4(x-4)}=16$

or $\frac{x+3}{2}+\frac{x+4}{3}+\frac{x+5}{4}=16$ ∴ $6(x+3)+4(x+4)+3(x+5)=192$ or $6x+18+4x+16+3x+15=192$ or $13x+49=192$

∴ $13x=143$ ∴ $x=11$

170. Adding the two fractions on the left hand side we have

$\frac{1}{x+a+b}\left\{\frac{2x}{x^2-(a-b)^2}\right\}=\frac{1}{x^2-(a+b)^2+x^2-(a-b)^2}$

∴ $\frac{1}{x+a+b}\cdot\frac{x-(a+b)}{x^2-(a-b)^2}=\frac{1}{x^2-(a+b)^2}\cdot\frac{x-(a+b)}{x^2-(a-b)^2}$

$=\frac{1}{x-(a+b)}$ ∴ $\{x-(a+b)\}^2=x^2-(a-b)^2$ ∴ $2x(a+b)$

$=(a+b)^2+(a-b)^2$ ∴ $x=\frac{a^2+b^2}{a+b}$.

171. $31\left\{\frac{24-5x}{x+1}+\frac{5-6x}{x+4}+11\right\}=29\left\{\frac{17-7x}{x+3}+\frac{8x+55}{x+3}-1\right\}$

∴ $31\left\{\frac{24-5x}{x+1}+5+\frac{5-6x}{x+4}+6\right\}=29\left\{\frac{17-7x}{x+2}+7+\frac{8x+55}{x+3}-8\right\}$

$\therefore 31\left\{\frac{29}{x+1}+\frac{29}{x+4}\right\} = 29\left\{\frac{31}{x+2}+\frac{31}{x+3}\right\}$; $\because \frac{1}{x+1}+\frac{1}{x+4}$ $=\frac{1}{x+2}+\frac{1}{x+3}$ $\therefore \frac{1}{x+1}-\frac{1}{x+2}=\frac{1}{x+3}-\frac{1}{x+4}$ $\therefore (x+1)(x+2)$ $=(x+3)(x+4)$ $\therefore 3x+2=7x+12$ $\therefore 4x=-10$ $\therefore x=-2\frac{1}{2}$.

172. $x\frac{x+3a}{c+3x}=a^{\frac{1}{2}}c^{\frac{1}{2}}.\frac{a+3x}{x+3c}$ $\therefore \frac{x^{\frac{1}{2}}(x+3a)}{a^{\frac{1}{2}}a+3x}=\frac{c^{\frac{1}{2}}(c+3x)}{x^{\frac{1}{2}}(x+3c)}$ $\therefore \frac{x^{\frac{3}{2}}+3ax^{\frac{1}{2}}}{a^{2}+3a^{\frac{1}{2}}x}$ $=\frac{c^{\frac{3}{2}}+3c^{\frac{1}{2}}x}{x^{\frac{3}{2}}+3cx^{\frac{1}{2}}}$ $\therefore \frac{\left(x^{\frac{3}{2}}+3ax^{\frac{1}{2}}\right)+\left(a^{\frac{3}{2}}+3a^{\frac{1}{2}}x\right)}{\left(x^{\frac{3}{2}}+3ax^{\frac{1}{2}}\right)-\left(a^{\frac{3}{2}}+3a^{\frac{1}{2}}x\right)}$ $=\frac{\left(c^{\frac{3}{2}}+3c^{\frac{1}{2}}x\right)+\left(x^{\frac{3}{2}}+3cx^{\frac{1}{2}}\right)}{\left(c^{\frac{3}{2}}+3c^{\frac{1}{2}}x\right)-\left(x^{\frac{3}{2}}+3cx^{\frac{1}{2}}\right)}$ or $\frac{\left(x^{\frac{1}{2}}+a^{\frac{1}{2}}\right)^3}{\left(x^{\frac{1}{2}}-a^{\frac{1}{2}}\right)^3}=\frac{\left(c^{\frac{1}{2}}+x^{\frac{1}{2}}\right)^3}{\left(c^{\frac{1}{2}}-x^{\frac{1}{2}}\right)^3}$ or $\frac{x^{\frac{1}{2}}+a^{\frac{1}{2}}}{x^{\frac{1}{2}}-a^{\frac{1}{2}}}=\frac{c^{\frac{1}{2}}+x^{\frac{1}{2}}}{c^{\frac{1}{2}}-x^{\frac{1}{2}}}$ $\therefore \frac{x^{\frac{1}{2}}}{a^{\frac{1}{2}}}=\frac{c^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ or $x=a^{\frac{1}{2}}c^{\frac{1}{2}}=\sqrt{ac}$.

173. $\frac{x-4-x+2}{(x-2)(x-4)}=\frac{x-8-x+6}{(x-6)(x-8)}$ or $\frac{-2}{(x-2)(x-4)}=\frac{-2}{(x-6)(x-8)}$ or $(x-2)(x-4)=(x-6)(x-8)$ or $x^2-6x+8=x^2-14x+48$ $\therefore 8x=40$ $\therefore x=5$.

174. $\frac{(x+1)^2}{x}=\frac{9}{x}$ (addition and subtraction) $\therefore (x+1)^2=9=3^2$ $\therefore x+1=3$ $\therefore x=2$.

175. $\left(1-\frac{2(a+b)}{x+2b}\right)^3=1-\frac{6(a+b)}{x+4b+2a}=1-\frac{3\times 2\frac{(a+b)}{x+2b}}{1+\frac{2(a+b)}{x+2b}}$.

Let $\frac{2(a+b)}{x+2b}=m$ then $(1-m)^3=1-\frac{3m}{1+m}$ $\therefore 1-3m+3m^2-m^3$

$=1-\frac{3m}{1+m}$ or $-3m+3m^2-m^3=-\frac{3m}{1+m}$ ∴ $-3+3m-m^2=-\frac{3}{1+m}$ ∴ $2m^2=m^3$ ∴ $m=2$ i. e. $\frac{2(a+b)}{(x+b)}=2$ ∴ $x+2b=a+b$ ∴ $x=a-b$.

176. $\frac{5x^4+10x^2+1}{x(x^4+10x^2+5)}=\frac{a(a^4+10a^2+5)}{5a^4+10a^2+1}$ or $\frac{5x^4+10x^2+1}{x^5+10x^3+5x}=\frac{a^5+10a^3+5a}{5a^4+10a^2+1}$ ∴ $\frac{(5x^4+10x^2+1)+(x^5+10x^3+5x)}{(5x^4+10x^2+1)-(x^5+10x^3+5x)}=\frac{(a^5+10a^3+5a)+(5a^4+10a^2+1)}{(a^5+10a^3+5a)-(5a^4+10a^2+1)}$ or $\left(\frac{x+1}{x-1}\right)^5=\left(\frac{1+a}{1-a}\right)^5$ ∴ $\frac{x+1}{x-1}=\frac{1+a}{1-a}$ ∴ $x=\frac{1}{a}$.

177. Dividing the numerator by the denominator we have $1+\frac{a-b}{x+b}=\left(1+\frac{a-b}{2x+b+c}\right)^2=1+\frac{2(a-b)}{2x+b+c}+\frac{(a-b)^2}{(2x+b+c)^2}$ ∴ $\frac{a-b}{x+b}=\frac{2(a-b)}{2x+b+c}+\frac{(a-b)^2}{(2x+b+c)^2}$ ∴ $\frac{1}{x+b}=\frac{2}{2x+b+c}+\frac{a-b}{(2x+b+c)^2}$ ∴ $\frac{1}{x+b}-\frac{2}{2x+b+c}=\frac{a-b}{(2x+b+c)^2}$ or $\frac{2x+b+c-2x-2b}{(2x+b+c)(x+b)}=\frac{a-b}{(2x+b+c)^2}$ ∴ $\frac{c-b}{x+b}=\frac{a-b}{2x+b+c}$ ∴ $2cx-2bx-b^2+c^2=ax-bx+ab-b^2$ ∴ $(a+b-2c)x=c^2-ab$ ∴ $x=\frac{c^2-ab}{a+b-2c}$

178. $\frac{(x^2+12x)+(6x^2+8)}{(x^2+12x)-(6x^2+8)}=\frac{126+124}{126-124}$ or $\left(\frac{x+2}{x-2}\right)^3=125$ ∴ $\frac{x+2}{x-2}=5$ ∴ $\frac{x}{2}=\frac{5+1}{5-1}=\frac{6}{4}$ ∴ $x=3$.

179. By cross multiplication we have $apx^3+bpx^2+cpx+aqx^2+bqx+qc=apx^3+aqx^2+arx+bpx^2+bqx+br$ or $(cp-ar)x=br-cq$ ∴ $x=\frac{br-cq}{cp-ar}$.

180. Transposing we have $\left\{3c+\frac{b}{a}-\frac{(2a+b)b^2}{a(a+b)^2}\right\}x=\frac{a^2b^2+3abc(a+b)^2}{(a+b)^3}$ or $x=\frac{ab}{a+b}$.

181. By transposition $m\left(\frac{x+a}{x+b}-1\right)+n\left(\frac{x+b}{x+a}-1\right)=0$ or $m\frac{a-b}{x+b}-n\frac{a-b}{x+a}=0$ ∴ $\frac{m}{x+b}=\frac{n}{x+a}$ or $mx+ma=nx+bn$ ∴ $(m-n)x=bx-ma$ ∴ $x=\frac{bn-am}{m-n}$.

182. Dividing the numerator by the denominator of each of the fractions we have $2x+\frac{2x+1}{2x^2+2x+3}=2x+\frac{1}{x+1}$ ∴ $\frac{2x+1}{2x^2+2x+3}=\frac{1}{x+1}$ ∴ $2x^2+3x+1=2x^2+2x+3$ ∴ $x=2$.

183. If $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ then $\frac{a}{b}=\frac{c}{d}$ Now putting the fractions in this form we have $\frac{(x^3+c)+(ax^2-bx)}{(x^3+c)-(ax^2-bx)}=\frac{x^2+(ax-b)}{x^2-(ax-b)}$ or $\frac{x^3+c}{ax^2-bx}=\frac{x^2}{ax-b}$

$\frac{x^3+c}{(ax-b)}-\frac{x^2}{ax-b}=0$ ∴ $\frac{1}{ax-b}\left\{\frac{x^3+c}{x}-x^2\right\}=0$

∴ $\frac{1}{ax-b}=0$ ∴ $ax-b=0$ ∴ $x=\frac{b}{a}$.

184. Simplifying the left hand side of the equation we have the result $=\frac{1}{x+1}$ ∴ $\frac{1}{x+1}=\frac{1}{6}$ ∴ $x+1=6$ $x=5$.

185. $\left(1+\frac{2}{x-2}\right)+\left(1-\frac{2}{x-7}\right)=\left(1+\frac{2}{x-1}\right)+\left(1-\frac{2}{x-6}\right)$ or $\frac{1}{x-2}-\frac{1}{x-7}=\frac{1}{x-1}-\frac{1}{x-6}$ or $\frac{x-7-x+2}{(x-2)(x-7)}=\frac{x-6-x+1}{(x-1)(x-6)}$ or $\frac{-5}{(x-2)(x-7)}=\frac{-5}{(x-1)(x-6)}$ or $(x-2)(x-7)=(x-1)(x-6)$ or $x^2-9x+14=x^2-7x+6$ ∴ $x=4$.

186. Squaring both sides $x-9=4$ ∴ $x=9+4=13$.

187. Squaring both sides $x+9=4x-12\sqrt{x}+9$ ∴ $12\sqrt{x}=3x$ ∴ $4\sqrt{x}=x$ ∴ $16x=x^2$ ∴ $x=16$.

188. $\sqrt{(4x-7)}=7-\sqrt{(4x)}$ ∴ $4x-7=49-14\sqrt{(4x)}+4x$ ∴ $14\sqrt{(4x)}=56$ ∴ $\sqrt{(4x)}=4$ ∴ $4x=16$ ∴ $x=4$.

Otherwise $4x-(4x-7)=7$ dividing this by the given eqn we have $\sqrt{(4x)}-\sqrt{(4x-7)}=1$ adding this with the given eqn we have $2\sqrt{(4x)}=8$ ∴ $\sqrt{(4x)}=4$ ∴ $4x=16$ ∴ $x=4$.

189 $x+12=(2+\sqrt{x})^2=4+4\sqrt{(x)}+x$ ∴ $4\sqrt{x}=8$ ∴ $\sqrt{x}=2$ ∴ $x=4$.

190. $\sqrt{(x-16)}=8-\sqrt{x}$ ∴ $x-16=64-16\sqrt{x}+x$ ∴ $16\sqrt{x}=80$ ∴ $\sqrt{x}=5$ ∴ $x=25$.

191. $\sqrt{(x+14)}=14-\sqrt{(x-14)}$ ∴ $x+14=196-28\sqrt{(x-14)}+x-14$ ∴ $28\sqrt{(x-14)}=168$ ∴ $\sqrt{(x-14)}=6$ ∴ $x-14=36$ ∴ $x=50$.

192. $3x+4=16$ ∴ $3x=12$ ∴ $x=4$.

193. $\sqrt{(2ax+x^2)}=b-(x+a)$ ∴ $2ax+x^2=b^2-2b(x+a)+(x+a)^2$ ∴ $2bx=(a-b)^2$ ∴ $x=\dfrac{(b-a)^2}{2b}$.

194. $\sqrt{(x+11)}=10-\sqrt{(x-9)}$ ∴ $x+11=100-20\sqrt{(x-9)}+x-9$ ∴ $20\sqrt{(x-9)}=80$ ∴ $\sqrt{(x-9)}=4$ ∴ $x-9=16$ ∴ $x=25$.

195. $\sqrt{(x-5)}=6-\sqrt{(x+7)}$ ∴ $x-5=36-12\sqrt{(x+7)}+x+7$ ∴ $12\sqrt{(x+7)}=48$ ∴ $\sqrt{(x+7)}=4$ ∴ $x+7=16$ ∴ $x=9$.

196. Squaring both sides we have $4m+x=4(n+x)-4\sqrt{(nx+x^2)}+x$ ∴ $4\sqrt{(nx+x^2)}=4(m-n-x)$ ∴ $\sqrt{(nx+x^2)}=m-n-x$ ∴ $nx+x^2=(m-n)^2-2(m-n)x+x^2$ ∴ $2mx-nx=(m-n)^2=(n-m)^2$

∴ $x=\dfrac{(n-m)^2}{2m-n}$.

197. $x+34\sqrt{x}+168=x+42\sqrt{x}+152$ ∴ $8\sqrt{x}=16$ ∴ $\sqrt{x}=2$ ∴ $x=4$.

198. Multiply both sides by $\sqrt{(5+x)}$ the denominator, then $5+x+\sqrt{(5x+x^2)}=15$ ∴ $\sqrt{(5x+x^2)}=10-x$ squaring $5x+x^2=100-20x+x^2$ ∴ $25x=100$ ∴ $x=4$.

199. Squaring, $x+4mn=4m^2-4m\sqrt{x}+x \quad \therefore \quad 4m\sqrt{x}=4m^2-4mn$ or $\sqrt{x}=m-n \quad \therefore \quad x=(m-n)^2$.

200. In the left hand side of the eqn divide the numerator by the denominator then $(\sqrt{(3x)}-1)-1=\frac{\sqrt{(3x)}-1}{2}$ or $\sqrt{(3x)}-2=\frac{\sqrt{(3x)}-1}{2}$ $\therefore \quad 2\sqrt{(3x)}-4=\sqrt{(3x)}-1 \quad \therefore \quad \sqrt{(3x)}=3 \quad \therefore \quad 3x=9 \quad \therefore \quad x=3$

201. Squaring $x-2\sqrt{(2x)}+2=x-2 \quad \therefore \quad -2\sqrt{(2x)}=-4$
$\therefore \quad \sqrt{(2x)}=2$ or $2x=4 \quad \therefore \quad x=2$

202. Squaring both sides $x-m+2\sqrt{(x-m)(x-n)}+x-n=m-n$
$\therefore \quad 2\sqrt{(x-m)(x-n)}=2m-2x$ or $\sqrt{(x-m)(x-n)}=m-x$
Squaring $x^2-(m+n)x+mn=m^2-2mx+x^2 \quad \therefore \quad (m-n)x$
$=m^2-mn=m(m-n) \quad \therefore \quad x=m$

203. Squaring both sides $12+x=4+4x^{\frac{1}{2}}+x$ or $4x^{\frac{1}{2}}=8 \therefore x^{\frac{1}{2}}=2$
$\therefore \quad x=4$

204. $\sqrt{(x+m)}=p-\sqrt{(x+n)} \quad \therefore \quad x+m=p^2-2p\sqrt{(x+n)}+x+n$
$\therefore \quad 2p\sqrt{(x+n)}=p^2+n-m \quad \therefore \sqrt{(x+n)}=\frac{p^2+n-m}{2p} \therefore x+n=\left(\frac{p^2+n-m}{2p}\right)^2$

205. $\sqrt{\frac{b}{a+x}}+\sqrt{\frac{c}{a-x}}=\sqrt{\frac{4bc}{a^2-x^2}}$ squaring, $\frac{b}{a+x}+2\sqrt{\frac{bc}{a^2-x^2}}$ $+\frac{c}{a-x}=\sqrt{\frac{4bc}{a^2-x^2}}=2\sqrt{\frac{bc}{a^2-x^2}} \quad \therefore \quad \frac{b}{a+x}+\frac{c}{a-x}=0$ or $b(a-x)+c(a+x)$ $=0$ or $ab-bx+ac+cx=0$ or $(b-c)x=ab+ac \quad \therefore \quad x=\frac{ab+ac}{b-c}$

206. Square both sides, then $x^2+40=x^2+8x+16 \therefore 8x=24 \therefore x=3$

207. Square both sides, $5x+10=5x+4(5x)^{\frac{1}{2}}+4 \quad \therefore \quad 4(5x)^{\frac{1}{2}}=6$
$\therefore \quad (5x)^{\frac{1}{2}}=\frac{6}{4}=\frac{3}{2} \quad \therefore \quad 5x=\frac{9}{4} \quad \therefore \quad x=\frac{9}{20}$

208. Multiply by $\sqrt{(3+x)}$ then $3+x+\sqrt{(3x+x^2)}=6 \quad \therefore \sqrt{(3x+x^2)}$
$=3-x \quad \therefore \quad 3x+x^2=9-6x+x^2 \quad \therefore \quad 9x=9 \quad \therefore \quad x=1$

209. $9x+30x^{\frac{1}{2}}+25=108+9x \therefore 30x^{\frac{1}{2}}=83 \quad \therefore \quad x^{\frac{1}{2}}=\frac{83}{30} \quad \therefore \quad x=(\frac{83}{30})^2$

210. Multiply the whole eqn by the L. C. M. of the denominators *i. e.* by $\sqrt{(x^2-1)}$ then $x+1+x-1=\sqrt{5(x^2-1)}$ $\therefore$ $\sqrt{5(x^2-1)}$ $=2x$ $\therefore$ $5(x^2-1)=4x^2$ $\therefore$ $x^2=5$ $\therefore$ $x=\sqrt{5}$

211. $(p^2+px)-(p^2-px)=2px$ dividing this by the given eqn we have $\sqrt{(p^2+px)}-\sqrt{(p^2-px)}=2x$ adding this with the given eqn we have $2\sqrt{(p^2+px)}=2x+p$; squaring $4(p^2+px)$

$$=4x^2+4px+p^2 \;\therefore\; 4x^2=3p^2 \;\therefore\; x=\frac{\sqrt{3}p}{2}$$

212. $\sqrt{(x-5)}-7=\sqrt{(x-12)}$; square, then $x-5-14\sqrt{(x-5)}+49$ $=x-12$ $\therefore$ $14\sqrt{(x-5)}=56$ $\therefore$ $\sqrt{(x-5)}=4$ $\therefore x-5=16$ $\therefore$ $x=21.$

213. $\sqrt{(4x+1)}=3$ $\therefore$ $4x+1=9$ $\therefore$ $x=2.$ 214. $\sqrt{(x+16)}=\sqrt{x}+2$
$\therefore$ $x+16=x+4\sqrt{x}+4$ $\therefore$ $4\sqrt{x}=12$ $\therefore$ $\sqrt{x}=3$ $\therefore$ $x=9.$

215. $\sqrt{(4x+9)}=2\sqrt{x}+1$ $\therefore 4x+9=4x+4\sqrt{x}+1$ $\therefore 4\sqrt{x}=8$ $\therefore x=4$

216. $x-7=-\sqrt{(x^2-7)}$ Square then $x^2-14x+49=x^2-7$ $\therefore$ $-14x$
$=-56$ $\therefore$ $x=4.$ 217. $\sqrt{(8x+x^2)}=4-x$ $\therefore$ $8x+x^2=16-8x+x^2$
$\therefore 16x=16$ $\therefore x=1.$ 218. Raise both sides to the $(2m)$ th power $\therefore$
$(4+x)^2=x^2+32x+4$ or $x^2+8x+16=x^2+32x+4$ $\therefore 24x=12$ $\therefore$ $x=\frac{1}{2}.$

219. Divide the numerator by the denominator in the left hand

side of the eqn then $\sqrt{(5x)}-3-1=\dfrac{\sqrt{(5x)}-3}{2}$ $\therefore$ $2\sqrt{(5x)}-8$

$=\sqrt{(5x)}-3$ $\therefore$ $\sqrt{(5x)}=5$ $\therefore$ $x=5.$ 220. Multiply the eqn by $\sqrt{x}$ then
$x=\sqrt{(x^2+x)}+1$ $\therefore$ $x^2+x=x^2-2x+1$ $\therefore$ $3x=1$ $\therefore$ $x=\frac{1}{3}.$

221. Multiply the eqn by $\sqrt{x}$ then $x-\sqrt{2}=\sqrt{(2x+x^2)}$, square then $x^2-2\sqrt{2}x+2=2x+x^2$ $\therefore$ $2x+2\sqrt{2}x=2$ or $x(\sqrt{2}+1)=1$

$$\therefore x=\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}=\sqrt{2}-1$$

222. $x-4\sqrt{x}+4=13-4\sqrt{x}$. or $x=9$

223. $\sqrt{(5x+10)}=\sqrt{(5x)}+2$ or $5x+10=5x+4\sqrt{(5x)}+4$ $\therefore$ $4\sqrt{(5x)}$ $=6$ or $2\sqrt{5x}=3$ $\therefore$ $\sqrt{(5x)}=\frac{3}{2}$ $\therefore$ $5x=\frac{9}{4}$ $\therefore$ $x=\frac{9}{20}.$

224. $\sqrt{(x^2+m^2)}=b-x$ $\therefore$ $x^2+m^2=b^2-2bx+x^2$ $\therefore$ $2bx=b^2-m^2$

225. $(5+x)-(5-x)=2x$, divide this by the given eqn then
$(5+x)^{\frac{1}{2}}-(5-x)^{\frac{1}{2}}=\sqrt{x}$ add this with the given eqn then $2(5+x)^{\frac{1}{2}}$
$=\sqrt{x}+2\sqrt{x}=3\sqrt{x}$ $\therefore 4(5+x)=9x$ $\therefore 5x=20$ $\therefore x=4$

226. $(\sqrt{x}-4)(\sqrt{x}+4)=259-10x$ or $x-16=259-10x$ $\therefore 11x$
$=275 \therefore x=25$. 227. Multiply by $\sqrt{(x-9)}$ then $x-9=36-\sqrt{(x^2-9x)}$
$\therefore \sqrt{(x^2-9x)}=45-x$ $\therefore x^2-9x=45^2-90x+x^2$ $\therefore 81x=45^2$ $\therefore x=25$

228. Multiply by $\sqrt{(x+m)}$, then $\sqrt{(x^2+mx)}+x+m=2m$
$\therefore \sqrt{(x^2+mx)}=m-x$ $\therefore x^2+mx=m^2-2mx+x^2$ $\therefore 3mx=m^2$
$\therefore x=\frac{m^2}{3m}=\frac{m}{3}$ 229. Square, $5+\sqrt{(x-4)}=9 \therefore \sqrt{(x-4)}=4$ $\therefore x-4$
$=16$ $\therefore x=20$. 230. By cross multiplication $x+19\sqrt{x}-42=x+4$
$\sqrt{x}+3$ $\therefore 15\sqrt{x}=45$ $\therefore \sqrt{x}=3$ $\therefore x=9$.

231. Squaring, $4a+x+a+x+2\sqrt{(a+x)(4a+x)}=4(x-2a)$
$=4x-8a$ $\therefore 2\sqrt{(a^2+5ax+x^2)}=2x-13a$ $\therefore 4(4a^2+5ax+x^2)=4x^2$
$-52ax+169a^2$ $\therefore 72ax=153a^2$ $\therefore x=\frac{153a^2}{72a}=\frac{17a}{8}$

232. $\sqrt{x-\sqrt{(1-x)}}=1-\sqrt{x}$ $\therefore x-\sqrt{(1-x)}=1-2\sqrt{x}+x$
$-\sqrt{(1-x)}=1-2\sqrt{x}$ $\therefore 1-x=1-4\sqrt{x}+4x$ $\therefore 5x=4\sqrt{x}$
or $5\sqrt{x}=4$ $\therefore x=\frac{16}{25}$.

233. $3(x^{\frac{1}{3}}-6)=2-2x^{\frac{1}{3}}$ $\therefore 5x^{\frac{1}{3}}=20$ $\therefore x^{\frac{1}{3}}=4$ $\therefore x=64$

234. Proceed as in ex 218.

235. Square then $p-\sqrt{(px+x^2)}=p-2\sqrt{px}+x \therefore -\sqrt{(px+x^2)}=$
$-2\sqrt{(px)}+x$, square, then $px+x^2=4px+x^2-4x\sqrt{(px)}$ $\therefore 4x\sqrt{(px)}$
$=3(px)$ or $4\sqrt{(px)}=3p \therefore \sqrt{(px)}=\frac{3p}{4} \therefore px=\frac{9p^2}{16} \therefore x=\frac{9p}{16}$

236. Square, then $5x-1-2\sqrt{(5x-1)}+1=5x-2$ $\therefore -2\sqrt{(5x-1)}$
$=-2$or $\sqrt{(5x-1)}=1$ $\therefore 5x-1=1$ $\therefore 5x=2$ $\therefore x=\frac{2}{5}$

237. $\sqrt{(x+9)}=1+\sqrt{x}$, square, then $x+9=1+2\sqrt{x}+x$ $\therefore 2\sqrt{x}$
$=8$ $\therefore x=16$.

238. Proceed as in ex 237. 239. Proceed as in ex 237.

240. See ex 197. 241. Square then $n+x+n-x+2\sqrt{(n^2-x^2)}=m^2\sqrt{(n^2-x^2)}$ × $\underline{\therefore (2-m^2)\sqrt{(n^2-x^2)}=-2n \therefore}$ $(2-m^2)^2(n^2-x^2)=4n^2$

$$\therefore n^2-x^2=\frac{4n^2}{(2-m^2)^2} \quad \therefore x^2=n^2-\frac{4n^2}{(2-m^2)^2}=n^2\left\{1-\frac{4}{(2-m^2)^2}\right\}=n^2$$

$$\left\{\frac{m^2(m^2-4)}{(m^2-2)^2}\right\} \quad \therefore x=\frac{mn}{m^2-2}\sqrt{(m^2-4)}$$

242 Square, then $4+\sqrt{(x^4-x^2)}=x^2-4x+4 \quad \therefore \sqrt{(x^4-x^2)}=x^2-4x \quad \therefore x^4-x^2=x^4-8x^3+16x^2 \quad \therefore 8x^3=17x^2$ or $8x=17 \quad \therefore x=\frac{17}{8}=2\frac{1}{8}$. 243. Cube both sides, then $\frac{ax+b}{mx+n}=1$ or $ax+b=mx+n$ $\therefore (a-m)x=n-b$.

244. $\sqrt{(3+x)}=\sqrt{x}+\sqrt{2}$ square, then $3+x=x+2\sqrt{(2x)}+2$ $\therefore 2\sqrt{(2x)}=1 \quad \therefore 8x=1$.

245. $\sqrt{(x^2+9)}+\sqrt{(x^2-9)}=4+\sqrt{34}$. square, then $x^2+9+x^2-9+2\sqrt{(x^4-81)}=16+34+8\sqrt{(34)}$ or $2x^2+2\sqrt{(x^4-81)}=50+8\sqrt{(34)}$, making the rational parts equal, we have $2x^2=50 \quad \therefore x^2=25$ or $x=5$

246. Multiply the eqn by $\sqrt{(x+m)}$ then $x+m=n-\sqrt{(x^2-m^2)}$ $\therefore \sqrt{(x^2-m^2)}=n-m-x \quad \therefore x^2-m^2=(n-m-x)^2=n^2+m^2+x^2-2mn-2nx+2mx \quad \therefore 2(n-m)x=2m^2+n^2-2mn$.

247. Square, then $1+x+x^2=4-4\sqrt{(1-x+x^2)}+1-x+x^2$ $\therefore 4\sqrt{(1-x+x^2)}=4-2x \quad \therefore 2\sqrt{(1-x+x^2)}=2-x \quad \therefore 4(1-x+x^2)=4-4x+x^2$ or $3x^2=0 \quad \therefore x=0$.

248. $\sqrt{1-x+\sqrt{(1+x)}}=1+\sqrt{(1-x)} \quad \therefore 1-x+\sqrt{(1+x)}=1+2\sqrt{(1-x)}+1-x \quad \therefore \sqrt{(1+x)}=1+2\sqrt{(1-x)} \quad \therefore 1+x=1+4\sqrt{(1-x)}+4(1-x) \quad \therefore 4\sqrt{(1-x)}=5x-4 \quad \therefore 16-16x=(5x-4)^2=25x^2-40x+16 \quad \therefore 25x^2=24x \quad \therefore x=\frac{24}{25}$.

249. $\sqrt{(x+m)}+\sqrt{(x-m)}=2n$, square, then $x+m+x-m+2\sqrt{(x^2-m^2)}=4n^2 \quad \therefore 2\sqrt{(x^2-m^2)}=4n^2-2x$ or $\sqrt{(x^2-m^2)}=2n^2-x$

$$\therefore x^2-m^2=4n^4-4n^2x+x^2 \quad \therefore 4n^2x=4n^4+m^2$$

250. $\sqrt{(x^2+4x)}=x+1 \quad \therefore x^2+4x=x^2+2x+1 \quad \therefore 2x=1$

251. Proceed as in ex 200. 252. Multiply the eqn by $\sqrt{(1+\sqrt{x})}$

× Should be minutely written.

then $1+\sqrt{x}-\sqrt{(1-x)}=m\sqrt{x}$ or $\sqrt{(1-x)}=1+\sqrt{x}-m\sqrt{x}$. square then, $1-x=1+x+m^2x+2\sqrt{x}-2m\sqrt{x}-2mx$ $\therefore$ $2(m-1)\sqrt{x}=(m^2-2m+1)$ $x=(m-1)^2x$ $\therefore$ $2=(m-1)\sqrt{x}$ $\therefore$ $\sqrt{x}=\frac{2}{m-1}$

253. Square both sides, then $1+x+x^2=9-6(1-x+x^2)^{\frac{1}{2}}$

$$+1-x+x^2 \therefore 6(1-x+x^2)^{\frac{1}{2}}=9-2x \therefore 36(1-x+x^2)$$
$$=81-36x+4x^2 \therefore 32x^2=45 \therefore x^2=\tfrac{45}{32} \therefore x=\frac{3\sqrt{5}}{2\sqrt{8}}$$

254. $\sqrt{(m+x)}\left(\frac{1}{m}+\frac{1}{x}\right)=\frac{1}{x}\sqrt{x}$ or $\sqrt{(m+x)}\frac{m+x}{mx}=\frac{x^{\frac{1}{2}}}{x}$

$$\text{or } \frac{(m+x)^{\frac{3}{2}}}{mx}=\frac{x^{\frac{1}{2}}}{x} \therefore \frac{(m+x)^{\frac{3}{2}}}{x^{\frac{3}{2}}}=\frac{m}{x} \therefore \frac{m+x}{x}=\frac{m^{\frac{2}{3}}}{x^{\frac{2}{3}}} \text{ or } \frac{m}{x}+1$$

$$=\frac{m^{\frac{2}{3}}}{x^{\frac{2}{3}}} \therefore \frac{m}{x}=\frac{m^{\frac{2}{3}}}{x^{\frac{2}{3}}}-1=\frac{m^{\frac{2}{3}}-x^{\frac{2}{3}}}{x^{\frac{2}{3}}} \text{ or } \frac{x}{m}=\frac{x^{\frac{2}{3}}}{m^{\frac{2}{3}}-x^{\frac{2}{3}}}$$

255. Squaring $m^2+x^2+m^2-x^2-2\sqrt{(m^4-x^4)}=p^2 \therefore -2\sqrt{(m^4-x^4)}$ $=p^2-2m^2 \therefore 4(m^4-x^4)=p^4-4m^2p^2+4m^4 \therefore 4x^4=4m^2p^2-p^4$

$$=p^4\left(\frac{4m^2}{p^2}-1\right)=p^4\left(\frac{4m^2-p^2}{p^2}\right) \therefore x^4=p^4\left(\frac{4m^2-p^2}{4p^2}\right) \therefore x=p\left(\frac{4m^2-p^2}{4p^2}\right)^{\frac{1}{4}}$$

256. $\sqrt{(ax)}-b=\frac{\sqrt{(ax)}-b-np}{n} \therefore n\sqrt{(ax)}-bn=\sqrt{(ax)}-b-np$

$$\therefore (n-1)\sqrt{(ax)}=bn-b-np=b(n-1)-np$$
$$\therefore \sqrt{(ax)}=\frac{b(n-1)-np}{n-1} \therefore ax=\left(\frac{b(n-1)-np}{n-1}\right)^2$$

257. $\sqrt{(1+x+x^2)}=px-\sqrt{(1-x+x^2)}$, square both sides then $1+x+x^2=p^2x^2+1-x+x^2-2px\sqrt{(1-x+x^2)} \therefore 2px\sqrt{(1-x+x^2)}$ $=p^2x^2-2x \therefore 2p\sqrt{(1-x+x^2)}=p^2x-2 \therefore 4p^2-4p^2x+4p^2x^2$ $=p^4x^2-4p^2x+4 \therefore (p^4-4p^2)x^2=4p^2-4 \therefore x^2=\frac{4}{p^2}\cdot\frac{p^2-1}{p^2-4}$

258. Proceed as in ex 200

259. By transposition $\sqrt{(2m+x)^2+n^2}+\sqrt{(2m-x)^2+n^2}=2m$.(1) also $\{(2m+x)^2+n^2\}-\{(2m-x)^2+n^2\}=8mx$, divide this by (1) then $\sqrt{(2m+x)^2+n^2}-\sqrt{(2m-x)^2+n^2}=4x$, add this with equ (1) then $2\sqrt{(2m+x)^2+n^2}=2m+4x=2(m+2x) \therefore \sqrt{(2m+x)^2+n^2}=m+2x$

$$\therefore (2m+x)^2+n^2=m^2+4mx+4x^2 \therefore 3x^2=3m^2+n^2$$

$$\therefore x^2=\frac{3m^2+n^2}{3}=m^2+\frac{n^2}{3}.$$

260. If $\frac{a+b}{a-b}=\frac{c}{d}$, it can be proved that $\frac{a}{b}=\frac{c+d}{c-d}$ from this formula the proposed eqn can be reduced to $\frac{\sqrt{4+x)}}{\sqrt{(4-x)}}=\frac{m+1}{m-1} \therefore \frac{4+x}{4-x}$ $=\frac{(m^2+1)+2m}{(m^2+1)-2m} \therefore \frac{4}{x}=\frac{m^2+1}{2m} \therefore \frac{x}{4}=\frac{2m}{m^2+1} \therefore x=\frac{8m}{1+m^2}$

261. $\left(\frac{m^2}{x}+n\right)^{\frac{1}{2}}-\left(\frac{m^2}{x}-n\right)^{\frac{1}{2}}=p^{\frac{1}{2}}$, squaring we have

$$\frac{m^2}{x}+n+\frac{m^2}{x}-n-2\left(\frac{m^4}{x^2}-n^2\right)^{\frac{1}{2}}=p \therefore \left(\frac{m^4}{x^2}-n^2\right)^{\frac{1}{2}}=\frac{m^2}{x}-\frac{p}{2}$$

$$\therefore \frac{m^4}{x^2}-n^2=\frac{m^4}{x^2}+\frac{p^2}{4}-\frac{m^2p}{x} \therefore \frac{m^2p}{x}=\frac{p^2}{4}+n^2=\frac{p^2+4n^2}{4}$$

$\therefore x=\frac{4m^2p}{4n^2+p^2}$ 262. $\sqrt{y(1-a)}=\frac{1}{\sqrt{y}} \therefore y=\frac{1}{1-a}$

263. By cross multiplication $3ax+5b\sqrt{(ax)}-8b^2=3ax-2b\sqrt{(ax)}$ $-5b^2 \therefore 7\sqrt{(ax)}=3b \therefore x=\frac{9b^2}{49a}$

264. By formula quoted in ex 260 we have $\frac{1+\sqrt{(1-x)}}{1-\sqrt{(1-x)}}=\frac{1}{p}$

$$\therefore \frac{1}{\sqrt{(1-x)}}=\frac{1+p}{1-p} \therefore \frac{1}{1-x}=\left(\frac{1+p}{1-p}\right)^2 \therefore 1-x=\left(\frac{1-p}{1+p}\right)^2$$

$$\therefore x=1-\left(\frac{1-p}{1+p}\right)^2=\frac{(1+p)^2-(1-p)^2}{(1+p)^2}=\frac{4p}{(1+p}$$

265. Using the formula in ex 260 we have,

$$\frac{\sqrt{(12x+1)}}{\sqrt{(12x)}}=\frac{18+1}{18-1}=\tfrac{19}{17}\ \therefore\ \frac{12x+1}{12x}=\frac{361}{289}\text{ or }1+\frac{1}{12x}=\frac{361}{289}\ \therefore\ \frac{1}{12x}$$
$$=\tfrac{72}{289}\text{ or }12x=\tfrac{289}{72}\ \therefore\ x=\tfrac{289}{864}.$$

266. Proceed as in ex 265. 267. Proceed as in ex 265.

268. Cube both sides, then $p+\sqrt{x}+p-\sqrt{x}+3(p^2-x)^{\frac{1}{3}}q^{\frac{1}{3}}=q$

$\therefore\ 3(p^2-x)^{\frac{1}{3}}q^{\frac{1}{3}}=q-2p$, again cube both sides, then $27\ (p^2q-px)$
$=q^3-8p^3-6pq^2+12p^2q\ \therefore\ 27qx=15p^2q+6pq^2+8p^3-q^3.$

269. Multiply by $x^{\frac{1}{3}}$ then $(m+x^{\frac{1}{2}})^{\frac{1}{2}}+(m-x^{\frac{1}{2}})^{\frac{1}{2}}=x^{\frac{1}{2}}$.

270. $(p+x)^{\frac{1}{2}}-\left(\dfrac{p}{p+x}\right)^{\frac{1}{2}}=(2p+x)^{\frac{1}{2}}$, squaring we have $p+x+\dfrac{p}{p+x}$

$$-2p^{\frac{1}{2}}=2p+x\ \therefore\ \frac{p}{p+x}=p+2p^{\frac{1}{2}}\ \therefore\ \frac{p^{\frac{1}{2}}}{p+x}=p^{\frac{1}{2}}+2\ \therefore\ p+x=\frac{p^{\frac{1}{2}}}{p^{\frac{1}{2}}+2}$$

$$\therefore x=\frac{p^{\frac{1}{2}}}{p^{\frac{1}{2}}+2}-p=\frac{p^{\frac{1}{2}}-2p-p^{\frac{3}{2}}}{p^{\frac{1}{2}}+2}=p^{\frac{1}{2}}\left(\frac{1-2p^{\frac{1}{2}}-p}{p^{\frac{1}{2}}+2}\right)$$

271. $\dfrac{1+x}{1-x}=\dfrac{1+2ax+a^2x^2}{1-2ax+a^2x^2}\ \therefore\ \tfrac{1}{x}=\dfrac{1+a^2x^2}{2ax}$ or $\dfrac{1+a^2x^2}{2a}=1\ \therefore\ 1+a^2x^2$

$$=2a\ \therefore\ a^2x^2=2a-1\ \therefore\ x^2=\frac{2a-1}{a^2}.$$

272. Squaring both sides, $x^{\frac{1}{2}}+3m^2+x^{\frac{1}{2}}-3m^2-2(x-9m^4)^{\frac{1}{2}}$

$$=\frac{4mx^{\frac{1}{2}}}{n}\ \therefore\ 2\,(x-9m^4)^{\frac{1}{2}}=2x^{\frac{1}{2}}-\frac{4mx^{\frac{1}{2}}}{n}\text{ or }(x-9m^4)^{\frac{1}{2}}=x^{\frac{1}{2}}-\frac{2mx^{\frac{1}{2}}}{n},$$

$$\therefore\ x-9m^4=x-\frac{4mx}{n}+\frac{4m^2x}{n^2}\ \therefore\ 4\left(\frac{m}{n}-\frac{m^2}{n^2}\right)x=9m^4\text{ or }4\left(\frac{1}{n}-\frac{m}{n^2}\right)x=9m^3$$

$$\text{or }4\left(\frac{n-m}{n^2}\right)x=9m^3\ \therefore\ x=\frac{9m^3n^2}{4(n-m)}$$

273. Proceeding as in ex 260 we have $\frac{3x}{\sqrt{(4x-x^2)}}=\frac{1+2}{1-2}=-3$

∴ $\frac{9x^2}{4x-x^2}=9$ ∴ $9x^2=36x-9x^2$ ∴ $18x^2=36x$ or $18x=36$ ∴ $x=2$.

274. $\frac{(nx+m^2)^{\frac{1}{2}}}{(nx-m^2)^{\frac{1}{2}}}=\frac{r+1}{r-1}$ ∴ $\frac{nx+m^2}{nx-m^2}=\left(\frac{r+1}{r-1}\right)^2$ ∴ $\frac{nx}{m^2}$

$=\frac{(r+1)^2+(r-1)^2}{(r+1)^2-(r-1)^2}=\frac{2(r^2+1)}{4r}=\frac{r^2+1}{2r}$ ∴ $x=\frac{m^2(r^2+1)}{2nr}$.

275. $\frac{m(c^2-x^2)^{\frac{1}{2}}-n(c+x)}{m(c^2-x^2)^{\frac{1}{2}}+n(a+x)}=\frac{ma-nb}{ma+nb}$ or $\frac{m(c^2-x^2)^{\frac{1}{2}}}{n(c+x)}=\frac{ma}{nb}$

∴ $\frac{(c^2-x^2)^{\frac{1}{2}}}{c+x}=\frac{a}{b}$ or $\frac{(c+x)^{\frac{1}{2}}.\,(c-x)^{\frac{1}{2}}}{(c+x)^{\frac{1}{3}}.\,(c+x)^{\frac{1}{2}}}=\frac{a}{b}$ or $\frac{(c-x)^{\frac{1}{2}}}{(c+x)^{\frac{1}{2}}}=\frac{a}{b}$

∴ $\frac{c-x}{c+x}=\frac{a^2}{b^2}$ ∴ $\frac{c}{x}=\frac{b^2+a^2}{b^2-a^2}$ or $\frac{x}{c}=\frac{b^2-a^2}{b^2+a^2}$ ∴ $x=\frac{c(b^2-a^2)}{b^2+a^2}$.

276. Cubing both sides, $a^2+\sqrt{x}+a^2-\sqrt{x}+3\sqrt[3]{(a^4-x)}\,a^{\frac{2}{3}}=a^2$

or $3\sqrt[3]{(a^4-x)}.\,a^{\frac{2}{3}}=-a^2$ ∴ $27(a^4-x)a^2=-a^6$ ∴ $27a^2x=27a^6+a^6=28a$

$$x=\frac{28a^6}{27a^2}=\frac{28a^4}{27}$$

277. $\frac{(1+\sqrt{2x+x^2})+x}{(1+\sqrt{2x+x^2})-x}=1-ax$ ∴ $\frac{1+\sqrt{(2x+x^2)}}{x}=\frac{2-ax}{-ax}$

or $1+\sqrt{(2x+x^2}=\frac{2-ax}{-a}=\frac{ax-2}{a}$ ∴ $\sqrt{(2x+x^2)}=\frac{ax-2}{a}-1$

$=\frac{ax-2-a}{a}$ ∴ $2x+x^2=\frac{a^2x^2+4+a^2-4ax-2a^2x+4a}{a^2}$ or $2a^2x+a^2x^2$

$=a^2x^2+4+a^2-4ax-2a^2x+4a$ ∴ $(4a^2+4a)x=a^2+4a+4$

or $x=\frac{a^2+4a+4}{4a(a+1)}$.

278. Proceed as in ex 260. 279 Proceed as in ex 268.

280. $\frac{\sqrt{(36x+1)}+\sqrt{(36x)}}{\sqrt{(36x+1)}-\sqrt{(36x)}}=9=\frac{9}{1}$ $\therefore$ $\frac{\sqrt{(36x+1)}}{\sqrt{(36x)}}=\frac{9+1}{9-1}=\frac{10}{8}=\frac{5}{4}$

$\therefore$ $\frac{36x+1}{36x}=\frac{25}{16}$ or $1+\frac{1}{36x}=\frac{25}{16}$ $\therefore$ $\frac{1}{36x}=\frac{25}{16}-1=\frac{9}{16}$ $\therefore$ $x=\frac{16}{9\times36}=\frac{4}{81}$.

281 $\frac{(1+x+x^2)(1-x)}{(1+x)(1-x+x^2)}=\frac{62}{63}$ or $\frac{1-x^3}{1+x^3}=\frac{62}{63}$ or $\frac{1+x^3}{1-x^3}=\frac{63}{62}$

$\therefore$ $\frac{1}{x^3}=\frac{63+62}{63-62}=125$ $\therefore$ $x^3=\frac{1}{125}$ $\therefore$ $x=\frac{1}{5}$

282 $1+\frac{px}{mn}=\left(1+\frac{2x}{m-x}\right)^2=1+\frac{4x}{m-x}+\frac{4x^2}{(m-x)^2}$; $\therefore$ $\frac{p}{mn}=\frac{4}{m-x}$ $+\frac{4x}{(m-x)^2}=\frac{4m}{(m-x)^2}$ $\frac{p}{n}=\frac{4m^2}{(m-x)^2}$ $\therefore$ $(m-x)^2=\frac{4m^2n}{p}$ $\therefore$ $m-x$ $=-2m\sqrt{\frac{n}{p}}$. $x=m+2m\sqrt{\frac{n}{p}}=m\left(1+2\sqrt{\frac{n}{p}}\right)$.

283. $\frac{1+x^3}{(1+x)^2}+\frac{1-x^3}{(1-x)^2}=m$ $\therefore$ $\frac{(1+x)(1-x+x^2)}{(1+x)^2}+\frac{(1-x)(1+x+x^2)}{(1-x)^2}$ $=m$ or $\frac{1-x+x^2}{1+x}+\frac{1+x+x^2}{1-x}=m$ $\therefore$ $(1-x)^2+x^2(1-x)+(1+x)^2$ $+x^2(1+x)=m(1-x^2)$ or $2+4x^2=m-mx^2$ $\therefore$ $(m+4)x^2$ $=m-2$ $\therefore$ $x^2=\frac{m-2}{m+1}$

284. $\frac{\sqrt{(1-x)}-1}{(1-x)-1}+\frac{\sqrt{(1+x)}+1}{(1+x)-1}=\frac{1}{x}$ $\therefore$ $\frac{\sqrt{(1-x)}-1}{-x}+\frac{\sqrt{(1+x)}+1}{x}$ $=\frac{1}{x}$ $\therefore$ $1-\sqrt{(1-x)}+\sqrt{(1+x)}+1=1$ $\therefore$ $\sqrt{(1+x)}-\sqrt{(1-x)}=-1$ $\therefore$ 1 $+x+1-x-2\sqrt{(1-x^2)}=1$ $\therefore$ $2\sqrt{(1-x^2)}=1$ $\therefore$ $4(1-x^2)=1$ $\therefore$ $4x^2=3$ $\therefore$ $x=\frac{1}{2}\sqrt{3}$

285. $\frac{1+x}{1+x+\sqrt{(1+x^2)}}+\frac{1-x}{1-x+\sqrt{(1+x^2)}}=4$, multiply the whole eqn by $2+2\sqrt{(1+x^2)}$ the product of the denominators, then $1-x^2+(1+x)$

$\sqrt{(1+x^2)}+1-x^2+(1-x)\sqrt{(1+x^2)}=4(2+2\sqrt{1+x^2})$ $\therefore$ $2(1-x^2)+2\sqrt{(1+x^2)}=8(1+\sqrt{1+x^2})$ $\therefore$ $1-x^2+\sqrt{(1+x^2)}=4+4\sqrt{(1+x^2)}$ $\therefore$ $3\sqrt{(1+x^2)}=3+x^2$ $\therefore$ $9+9x^2=9+6x^2+x^4$ $\therefore$ $x^4=3x^2$ $\therefore$ $x^2=3$

286. Let $\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}=m$ $\therefore$ $\left(\frac{1-x}{1+x}\right)^{\frac{1}{4}}=\frac{1}{m}$ then the given eqn $=(1+p^2)^{\frac{1}{2}}m+(1-p^2)^{\frac{1}{2}}\cdot\frac{1}{m}=2(1-p^4)^{\frac{1}{4}}$ $\therefore$ $(1+p^2)^{\frac{1}{2}}m^2-2(1-p^4)^{\frac{1}{4}}m+(1-p^2)^{\frac{1}{2}}=0$ $\therefore$ $(1+p^2)^{\frac{1}{4}}m-(1-p^2)^{\frac{1}{4}}=0$ $\therefore m=\left(\frac{1-p^2}{1+p^2}\right)^{\frac{1}{4}}$ $\therefore$ $\frac{1+x}{1-x}=m^4=\frac{1-p^2}{1+p^2}$ $\therefore$ $x=-p^2$.

287. Proceed as in ex. 260.

288. Squaring, $m+x+m-x+2\sqrt{(m^2-x^2)}=\sqrt{(m^2+x^2)}+\sqrt{(m^2-x^2)}+2\sqrt[4]{(m^4-x^4)}$ $\therefore$ $2m-2\sqrt[4]{(m^4-x^4)}=\sqrt{(m^2+x^2)}-\sqrt{(m^2-x^2)}$ $\therefore$ $4m^2-8m\sqrt[4]{(m^4-x^4)}+4\sqrt{(m^4-x^4)}=2m^2-2\sqrt{(m^4-x^4)}$ $\therefore$ $m^2+3\sqrt{(m^4-x^4)}=4m\sqrt[4]{(m^4-x^4)}$, squaring, $10m^4-9x^4=10m^2\sqrt{(m^4-x^4)}$, squaring again and equating the results we have $81x^8=80m^4x^4$ $\therefore$ $x=\frac{2m}{3}\sqrt[4]{5}$.

289. $\frac{(m+x)(\sqrt{m}-\sqrt{m+x})}{m-(m+x)}+\frac{(m-x)(\sqrt{m}-\sqrt{(m-x)})}{m-(m-x)}=m^{\frac{1}{2}}$ or $(m+x)(\sqrt{(m+x)}+\sqrt{m})+(m-x)(\sqrt{m}-\sqrt{m-x})=xm^{\frac{1}{2}}$ $\therefore$ $(m+x)^{\frac{3}{2}}-(m-x)^{\frac{3}{2}}=3xm^{\frac{1}{2}}$ $\therefore$ $(m+x)^3+(m-x)^3-2(m^2-x^2)^{\frac{3}{2}}=9mx^2$ $\therefore$ $2m^3+6mx^2-9mx^2=2(m^2-x^2)^{\frac{3}{2}}$ or $2m^3-3mx^2=2(m^2-x^2)^{\frac{3}{2}}$ $\therefore 4m^6-12m^4x^4+9m^2x^4=4m^6-12m^4x^4+12m^2x^4-4x^6 \therefore 4x^6=3m^2x^4$ $\therefore 4x^2=3m^2 \therefore 2x=m\sqrt{3}$.

290. Squaring $\sqrt[4]{(y^4-1)}=2y^6-y^2-2y^4\sqrt{(y^4-1)}$ $\therefore$ $y^2+\sqrt{(y^4-1)}=2y^4(y^2+\sqrt{y^4-1})$ $\therefore$ $(y^2+\sqrt{y^4-1})^2=2y^4$ $\therefore$ $y^2+\sqrt{(y^4-1)}=\sqrt{2}y^2$ $\therefore$ $\sqrt{(y^4-1)}=(\sqrt{2}-1)y^2$ $\therefore$ $y^4-1=(3-2\sqrt{2})y^4$ $\therefore$ y^4

$$=\frac{1}{2\sqrt{2}-2}=\frac{1}{2(\sqrt{2}-1)}=\frac{\sqrt{2}+1}{2(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{1+\sqrt{2}}{2}$$

291. The eqn can be put in this form $(x^2+39x+374)^{\frac{1}{2}}$ $-\frac{19}{2}\left(\frac{x+22}{x+17}\right)^{\frac{1}{2}}=(x^2+20x+51)^{\frac{1}{2}}$ squaring and observing that $x^2+39x+374=(x+22)(x+17)$ we have $x^2+39x+374-19(x+22)$ $+\frac{361}{4}\cdot\frac{x+22}{x+17}=x^2+20x+51$ $\therefore\ \frac{361}{4}\cdot\frac{x+22}{x+17}=51+(19-17)\times 22=95$

$$\therefore\ \frac{x+22}{x+17}=\frac{4}{361}\times 95=\frac{20}{19}\ \therefore\ x=78.$$

292. Assume $\sqrt[3]{x}+\sqrt[3]{m}=\sqrt[3]{y}$ them cubing these equals $x+m+3$ $\sqrt[3]{(mx)}(\sqrt[3]{x}+\sqrt[3]{m})=y$ $\therefore\ x+m+3\ \sqrt[3]{(myx)}=y$. Now comparing this with the original eqn it appears that $y=n$, $\therefore\ \sqrt[3]{x}+\sqrt[3]{m}=\sqrt[3]{n}$ $\therefore\ \sqrt[3]{x}=\sqrt[3]{n}-\sqrt[3]{m}$.

293. $x+a=\sqrt{a^2+x\sqrt{(b^2+x^2)}}\ \therefore\ x^2+2ax+a^2=a^2+x\sqrt{(b^2+x^2)}$

$$\therefore\ x+2a=\sqrt{(b^2+x^2)}\ \therefore x^2+4ax+4a^2=b^2+x^2\ \therefore x=\frac{b^2-4a^2}{4a}$$

294. $$\frac{\{(m^2+x)^{\frac{1}{2}}+(m^2-x)^{\frac{1}{2}}\}^2}{\{(m^2+x)^{\frac{1}{2}}-(m^2-x)^{\frac{1}{2}}\}^2}=p^3\,\frac{(m^2+x)^{\frac{1}{2}}-(m^2-x)^{\frac{1}{2}}}{(m^2+x)^{\frac{1}{2}}+(m^2-x)^{\frac{1}{2}}}$$

$$\therefore\ \left(\frac{(m^2+x)^{\frac{1}{2}}+(m^2-x)^{\frac{1}{2}}}{(m^2+x)^{\frac{1}{2}}-(m^2-x)^{\frac{1}{2}}}\right)^3=p^3\ \therefore\ \frac{(m^2+x)^{\frac{1}{2}}+(m^2-x)^{\frac{1}{2}}}{(m^2+x)^{\frac{1}{2}}-(m^2-x)^{\frac{1}{2}}}=p$$

$$\therefore\ \frac{(m^2+x)^{\frac{1}{2}}}{(m^2-x)^{\frac{1}{2}}}=\frac{p+1}{p-1}\ \therefore\frac{m^2+x}{m^2-x}=\left(\frac{p+1}{p-1}\right)^2\ \therefore\ \frac{m^2}{x}=\frac{2(p^2+1)}{4p}=\frac{p^2+1}{2p}$$

295. $$\frac{x+\sqrt{(x^2-9)}}{x-\sqrt{(x^2-9)}}=9\sqrt{3}\left(\frac{\sqrt{(x+3)}-\sqrt{(x-3)}}{\sqrt{(x+3)}+\sqrt{(x-3)}}\right)^{\frac{1}{2}}$$

$$\text{or}\ \left(\frac{\sqrt{(x+3)}+\sqrt{(x-3)}}{\sqrt{(x+3)}-\sqrt{(x-3)}}\right)^2=9\sqrt{3}\left(\frac{\sqrt{(x+3)}-\sqrt{(x-3)}}{\sqrt{(x+3)}+\sqrt{(x-3)}}\right)^{\frac{1}{2}}$$

$$\therefore\ \frac{(\sqrt{(x+3)}+\sqrt{(x-3)})^{\frac{5}{2}}}{(\sqrt{(x+3)}-\sqrt{(x-3)})^{\frac{5}{2}}}=9\sqrt{3}=(\sqrt{3})^5\ \therefore\left(\frac{\sqrt{(x+3)}+\sqrt{(x-3)}}{\sqrt{(x+3)}-\sqrt{(x-3)}}\right)^{\frac{1}{2}}=\sqrt{3}$$

$$\frac{\sqrt{(x+3)}+\sqrt{(x-3)}}{\sqrt{(x+3)}-\sqrt{(x-3)}}=3 \;\therefore\; \frac{\sqrt{(x+3)}}{\sqrt{(x-3)}}=\frac{3+1}{3-1}=2 \;\therefore\; \frac{x+3}{x-3}=4$$

$$\tfrac{x}{3}=\tfrac{4+1}{4-1}=\tfrac{5}{3} \;\therefore\; x=5.$$

PROBLEMS

Ex. 21. 1. Let $x =$ no required then $8x+18=90 \therefore 8x=72 \therefore x=9$

2 Let $x =$ no required, then $\frac{x}{7}+\frac{x}{5}=24 \therefore 12x=35\times 24 \therefore x=70$

3. Let $x=$ no required then $x-25$ is the remainder $\therefore \frac{2}{3}(x-2)=50 \therefore x=100$ 4. Let $x=$ no required then $\frac{x}{12}-\frac{x}{15}=1 \therefore x=60$.

5. Let $x=$ no required then $\frac{3(x-8)+65}{4}=x \therefore x=41$.

6. Let $x=$ one number then $x+28$ is the other number ($\because$ the diff $=28$) $\therefore x+(x+28)=70 \therefore x=21$.

7. Let $x=$ age of the father then the age of the eldest is $\frac{4}{7}x$ and of the youngest is $\frac{1}{4}$ of $\frac{4x}{7}=\frac{x}{7} \therefore x+\frac{4x}{7}+\frac{x}{7}=84 \therefore x=49$.

8. Let $x=$ age of Ram then $3x=$ age of Sham $\therefore x-15$, and $3x-15$ were their respective ages 15 years ago $\therefore 3x-15=6(x-15)$ $\therefore x=25$. 9. Let $x=$ greater no then $\frac{96}{x}=$ less no $\therefore (x-1)\frac{96}{x}=96-8=88 \therefore 96x-96=88x$ or $8x=96 \therefore x=12$.

10. Let $x=$ required no then $x^{\frac{1}{3}}=\frac{1}{3}x^{\frac{1}{2}} \therefore x^2=\frac{1}{3^6}x^3$ or $1=\frac{1}{3^6}x$ $\therefore x=3^6$. 11. Let $x=$ no of men then $30-x=$ no of boys $\therefore x\times 10+(30-x)\times 5=14\times 16+6 \therefore x=16$.

12. Let $x=$ present age of the boy then $x+42=$ age after 42 years and $x-6=$ age 6 years since $\therefore x+42=7(x-6)$ or $x+42=7x-42$ $\therefore 6x=84 \therefore x=14$. 13. Let $x=$ no required then $x-10$, $x-40$, and $x-25$ are the remainders $\therefore \frac{x-10}{3}+\frac{x-40}{4}+\frac{x-40}{5}=60 \therefore x=100$

14. Let $x=$ no of hours indicated by the clock then $3x=$ age of the boy and $\therefore 9x=$ age of the father $\therefore 3x+9x=96$ or $x=8$.

15. Let $x =$ no of Rs. A had at first then $147 - x =$ no of Rs. B had $\therefore x + 12 - (147 - x - 12) = 47 \quad \therefore x = 85$

16. Let $x =$ no of Rs. each had begun with, then $x + 24 =$ amount which A had and $x - 24$ which B then had $\therefore \frac{3}{4}(x + 24) + 14 = x - 24 + \frac{1}{4}(x + 24 \quad \therefore x = 100$. 17. Let $x =$ greater part then $109 - x =$ less part $\therefore \frac{2}{3}(100 - x) + x = 89 \quad \therefore x = 67$.

18. Let $x =$ no of two anna pieces then $10x =$ no of eight anna prices $\therefore 2 \times x + 8 \times 10x = 328 \times 16 \quad \therefore x = 64$.

19. Let $x =$ A's share, then $2x =$ B's share $\therefore 6x =$ C's share $\therefore x + 2x + 6x = 360$ or $9x = 360 \quad \therefore x = 40$

20. Let $x =$ no of half crowns then $x + 1 =$ no of florins $\therefore 2(x + 1) + 2\frac{1}{2} \times x = 74$ or $x = 16 \quad \therefore x + 1 = 17$ no of florins.

✓ 21. Let $x = 1^{st}$ part then $x + 3 = 2^{nd}$ part $- 3 \quad \therefore 2^{nd}$ part $= x + 6$, also $x + 3 = 3^{rd}$ part $\times 3 \quad \therefore 3^{rd}$ part $= \frac{x + 3}{3}$, again $x + 3 = \frac{4^{th} \text{ part}}{3}$

$\therefore 1$ part $= 3(x + 3) \quad \therefore$ sum of all the parts $= x + (x + 6) + \frac{x + 3}{3} + 3(x + 3)$ and this by the question $= 48 \quad \therefore x = 6$.

22. Let $x =$ money each had at first then $x + 180 = 3(x - 140)$ $\therefore x = 300$ 23. Let $x =$ no required then $\frac{x}{3} - 80 = 80 - \frac{x}{5} \quad \therefore x = 300$

24. Let $x =$ no of mangoes contained in each basket then $x - 50 = \frac{3}{2}(x - 75) \quad \therefore x = 125$ 25. Let $x = 1^{st}$ part then $240 - x =$ other part $\therefore \frac{2}{5}x + \frac{3}{7}(240 - x) = 100 \quad \therefore x = 100$

26. Let $x =$ distance in miles travelled, then $\frac{x}{12} =$ no of hours it took him to reach the distant place and $\frac{x}{6} =$ no of hours it took him to come back $\therefore \frac{x}{12} + \frac{x}{6} = 10 \quad \therefore x = 40$.

27. Let $x =$ no required then when it is divided into p parts each part $= \frac{x}{p}$ and when it is divided into $p + 1$ equal parts each part $= \frac{x}{p + 1}$

$\therefore$ product of the p equal parts $= \left(\frac{x}{p}\right)^p$ and the product of $p + 1$ equal parts $=\left(\frac{x}{p+1}\right)^{p+1}$ $\therefore$ by the question $\left(\frac{x}{p}\right)^{p}=p\left(\frac{x}{p+1}\right)^{p+1}$ $\therefore$ $\frac{x^p}{p^p}$ $=\frac{px^{p+1}}{(p+1)^{p+1}}\therefore\ \frac{1}{p^p}=\frac{px}{(p+1)^{p+1}}\ \therefore\ x=\frac{(p+1)^{p+1}}{p^p p}=\left(\frac{p+1}{p}\right)^{p+1}$

28. $\therefore$ the waterman can row down 18 miles in $1\frac{1}{2}$ hours $\therefore$ in one hour he can row 12 miles; let x = rate of the stream in the middle in miles per hour then $12-x$ = rate in miles per hour of his pulling in still water; now when he came back along the shore he was obliged to force his way against a current whose rate $=\frac{3}{5}x$ and $(12-x)-\frac{3}{5}x$ = no of miles he advanced in an hour and by the question

$$\frac{18}{(12-x)-\frac{3}{5}x}=2\tfrac{1}{4}\ \therefore\ x=2\tfrac{1}{2}.$$

29. The hand a describes the circle in 1 hour or 60 min $\therefore$ in 1 min it describes $\frac{1}{60}$ of the circle; the hand b describes the circle in $1\frac{1}{60}$ hour or in 61 min $\therefore$ in 1 min it describes $\frac{1}{61}$ of the circle $\therefore$ the hand a gains over b, $\frac{1}{60}-\frac{1}{61}=\frac{1}{60\times 61}$ of the circle in 1 min $\therefore$ in order that they may be together again the hand a should gain a complete circle; let x = no of mins required then $\frac{1}{60\times 61}\times x=1$ $\therefore$ $x=60\times 61$ min $=61$ hours; otherwise by common rule of three $\frac{1}{60\times 61}:1::1$ min $:x$ $\therefore$ $x=60\times 61$ min.

30. Let x = one no then $x+51$ = other no $\therefore$ $\frac{x+54}{x}=4$ $\therefore$ $x=18$

31. Let x = A's share then $3x$ = B's share, C's share $=2(x+3x)=8x$ and D's share $=\frac{1}{4}(x+3x+8x)=3x$ $\therefore$ sum of their shares $=x+3x+8x+3x$ which by the question $=1500$.

32. Let x = price of the pony chaise then x = price of the horse also, now $x-50$ would be the price of the horse at the deminished rate and $x+150$ = price of the chaise at the advanced rate $\therefore$ by the question $x-50=\frac{1}{2}(x+150)$ $\therefore$ $x=250$.

33. Let x = no of days he was idle then $45-x$ = no of days he was

engaged then the amount he earned for the no of days he was engaged is $5(45-x)$ and the amount he lost for being idle is $4x$

$\therefore 5(45-x)-4x=90 as \therefore x=15$,

34. Let x = no of minutes past 7 o'clock when the two hands will be together, then $\frac{x}{12}$ = no of minute divisions passed by the hour hand but the hour-hand is already 35 minutes in advance $\therefore 35+\frac{x}{12}=x$ $\therefore x=38\frac{2}{11}$. In the case in which the two hands will be at right angles to one another let x = no of minutes passed by the minute-hand then to be at right angles the two hands will be 15 minutes apart and therefore $x+15=35+\frac{x}{12} \therefore x=21\frac{9}{11}$. Lastly when the two hands would be opposite $x+30=35+\frac{x}{12} \therefore x=5\frac{5}{11}$

35. As in 4 o'clock the hour-hand is at $5\times4=20$ min divisions distant from 12, so in m. o'clock the hour-hand is at 5m. min divisions distant from 12 ; let x = no of min passed m o'clock when the min hand is d min divisions apart then by the reasoning shewn in the preceding problem we have $5m+\frac{x}{12}\pm d=x$ ($\pm$ $\because$ this may happen when the min hand is before or after the hour-hand) $\therefore 60m+x\pm12d=12x$

$$\therefore x=\frac{12(5m\pm d)}{11}.$$

36. Let x = amount of his habilities, then x = amount which remains unrealized. Now the sum realized on 4000lbs at 12 *as* is 3000Rs and the other sum realized is $x-4000$ .. whole sum realized is $3000+x-4000$ $\therefore$ whole sum realized is $3000+x-4000=x-1000$ but this will meet the whole expenditure that will be paid to the creditors + percentage of bankruptcy *i. e.* $x-1000=\frac{13\frac{1}{2}\times x}{16}+\frac{x}{20}=\frac{7x}{8} \therefore 8x-8000=7x$

$$\therefore x=8000 \text{ Rs}.$$

37. Let x = no of deals be won $\therefore 20-x$ = no he lost, also $2x$ = money won and $3(20-x)$ = money lost ; $\therefore 2x-3(20-x)=5 \therefore x=13$.

38. Let x = no. of Rs Hemachrish had, then $x+12$Rs = Kristoe's amount $-$ 12Rs $\therefore$ Kristo's amount $=x+24$ $\therefore$ Kristo's amount after winning his own money + 6 Rs is $(x+24)+6=x+30$ also $x-6$ is the

amount which will remain in the hand of Hemadrish after losing 6 Rs. $\therefore$ by the question $2\frac{1}{5}(x-6)=x+30 \therefore x=36$.

39. Let x = no of coins in 4 anna pieces then $18-x$ = no of coins in the 2 anna pieces $\therefore 4as \times x+2(18-x)=58as \therefore x=11$.

40. Let x = no of seers of the inferior quality of sugar that is to be mixed with, then $(60+x)\times 6\frac{3}{5}=60\times 7+6x \therefore x=40$ *seers* = one maund.

41. Let x = no of men in the original regiment then $\sqrt{x}$ = no of men in front of the solid square but when 1500 men were taken away the no was $x-1500 \therefore \sqrt{(x-1500)}$ = no of men in front of the 2nd solid square $\therefore \sqrt{x}=\sqrt{(x-1500)}+10$ or $\sqrt{(x-1500)}=\sqrt{x}-10 \therefore x-1500=x+16-20\sqrt{x} \therefore 20\sqrt{x}=1600 \therefore \sqrt{x}=\frac{1600}{20}=80 \therefore x=6400$

42. Let x = 1st part then $80-x$ = 2nd part then by the question $\frac{x}{3}=(80-x\ 3 \therefore x=72$

43. Let x = no of persons then $3x$ = amount he distributed $\therefore 3x+7$ = money he had $\therefore 3x+7+12=3x+19$ = amount he would have if 3 as were given him and this amount if distributed among the x persons, each will have 4 pice $\therefore \frac{3x+19}{x}=4 \therefore x=19$.

44. Let x = no of miles Ram travelled in a day then the whole distance travelled by Ram in 36 days $=36x$, also the whole distance travelled by Sham $=36\times 15=540$, now this distance 540 miles = distance travelled by Ram + 2 distance Ram would have travelled in 9 days $=36x+18x$ whence $54x=540 \therefore x=10$.

45. Let x = portion of sugar in one seer of the mixture then $1-x$ = portion of channa, then the cost price of the mixture $=6x+8(1-x)=8-2x$ and $\therefore$ at the rate of profit of 20 per cent the cost per seer will be $(8-2x)+\frac{1}{5}(8-2x)=\frac{6}{5}(8-2x)$ and this by the question $=9$as $\therefore \frac{6}{5}(8-2x)=9 \therefore 48-12x=45 \therefore 12x=3 \therefore x=\frac{1}{4}$.

46. Let x = distance in miles of the Howrah Stn. from the place whence the person started, then $\frac{x}{3}$ n would be the no of hours for reaching the Stn. but he performs $\frac{x}{3}$ at the rate of 3 miles an hour and $\frac{2x}{3}$ at the rate of 4 miles an hour $\therefore \frac{x}{3\times 3}+\frac{1}{4}+\frac{2x}{3\times 4}=\frac{x}{3} \therefore x=4\frac{1}{2}$

47. Let $x =$ no of seers of the 1st sort then $80 - x =$ no of seers of the 2nd sort ∴ $8x + 10(80 - x) = 9\frac{1}{5}(80 - x)$ ∴ $x = 32$

48. Let $x =$ circumference of the island then ∴ Soodeen has travelled $\frac{x}{2} - 5$ miles when he meets Hurry ∴ it will take $\frac{\frac{x}{2} - 5}{5}$ hours and ∵ Hurry has travelled 5 miles more than half the way his distance would be $\frac{x}{2} + 5$ and at the rate of 7 miles it will take him $\frac{\frac{x}{2} + 5}{7} = \frac{x + 10}{14}$ Now since the interval between starting and meeting of Soodeen and Hurry is the same ∴ $\frac{\frac{x}{2} - 5}{5} = \frac{x + 10}{14}$ ∴ $x = 60$

49. Let the whole distance $CD = d$, let $x =$ no of hours

elapsed from starting to meeting in the point E then $x + 4 =$ no of hours that were required by Alexander to complete the journey and $x + 9 =$ no of hours that were required by Napoleon to do the same ∴ rate per hour of Alexander is $\frac{d}{x + 4}$ and rate per hour of Nepolean is $\frac{d}{x + 9}$ ∴ distance DE travelled by Napolean in x hours is $\frac{dx}{x + 9}$, and distance CE travelled by Alexander in x hours is $\frac{dx}{x + 4}$ ∴ $\frac{dx}{x + 4} + \frac{dx}{x + 9}$ = whole distance $CD = d$ ∴ $\frac{x}{x + 4} + \frac{x}{x + 9} = 1$ ∴ $x(x + 9) + x(x + 4) = (x + 4)(x + 9)$ or $2x^2 + 13x = x^2 + 13x + 36$ ∴ $x^2 = 36$ ∴ $x = 6$ ∴ time of A for the whole journey is $x + 4 = 10$ hours and time of B for the same is $x + 9 = 15$ hours.

50. Let $x =$ no of marbles C had at first then $50 - x =$ no D had, C wins ½ D's, he is in possession of $x + \frac{50 - x}{2}$, if he now loses 5 he will be in possession of $x + \frac{50 - x}{2} - 5$ and this by the question = D's original money $= 50 - x$ ∴ $x = 20$.

51. Let $x=$ no of seers the weight of the whole crew exclusive of the manjee, then the average weight of each crew $=\frac{x}{8}$ and the average weight of the crew including the manjee is $\frac{x+72}{9}$ $\therefore$ by the question $\frac{x}{8}=\frac{x+72}{9}+2$ $\therefore$ $x=720$ seers $=18$ mds $\therefore$ weight including the manjee is 19 mds 32 seers.

52. Let $x=$ no of miles travelled by Saroda at first then when he returned he walked $2x$ miles $\therefore$ whole distance travelled by Saroda is $2x+50$, but in the time Saroda takes to walk $2x+50$ miles, Borada walks 50 miles. But Saroda's no of hours $=\frac{2x+50}{8}$ and Borada's no of hours $=\frac{50}{6}$ $\therefore$ $\frac{2x+50}{8}=\frac{50}{6}$ $\therefore$ $x=8\frac{1}{3}$.

53. It was $5\frac{5}{11}$ min after 10 when the steamer passed by Dacca and when it arrived at Narangunge it was $27\frac{3}{11}$ min after 11, so that $81\frac{9}{11}$ mins were required for the passage $\therefore$ $60 : 81\frac{9}{11} :: 11 : x$ $\therefore$ $x=15$.

51. Let $x=$ no of oranges he had at first then when he sells $\frac{1}{5}$ of the no, there remains $\frac{4x}{5}$ and from this 2 more are sold, *i. e.* the remainder $=\frac{4x}{5}-2=\frac{4x-10}{5}$; again when he sells $\frac{1}{2}$ this amount to the 2*nd* person there remains with him $\frac{1}{2}$ of $\frac{4x-10}{5}=\frac{2x-5}{5}$, from this one more is sold $\therefore$ remainder $=\frac{2x-5}{5}-1=\frac{2x-10}{5}$, again when he sells $\frac{2}{3}$ of this amount to the 3rd person there remains $\frac{1}{3}$ of the amount *i. e.*, $\frac{1}{3}$ of $\frac{2x-10}{5}=\frac{2x-10}{15}$, from this 3 more were sold $\therefore$ the remainder was $\frac{2x-10}{15}-3$ *i. e.* $\frac{2x-55}{15}$ and by the question this remainder $=3$ $\therefore$ $\frac{2x-55}{15}=3$ $\therefore$ $2x-55=45$ $\therefore$ $2x=100$ $\therefore$ $x=50$.

55. Let $x=$ no of persons that were in the ship and $1=$ value of each man's provision $\therefore$ whole provision for x persons is $30x$; now in

20 days $20x$ of the provisions were expended and in the following 2 days $2(x-6)$ provisions were expended $\therefore 20x+2(x-6)+\frac{5}{6}(x+12)\times 8=30x \therefore x=51$.

56. Let $x=$ no of seers the weight of the body, then weight of the head $=$ wt of tail $+\frac{1}{2}$ wt of the body $=9+\frac{1}{2}x$, by the question wt of the body $=$ wt of head $+$ wt of tail *ie*., $x=9+\frac{x}{2}+9=18+\frac{x}{2} \therefore \frac{x}{2}=18$ $\therefore x=36$ seers $\therefore$ wt of head $=9+18=27$ seers $\therefore$ whole wt $=9+36+27=72$ seers.

57. Let $x=$ no of men in the ship then $60x=$ no of lbs of biscuits that were in the ship, now $20x$ is expended in 20 days and when 5 men were washed away there remained $x-5$ and they will have to sustain for 64 days $\therefore 20x+\frac{5}{7}\times 64(x-5)=60x \therefore x=40$.

58, Let $x=$ no of men in a side of the solid square, then x^2+6 would be the total no of men also, if $x+1$ be the no of men in one side $\therefore (x+1)^2-19$ will be the total no of men $\therefore (x+1)^2-19=x^2+6$ $\therefore x=12 \therefore x^2+6=144+6=150$.

59. Let $x=$ no of men in front of the hollow square then $x-16$ would have been the no of persons in the hollow portion $\therefore x^2-(x-16)^2=1024 \therefore x=40$.

60. Let $x=$ no of men in front of the solid square at first made, then x^2+39 would be the no of men, also if the side of the square be increased by 1 man the no will be $(x+1)^2$ but then there would be a deficiency of 50 men, hence $(x+1)^2-50=$ total no $\therefore x^2+39=(x+1)^2-50 \therefore x^2+39=x^2+2x+1-50 \therefore x=44 \therefore x^2+39=1975$.

61. Let $x=$ distance in miles of Bally from Howrah then $\because$ he had to wait 6 minutes for a train which was 3 miles off, it is evident that the rate of the train is 30 miles per hour. Now whole time occupied for his journey from the seminary to Bally and back $=$ time of journey from the seminary to Howrah $+$ time of journey by foot from Howrah to Bally $+$ stay at Bally for 6 min $+$ time of his journey by the train $+$ time of travelling from Howrah to the Seminary $=\frac{2}{7}+\frac{2x}{7}+\frac{1}{10}+\frac{x}{30}+\frac{2}{7}$ and this time by the question $=2$ *hrs*. $35\frac{1}{7}$ *m*. $=\frac{543}{210}$ *hrs*. $\therefore \frac{4}{7}+\frac{2x}{7}+\frac{1}{10}+\frac{x}{30}=\frac{543}{210} \therefore x=6$ miles.

62. $S = 16.1 \times t^2 = 16.1 \times (1\frac{1}{2})^2 = 16.1 \times \frac{9}{4} = 36.225$ ft.

63. Let x = periphery of the forewheel in ft. then $x+3$ = periphery of the hind wheel in ft $\therefore$ forewheel will make $\frac{180}{x}$ turns and the hind wheel $\frac{180}{x+3}$ turns and by the question $\frac{180}{x} = \frac{180}{x+3} + 3$ $\therefore$ $180x + 540 = 180x + 3x^2 + 9x$ $\therefore$ $x^2 + 3x + \frac{9}{4} = \frac{9}{4} + 180 = \frac{729}{4}$ $\therefore$ $x + \frac{3}{2} = \frac{27}{2}$ $\therefore$ $x = 12$.

64. Let x = no pence A spent per day $\therefore$ $x+6$ = no of pence B spent per day $\therefore$ saving of A per day $= 60 - x$ and saving of B per day $= 60 - (x+6) = 54 - x$ and by the question, $50(60 - x) = 2 \times 50(54 - x) + 2x$, whence $x = 50$.

65. Let $3x$ = no of leaps the greyhound must take $\therefore$ $4x$ = number the hare takes in the same time; $\therefore$ $4x + 50$ = whole no she takes and $2 : 3 :: 3x : 4x + 50$ $\therefore$ $x = 100$

66. Let x = 1*st* part then $x + 5$ = 2*nd* part -4 $\therefore$ 2*nd* part $x + 9$; also $x + 5$ = 3*rd* part $\times 3$ $\therefore$ 3*rd* part $= \frac{x+5}{3}$ and $x + 5 = \frac{4th\ \text{part}}{2}$ $\therefore$ 4*th* part $= 2(x+5)$ $\therefore$ sum of all the parts $= 116$ or $x + x + 9 + \frac{x+5}{3} + 2(x+5 = 116$ $\therefore$ $x = 22$.

67. Let x = no of Rs each had at first, then $x + 40$ = sum Kadar had after the 1*st* year, and $x - 40$ = sum Baykonto had, also $\frac{2}{3}(x+40)$ = sum Kadar had after the 2*nd* year and $x - 40 + \frac{2}{3}(x+40) - 40$ = sum Baykonto had $\therefore$ $\frac{4}{3}(x+40) = x - 40 + \frac{2}{3}(x+40) - 40$ $\therefore$ $x = 320$.

68. Let $2x$ = no of cards cut off by Sham, then $52 - 2x$ = no he left and x = no Ram left $\therefore$ $52 - x$ = no he cut off whence $52 - x = 7(52 - 2x)$ $\therefore$ $x = 24$.

69. Let x = no of Rs. he began with, then $3x$ = sum he had after winning $2x$ and $3x - 16$ = sum remaining after the next loss. Now since he lost $\frac{4}{5}$ of this, $\frac{3x-16}{5}$ = sum remaining $\therefore$ $\frac{3x-16}{5} + x = 80$ $\therefore$ $x = 52$.

70. Let x = no then $x - 70 : x + 150 :: 1 : 3$ $\therefore$ $x - 70 : 220 :: 1 : 2$ whence $x = 180$.

71. Let x = distance in miles the thief will run, then the distance

the constable had to run is $x+\frac{1}{8}=\frac{8x+1}{8}$ the time the thief took to walk the distance is $\frac{x}{4\frac{1}{4}}$ and the time the constable took is $\frac{8x+1}{8\times 5}$; Now since their times from starting to meeting must be equal we have

$$\frac{x}{4\frac{1}{4}}=\frac{8x+1}{8\times 5}\therefore\ x=9\tfrac{1}{2}.$$

72. Proceed as in ex. 66

73. Let $x=$ no of maunds required then when the price is at 2 Rs. a md. the whole rent $=35+2x$ and when the price is 3 Rs. a md the whole rent $=35+3x$ $\therefore$ $35+3x : 35+2x :: 100 : 90$ or

$$\frac{36+3x}{35+2x}=\frac{100}{90}=\frac{10}{9}\ \therefore\ x=5 \text{ mds.}$$

74 Finding the difference of the two interests we have $202\frac{1}{2}$ Rs. due to $1\frac{1}{2}$ per cent interest $\therefore$ for 405 Rs. interest, the rate ought to be 3 per cent $\therefore$ $3 : 405 :: 100 : x$ $\therefore$ $x=13500$ Rs.

75. Let us first ascertain the rate of the train ; $\because$ it gains 88 yds over Naran in every 10 secs, it gains 18 miles in an hour by the following proportion $10 : 60\times 60 :: 18 : x$; $\therefore$ the train runs 18 miles more than Naran in an hour whose rate $=4$ miles $\therefore$ the rate of the train $=22$ miles ; again when the train overtakes Ram and passes him in 9 secs, we see that Ram lags behind the train at the rate of 20 miles in an hour, but the rate of the train is 22 miles an hour $\therefore$ Ram's rate $=2$ miles ; lastly we must ascertain at what distance was Ram when the train overtook Naran ; we know that in each hour the separation between Ram and the train is 20 miles $\therefore$ distance due to 20 mins or $\frac{1}{3}$ of an hour is $\frac{20}{3}$ miles ; now Naran gains over Ram 2 miles in each hour $\therefore$ $2 : \frac{20}{3} :: 1 : x$ $\therefore$ $x=3\frac{1}{3}$ hours.

76. Let $x=$ no of eggs she had, then $x+x+\frac{x}{2}+1\frac{1}{2}=104$ $\therefore$ $x=41$.

77. Let $x=$ no of oranges he buys at 2 a penny, then $4x=$ no he buys at 5*d* a dozen and $5x=$ no he buys 8*d* a 100 $\therefore$ cost price of all the oranges $=\frac{x}{2}+\frac{4x\times 5}{12}+\frac{5x\times 8}{20}=\frac{x}{2}+\frac{5x}{3}+2x=\frac{25x}{6}$ and the selling price

of the $10x$ no of oranges $=\frac{44\times 10x}{100}=\frac{22x}{5}\ \therefore\ \frac{22x}{5}-\frac{25x}{6}=42\ \therefore\ x=180$ $\therefore$ no of oranges $=1800$.

78. Let $x=$ distance of Ootterparah from Calcutta, then $\frac{x}{3}=$ no of hours it took him to walk from Calcutta to Ootterparah, and $\frac{x}{2}=$ no of hours it took him to come back to Calcutta $\therefore\ \frac{x}{3}+\frac{x}{2}=5\ \therefore\ x=6$.

79. Let $x=$ distance to the railway stn., then $\frac{x}{3}=$ no of hours it took him to reach the Station and during his return he walked part of the way in 1 hr 5 mins at the rate of 3 miles *i. e.*, he walked $3\frac{1}{4}$ miles $\therefore$ portion of the road he ran $=x-3\frac{1}{4}$ and by the question $\frac{x}{3}$ $+\frac{x-3\frac{1}{4}}{8\frac{1}{3}}+1\frac{1}{12}=2\frac{11}{15}\ \therefore\ x=4\frac{1}{2}$.

SIMPLE EQUATIONS OF TWO UNKNOWN QUANTITIES.

Ex. 22. 1. $4x+5y=23$

$$4x+8y=32 \text{ (multiplying 2nd equ by 4)}$$

$$\therefore\ 3y=9\ \therefore\ y=3\ \therefore\ x+2y=x+2\times 3=8 \text{ or } x=2$$

2. Multiply (1) by 2 and (2) by 3, we have $6x+4y=60$

$$9x+9y=75$$

$$5y=15\ \therefore y=3$$

$$\therefore\ 2x+3y=2x+9=25\ \therefore\ 2x=16\ \therefore\ x=8$$

13. Multiply (1) by m and (2) by a, we have

$$amx+bmy=cm$$

$$amx+apy=ad$$

Subtract $\quad (bm-ap)y=cm-ad \qquad y=\frac{cm-ad}{bm-ap}$,

again multiply (1) by p and the 2nd by b we have

$$apx+bpy=cp$$

$$bmx+bpy=bd$$

Subtract $\quad (ap-bm)x=cp-bd\ \therefore\ x=\frac{cp-bd}{ap-bm}$

14 $7x + 2y = 40$ .. (1) and $5xy = 5xy - 6x + 5y - 6$ or $5y - 6x = 6$ ... (2) multiply (1) by 5 and the (2) by 2, we have

$$35x + 10y = 200$$
$$10y - 12x = 12$$

Subtract $47x = 188 \therefore x = \frac{188}{47} = 4$ from (2) $y = \frac{6 + 6x}{5} = \frac{6 + 24}{5} = \frac{30}{5} =$

39. $y^{\frac{1}{2}} - (a - x)^{\frac{1}{2}} = (y - x)^{\frac{1}{2}}$ ... ... (1)

$2(y - x)^{\frac{1}{2}} = 3(a - x)^{\frac{1}{2}}$ ... ... (2)

from (1) we deduce $y^{\frac{1}{2}} - (a - x)^{\frac{1}{2}} = \frac{3}{2}(a - x)^{\frac{1}{2}}$

$\therefore 2y^{\frac{1}{2}} - 2(a - x)^{\frac{1}{2}} = 3(a - x)^{\frac{1}{2}} \therefore 2y^{\frac{1}{2}} = 5(a - x)^{\frac{1}{2}} \therefore 4y = 25a - 25x$

$\therefore 25x + 4y = 25a$. Again $3y^{\frac{1}{2}} - 3(a - x)^{\frac{1}{2}} = 3(y - x)^{\frac{1}{2}}$ and $3(a - x)^{\frac{1}{2}} = 2(y - x)^{\frac{1}{2}} \therefore 3y^{\frac{1}{2}} = 5(y - x)^{\frac{1}{2}} \therefore 9y = 25y - 25x \therefore 25x - 16y = 0$

We have $25x + 4y = 25a \therefore 20y = 25a$ (by subtraction) $\therefore y = \frac{5a}{4}$

Again $25x - 16y = 0 \therefore 25x = 16y \therefore 25x = 20a \therefore x = \frac{4a}{5}$

40. $\therefore 20x^2 - 40xy + 5x + 12xy - 24y^2 + 3y + 12x - 24y + 3 = 24x^2 - 24y^2 - 28xy + 4x$ multiplying both the equations by $2(5x + 3y + 3)$

$\therefore -28xy + 17x - 21y + 3 = -28xy + 4x \therefore 13x - 21y = -3$

Again $2\sqrt{(6 + x)} = 3\sqrt{(6 - y)} \therefore 4(6 + x) = 9(6 - y) \therefore 24 + 4x = 54 - 9y$

$\therefore 4x + 9y = 30$. But $13x - 21y = -3 \therefore 52x + 117y = 390$

and $52x - 84y = -12 \therefore 201y = 402 \therefore y = 2$

But $4x + 9y = 30 \therefore 4x + 18 = 30 \therefore 4x = 12 \therefore x = 3$.

41. $y = (a^{0})^n = 1^n = 1 \therefore x^{3mn} = c^m \therefore x^{3n} = c \therefore x = c^{\frac{1}{3n}}$

42. $\frac{x^{2p} - y^{2p}}{x^p + y^p} = \frac{m^2}{n}$ or $x^p - y^p = \frac{m^2}{n}$ ... (1) and $x^p + y^p = n$

$\therefore 2x^p = \frac{m^2}{n} + n = \frac{m^2 + n^2}{n} \therefore x^p = \frac{m^2 + n^2}{2n}$

By subtraction $2y^p = n - \frac{m^2}{n} = \frac{n^2 - m^2}{n} \therefore y^p = \frac{n^2 - m^2}{n}$

43. Squaring, $(x + y)^2 = 25$ or $x^2 + 2xy + y^2 = 25$ subtract $4xy = 24$ then $x^2 - 2xy + y^2 = 1 \therefore x - y = 1$ also $x + y = 5$ adding $2x = 6$

$\therefore x = 3$, subtracting $2y = 4 \therefore y = 2$

44. From (1) $(a^2 - b^2)3x + (a^2 - b^2)5y = 8a^2b - 2ab^2$ and from

$$(2)\ (a^2 - b^2)3x + 3(a + b + c)by = 5a^2b + 6ab^2 + \frac{3ab^2c}{a + b}$$

$$\therefore (5a^2 - 8b^2 - 3ab - 3bc)y = 5a^2b - 8ab^2 - \frac{3ab^2c}{a + b}$$

$$= ab\left\{5a - 8b - \frac{3bc}{a + b}\right\} = \frac{ab}{a + b}\left\{5a^2 - 8b^2 - 3ab - 3bc\right\} \therefore y = \frac{ab}{a + b}$$

Substituting the value of y in (1) we have $(a^2 - b^2)3x + (a - b)5ab = 8a^2b - 2ab^2$ or $(a^2 - b^2)3x = 8a^2b - 2ab^2 - 5a^2b + 5ab^2 = 3a^2b + 3ab^2$

$$= 3ab(a + b) \therefore x = \frac{ab}{a - b}$$

45. Squaring both sides, $x - 3 = \frac{25}{16}(x + y)$ or $16x - 48 = 25x + 25y$ but $25y = 5x \therefore 16x - 48 = 25x + 5x = 30x \therefore 14x = -46 \therefore x = 3\frac{3}{7}$

$$\therefore y = \frac{x}{5} = -\frac{48}{14 \times 5} = -\frac{24}{35}$$

46. From the 2nd equ $x^2 - y^2 = \frac{4ab}{a^2 - b^2}$ (1) substitute in the 1st then we have $a(x^2 + y^2) - b\frac{4ab}{a^2 - b^2} = 2a$ whence $x^2 + y^2 = \frac{2(a^2 + b^2)}{a^2 - b^2}$..(2); from (1) and (2) $2x^2 = \frac{2(a + b)^2}{a^2 - b^2}$ or $x^2 = \frac{a + b}{a - b} \therefore x = \sqrt{\frac{a + b}{a - b}}$ and the value of y may be found in like manner

47. From the 2nd equ $2\sqrt{(y - x)} = 3\sqrt{(20 - x)} \therefore \sqrt{(y - x)} = \frac{3}{2}\sqrt{(20 - x)}$, substitute this in the 1st then $\sqrt{y} - \sqrt{(20 - x)} = \frac{3}{2}\sqrt{(20 - x)}$

$\therefore \sqrt{y} = \frac{5}{2}\sqrt{(20 - x)} \therefore \sqrt{(20 - x)} = \frac{2}{5}\sqrt{y}$ ... (1) $\therefore 20 - x = \frac{4}{25}y$

$\therefore 500 - 25x = 4y$ .. (2) $\therefore$ the 1st equ is equivalent to $\sqrt{y} - \frac{2}{5}\sqrt{y} = \sqrt{(y - x)}$ or $\frac{3}{5}\sqrt{y} = \sqrt{(y - x)}$, squaring $\frac{9}{25}y = y - x$ or $25x = 16y$...(3) : from (2) and (3) $x = 16$ and $y = 25$.

48. Add the 2 equs then $x^2+2xy+y^2=81$ or $x+y=9$; from (1) $x(x+y)=45$ or $x\times 9=45$ $\therefore$ $x=5$; from (2) $y(x+y)=36$ or $y=4$.

49 Divide (2) by (1) then $x-y=2$, add this to (1) then $2x=6$ $\therefore$ $x=3$; again subtract, then $2y=4$ $\therefore$ $y=2$.

50. From (1) $x+y=5$ add this to (2) then $2x=6$ $\therefore$ $x=3$; again subtract (2) from the 1st result then $2y=4$ $\therefore y=2$.

51 From (2) $x^2+2xy+y^2=25$ (3) or $13+2xy=25$ $\therefore$ $2xy=12$ $\therefore$ $4xy=24$, subtract this from (3) then $x^2-2xy+y^2=1$ $\therefore$ $x-y=1$ add this to (2) then $2x=6$ $\therefore$ $x=3$; by subtraction y can be found.

52. $x^2-2xy+y^2=1$ $\therefore$ $x^2+y^2=1+2xy$ or $5=1+2xy$ $\therefore$ $2xy=4$ add this to (1) then $x^2+2xy+y^2=9$ or $x+y=3$ ... (1) add this to (2) then $2x=4$ $\therefore$ $x=2$. 53 Subtract (2) from (1) then $x^2-2xy+y^2=4$ $\therefore$ $x-y=2$, from (1) $x(x-y)=8$ or $x\times 2=8$ $\therefore$ $x=4$.

54. Cubing (2) we have $x^3+y^3+3xy(x+y)=125$ or $35+3xy\times 5=125$ $\therefore$ $15xy=90$ $\therefore$ $xy=6$ $\therefore$ $4xy=24$ ..(3) also $x^2+2xy+y^2=25$ subtract (3) from this then $x^2-2xy+y^2=1$ $\therefore$ $x-y=-1$ and $x+y=5$ $\therefore$ $2x=4$ $\therefore$ $x=2$. 55. Proceed as in ex 48.

56 $(x^2-xy+y^2)(x^2+y^2)=15$,...(1) and $(x^2-xy+y^2)(x^2+xy+y^2)=21$. (2); divide (2) by (1) then $\dfrac{x^2+xy+y^2}{x^2+y^2}=\frac{21}{15}=\frac{7}{5}$ or $1+\dfrac{xy}{x^2+y^2}=\frac{7}{5}$ $\therefore$ $\dfrac{xy}{x^2+y^2}=\frac{7}{5}-1=\frac{2}{5}$ $\therefore$ $\dfrac{2xy}{x^2+y^2}=\frac{4}{5}$ $\therefore$ $\dfrac{x^2+y^2}{2xy}=\frac{5}{4}$ $\therefore$ $\dfrac{x^2+2xy+y^2}{x^2-2xy+y^2}=\dfrac{5+4}{5-4}=\frac{9}{1}$ or $\dfrac{x+y}{x-y}=\frac{3}{1}$ $\therefore$ $\dfrac{x}{y}=\dfrac{3+1}{3-1}=\frac{4}{2}=2$ $\therefore$ $x=2y$ . . (3); substitute the value of x in the 1st equ, then $(4y^2-2y^2+y^2)(4y^2+y^2)=15$ or $3y^2\times 5y^2=15$ $\therefore$ $y^4=1$ $\therefore$ $y=1$ and from (3), $x=2y=2$.

57. From the 1st equ $5(w+z)^{\frac{1}{2}}z+5(w+z)^{\frac{1}{2}}w=\frac{27}{4}wz$...(a) and from the 2nd equ $3(w-z)^{\frac{1}{2}}w-3(w-z)^{\frac{1}{2}}z=\frac{3}{20}wz$...(b), from (a) $5(w+z)^{\frac{3}{2}}=\frac{27}{4}wz$ (c) and from (b) $3(w-z)^{\frac{3}{2}}=\frac{3}{20}wz$ ..(d), divide (c) by (d) then

$\frac{5}{3}\left(\frac{w+z}{w-z}\right)^{\frac{3}{2}}=\frac{27}{4}\times\frac{20}{3}=45 \therefore \left(\frac{w+z}{w+z}\right)^{\frac{3}{2}}=45\times\frac{3}{5}=27 \therefore \frac{w+z}{w-z}=27^{\frac{2}{3}}=9$

$\therefore \frac{w}{z}=\frac{9+1}{9-1}=\frac{10}{8}=\frac{5}{4} \therefore w=\frac{5z}{4}$; substitute the value of w in eqn (c)

then $5\left(\frac{5z}{4}+z\right)^{\frac{3}{2}}=\frac{27}{4}\times\frac{5z}{4}\times z$ or $\left(\frac{9z}{4}\right)^{\frac{3}{2}}=\frac{27z^2}{16} \therefore \left(\frac{9z}{4}\right)^3=\frac{27^2\times z^4}{4^4}$

$\therefore 9^3=\frac{27^2\times z}{4} \quad \therefore z=\frac{4\times 9^3}{27^2}=4 \therefore w=\frac{5z}{4}=\frac{5\times 4}{4}=5.$

SIMPLE EQUATIONS OF MORE THAN TWO UNKNOWNS.

Ex. 23. 1. Multiply the *1st* eqn. by 2 and the *2nd* by 5 and substract, we have then

$$\begin{array}{l} 10x+4y+2z=24 \\ 10x+15y+20z=100 \\ \hline -11y-18z=-76 \text{ or } 11y+18z=76\ldots\ldots(1) \end{array}$$

In the same manner multiply the *2nd* equation by 3 and the *3rd* by 2 we have

$$\begin{array}{l} 6x+9y+12z=60 \\ 6x+8y+10z=52 \\ \hline y+2z=8\ldots\ldots\ldots\ldots\ldots\ldots(2) \end{array}$$

Multiply (2) by 11 and subtract from the *1st* we have then

$-4z=-12 \therefore z=3$ and from (2) $y+6=8 \therefore y=2$ and from the *1st* eqn $5x+4+3=12 \therefore 5x=5 \therefore x=1.$

12. Adding the 3 eqns together we get

$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{3}{4}+\frac{5}{12}+\frac{2}{3}=\frac{11}{6} \therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{12}$ or from the 1st

eqn $\frac{3}{4}+\frac{1}{z}=\frac{11}{12} \therefore \frac{1}{z}=\frac{11}{12}-\frac{3}{4}=\frac{2}{12}=\frac{1}{6} \therefore z=6$ and so of the rest

13. $x+m^2=y+n^2=z+p^2$ ∴ each of them is $\frac{1}{3}$ of their sum ∴ $x+m^2=\frac{1}{3}(x+y+z+m^2+n^2+p^2)=\frac{1}{3}(m^2+n^2+p^2+m^2+n^2+p^2)$ (from the 1st eqn) $=\frac{2}{3}(m^2+n^2+p^2)$ ∴ $x=\frac{2}{3}(m^2+n^2+p^2)-m^2$ and so of the rest.

14. Multiply the 1st eqn by 3 and subtract the result from the 3rd eqn.

15. Adding the 3 eqns together we get $x+y+z=17$...(1) from (1) subtract the 1st eqn and the result is $2z=12$ ∴ $z=6$; in the same manner subtracting the 2*nd*, 3*rd* eqs from (1) we get y and x.

18. Multiply the 3 eqns together then $x^2y^2z^2=36$ ∴ $xyz=6$,
From (1) $2z=6$ ∴ $z=3$.

19 Multiply the 4 eqns together then $x^3y^3z^3w^3=2^3\times3^3$ ∴ $xyzw=6$ ∴ from (1) $2w=6$ ∴ $w=3$ and so of the rest.

20. Multiply the 3 equations then $x^4y^4z^4=6\times12\times18=6^4$ ∴ $xyz=6$ ∴ $x^2y^2z^2=36$, divide this result by the 1*st* eqn, we have $yz=6$ ∴ from the 1*st* eqn $x^2=\frac{6}{yz}=\frac{6}{6}=1$ ∴ $x=1$.

21. Multiply the 3 eqns then $x^3y^3z^3=36\times32\times12=24^3$ ∴ $xyz=24$ ∴ $x^2y^2z^2=24\times24$; divide this by the 1*st* eqn then $x^2z=\frac{24\times24}{36}=16$. and $z^2x\times x^2z=32\times16$ or $x^3z^3=32\times16=2^9$ ∴ $xz=2^3=8$ ∴ from the 2*nd* eqn $z=\frac{32}{xz}=\frac{32}{8}=4$.

22. Multiply the 1*st* and 2*nd* eqns then $x^{-3}z^3=\frac{3}{2}\times18=27$ ∴ $x^{-1}z=3$...(4); from the 2*nd* eqn $yz=\frac{18}{x^{-1}z}=\frac{18}{3}=6$ ∴ $y^2z^2=36$ and ∴ from the 3*rd* eqn $xz=\frac{108}{y^2z^2}=3$, ∵ $xyz^2=18$ ∴ $yz^2=\frac{18}{x}$ also from the 2*nd* eqn $yz^2=\frac{18}{x^{-1}}=18x$ ∴ $\frac{18}{x}=18x$ ∴ $x^2=1$ ∴ $x=1$ but $xz=3$ ∴ $z=3$ and $yz=6$ ∴ $y=2$.

23 Multiply the 3 eqns we have then $(x+y)^2(y+z)^2(x+z)^2 = 35\times 42\times 30 = 5^2.\,6^2\,7^2$ $\therefore$ $(x+y)\times 42 = 5.\,6\,7$ $\therefore$ $x+y=5$. (1) ; similarly $y+z=7$...(2) and $x+z=6$ (3) ; adding (1), (2) and (3) we have $2(x+y+z)=18$ $\therefore$ $x+y+z=9$ $\therefore$ from (1) $z=4$, from (2) $x=2$ and from (3) $y=3$.

24 $xy=1$, $yz=24^2.\,x^2$, $xz=16y^2$, multiply together, then $x^2y^2z^2 = 4^2.\,24^2\,x^2y^2$ $\therefore$ $xyz=4\,24.\,xy$ $\therefore$ $z=4\times 24=96$; from the 1st eqn $x=\frac{1}{y}$ and from the 3rd $x=\frac{16y^2}{z}=\frac{16y^2}{96}=\frac{y^2}{6}$ $\therefore$ $\frac{1}{y}=\frac{y^2}{6}$ $y^3=6$ $\therefore$ $y=\sqrt[3]{6}$

$\therefore$ $x=\frac{1}{y}=\frac{1}{\sqrt[3]{6}}$

25. Multiply the 1st eqn by 7 and the 3rd by 3 and subtract the 2nd result from the 1st we have then $28y-6z=94$ or $14y-3z=47$. (1) multiply the 2nd eqn by 7 and subtract the result from (1) we have then $4z=12$ $\therefore$ $x=3$; from the 2nd eqn $2y-3=5$ $\therefore$ $y=4$ and from the 1st eqn $3x+16=19$ $\therefore$ $3x=3$ $\therefore$ $x=1$

27 From the 1st eqn $\frac{x+y}{xy}=\frac{9}{20}$ or $\frac{1}{x}+\frac{1}{y}=\frac{9}{20}$; from the 2nd eqn $\frac{x+z}{xz}=\frac{7}{12}$ or $\frac{1}{z}+\frac{1}{x}=\frac{7}{12}$; from the 3rd eqn $\frac{y+z}{yz}=\frac{9}{15}$, then proceed as in ex 12.

29 Subtract the 3rd eqn from the 1st and we get $x=12$; from the 2nd eqn by substituting the value of x we get $2w+y+z=9$...(1) , multiply the 4th equation by 2 and subtract the result from (1) we thus get $3y+z=13$. (2). Subtracting the 3rd eqn from (1) we get $w=1$; substituting this value of w in (1) we get $y+z=7$ (3) , subtract (3) from (2) then $2y=6$ $\therefore$ $y=3$ substituting this value of y in (3) $z=4$.

30. Squaring the 1st eqn we have $x^2+y^2+z^2+2(xy+xz+yz) = 196$ or $14+2(xy+xz+yz)=196$ $\therefore 2(xy+y^2+yz)=196-14=182$ or $2y(x+y+z)=182$ or $2y\times 14=182$ $\therefore$ $y=4$ $\therefore$ from 1st eqn $x+z=10$

$\therefore\ x^2+2xz+z^2=100$ and $4xz=4y^2=64\ \therefore x^2-2xz+z^2=36\ \therefore x-z=6$ and $x+z=10\ \therefore\ 2x=16\ \therefore\ x=8$.

31. From the 1*st* eqn $x^3+y^3+z^3-3xyz=0\ \therefore\ (x+y+z)(x^2+y^2+z^2-xy-xz-yz)=0\ \therefore\ x+y+z=0\ldots(1)$. Now sum of $3m-x+z$, $3-y+x$ and $3p-z+y$ is equal to 3 times any one of them $\therefore$ they are all equal to one another; $\therefore\ (3m-x+z)+(3n-y+x)+(3p-z+y)=3(3m-x+z)$ or $=3(3n-y+x)$ or $=3(3p-z+y)=3(3m-x+z)$ or $m+n+p=3m-x+z\ \therefore\ z-x=n+p-2m\ \ldots(2)$ and $m+n+p=(3n-y+x)\ \therefore x-y=m+p-2n\ldots(3)$ also $m+n+p=3p-z+y$ $\therefore\ y-z=m+n-2p$. (4) Subtracting (4) from (2) we have $2z-(x+y)=3(p-m)$ but $x+y=-z$ from (1); $\therefore 3z=3(p-m)\ \therefore\ z=p-m$. Again subtract (3) from (4) we have then $2y-(x+z)=3(n-p$ but from (1) $x+z=-y\ \therefore 3y=3(n-p)$, $\therefore y=n-p$, again subtract (2) from (3) and thus x will be found.

32. Squaring the 3rd eqn we get $x^2+y^2+z^2+2(xy-xz-yz)=25$, substituting value of $x^2+y^2+z^2=29$ in the 1st eqn we have $2(xy-xz-yz)=-4\ \therefore\ xy-xz-yz=-2$; adding this to the 2*nd* eqn we get $2xy=24\ \therefore\ xy=12$; substitute in the 2nd and we get $xz+yz=14$ or $(x+y)z=14\ \therefore\ x+y=\frac{14}{z}$, but from the 3rd eqn $x+y=5+z\ \therefore\ \frac{14}{z}=5+z$ whence $z=2\ \therefore\ x+y=7\ \therefore\ x^2+2xy+y^2=49$ and $4xy=48\ \therefore$ $x^2-2xy+y^2=1\ \therefore\ x-y=1$ and $x+y=7\ \therefore\ x=4,\ y=3$.

33. $\frac{y^2+z^2}{xyz}=\frac{25}{24}$ or $\frac{y}{xy}+\frac{z}{xy}=\frac{25}{24}$. (1) $\frac{z^2+x^2}{xyz}=\frac{5}{6}$ or $\frac{z}{xy}+\frac{x}{yx}=\frac{5}{6}\ldots$ (2) $\frac{x^2+y^2}{xyz}=\frac{13}{24}\ \therefore\ \frac{x}{yz}+\frac{y}{xz}=\frac{13}{24}\ldots(3)$; adding (1), (2) and (3) we have $2\left(\frac{x}{yz}+\frac{y}{xz}+\frac{z}{xy}\right)=\frac{25}{24}+\frac{5}{6}+\frac{13}{24}=\frac{29}{12}\ \therefore\ \frac{x}{yz}+\frac{y}{xz}+\frac{z}{xy}=\frac{29}{24}$; from (1) $\frac{x}{yz}=\frac{29}{24}-\frac{25}{24}=\frac{4}{24}=\frac{1}{6}$; from (2) $\frac{y}{xz}=\frac{3}{8}$; and from (3) $\frac{z}{xy}=\frac{2}{3}\ \therefore\ \frac{y}{xz}\times\frac{z}{xy}$

$=\frac{3}{8}\times\frac{2}{3}=\frac{1}{4}$ or $\frac{1}{x^2}=\frac{1}{4}$ $\therefore$ $x=2$ again $\frac{x}{yz}\times\frac{y}{xz}=\frac{1}{6}\times\frac{3}{8}=\frac{1}{16}$

$\therefore$ $\frac{1}{z^2}=\frac{1}{16}$ $\therefore$ $z=4$.

34. Multiplying the 1st eqn by $n+p$ we have $(n+p)x+(n+p)y+(n+p)z=0$ $\therefore$ $(m-p)x+(m-n)y=0$; again multiplying the 1st eqn by np we have $npx+npy+npz=0$ but $mnx+mpy+npz=1$ $\therefore$ $(mn-np)x+(mp-np)y=1$ or $n(m-p)x+p(m-n)y=1$ again $p(m-p)x+p(m-n)y=0$ $\therefore$ $x\{n(m-p)-p(m-p\}=1$ $\therefore x=\frac{1}{n(m-p)-p(m-p)}=\frac{1}{(n-p)(m-p)}$ Similarly y and z can be found

35 From the 1st and 2nd eqns $(m^2-n^2)y-(m^4-n^4)z+m^6-n^6=0$ or $y-(m^2+n^2)z+m^4+m^2n^2+n^4=0$; from the 2nd and 3rd $y-(n^2+p^2)z+n^4+n^2p^2+p^4=0$ $\therefore$ $(m^2-p^2)z-(m^4-m^2n^2-n^2p^2-p^4)=0$ $\therefore$ $(m^2-p^2)z=m^4-p^4+(m^2-p^2)n^2$ $\therefore$ $z=m^2+n^2+p^2$ also $y=(m^2+n^2)z-(m^4+m^2n^2+n^4)=(m^2+n^2)(m^2+n^2+p^2)-m^4-m^2n^2-n^4=m^2n^2+m^2p^2+n^2p^2$.

36. $\frac{x+y}{xy}=\frac{1}{a}$ or $\frac{1}{y}+\frac{1}{x}=\frac{1}{a}$; $\frac{x+z}{xz}=\frac{1}{b}$ or $\frac{1}{z}+\frac{1}{x}=\frac{1}{b}$; $\frac{y+z}{yz}=\frac{1}{c}$ or $\frac{1}{z}+\frac{1}{y}=\frac{1}{c}$ then proceed as in ex 12.

37. Add the 3 eqns then $(x+y+z)^2=18+27+36=81$ $\therefore x+y+z=9$, from the 1st eqn $x\times 9=18$ $\therefore$ $x=2$; from the 2nd $y\times 9=27$ $\therefore$ $y=3$ and in like manner $z=4$.

38. Divide the 1st eqn by the 2nd and by the 3rd then $\frac{x}{z}=\frac{p}{m}$, $\frac{y}{z}=\frac{p}{n}$ or $x=\frac{pz}{m}$ and $y=\frac{pz}{n}$; substitute the values of x and y in the 1st, eqn $\therefore$ $\frac{pz}{m}\times\frac{pz}{n}=p\left(\frac{pz}{m}+\frac{pz}{n}+z\right)$ or $\frac{p^2z^2}{mn}=pz\left(\frac{p}{m}+\frac{p}{n}+1\right)=pz\left(\frac{np+mp+mn}{mn}\right)$ $\therefore$ $pz=mn+mp$ $\therefore z=\frac{mn+mp+np}{p}$ and similarly x and y can be found.

39. $\frac{yz - xz - xy}{xyz} = \frac{1}{m}$ or $\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = \frac{1}{m}$; $\frac{xz - xy - yz}{xyz} = \frac{1}{n}$ or $\frac{1}{y} - \frac{1}{z} - \frac{1}{x} = \frac{1}{n}$; $\frac{xy - yz - xz}{xyz} = \frac{1}{p}$ or $\frac{1}{z} - \frac{1}{x} - \frac{1}{y} = \frac{1}{p}$; adding 2nd and 3rd, $-\frac{2}{x} = \frac{1}{n} + \frac{1}{p}$ $\therefore x = -\frac{2np}{n+p}$, similarly $y = -\frac{2mp}{m+p}$.

40. From the 1st., $x + y + z = m + n + p \ldots(1)$, from the 2nd $nx + py + mz = px + my + nz$ and this from the 3rd eqn $= m^2 + n^2 + p^2 \ldots(2)$, multiply (1) by m and subtract the result from (2) then $(n - m)x + (p - m)y = n^2 + p^2 - m(n + p)$ and similary $(p - n)x + (m - n)y = m^2 + p^2 - n(m + p)$; multiply the former by $n - m$ and the latter by $p - m$ and add then $x\{(n - m)^2 + (p - m)(p - n)\} = (n - m)(n^2 + p^2 - mn - mp) + (p - m)(m^2 + p^2 - mn - np)$ $\therefore x(m^2 + n^2 + p^2 - mn - mp - np) = m^3 + n^3 + p^3 + 2m^2n + 2m^2p - 2n^2m - 2p^2m - mnp$ $\therefore$ by division $x = n + p - m$.

41. $x + y = \frac{1}{a}$, $x + z = \frac{1}{b}$, $y + z = \frac{1}{c}$ and then add.

42. Multiply 1st eqn by m^2 then $m^2x + m^2y + m^2z = 0$, subtract 2nd eqn from this, then $(m^2 - n^2)y + (m^2 - p^2)z = 0 \ldots(a)$; also multiply 1st, eqn by n^2p^2 then $n^2p^2x + n^2p^2y + n^2p^2z = 0$ subtract 3rd eqn from this then $p^2(n^2 - m^2)y + n^2(p^2 - m^2)z = -(n^2 - m^2)(n^2 - p^2)(p^2 - m^2)$ or $p^2(m^2 - n^2)y + n^2(m^2 - p^2)z = (n^2 - m^2)(n^2 - p^2)(p^2 - m^2)\ldots(b)$; multiply (a) by p^2 then $p^2(m^2 - n^2)y + p^2(m^2 - p^2)z = 0$; subtract (b) from this then $(p^2 - n^2)(m^2 - p^2)z = -(n^2 - m^2)(n^2 - p^2)(p^2 - m^2) = (m^2 - n^2)(p^2 - n^2)(m^2 - p^2)$ $\therefore z = m^2 - n^2$, similarly x and y can be found.

43. Multiplying the 3 eqns we have $x^4y^4z^4 = 50 \times 10 \times 20 = 10000 = 10^4$ $\therefore xyz = 10$ $\therefore x^3y^3z^3 = 10^3$. (a) but from the 1st and 2nd eqns $x^3y^3z^2 = 500$; divide (a) by this then $z = \frac{10^3}{500} = 2$ similarly x and y can be found.

44. Proceed as in ex 31.

45. Divide the 1st eqn by xy then $\frac{3}{y} + \frac{4}{x} = 2 \cdots (1)$; divide the 2nd

eqn by yz, then $\frac{4}{z}+\frac{3}{y}=3$...(2) ; divide the 3rd eqn by xz, then $\frac{6}{x}+\frac{5}{z}=4$ ...(3) ; subtract (1) from (2) $\frac{6}{z}-\frac{6}{x}=\frac{3}{2}$.. (4) ; add (3) and (4), $\frac{11}{z}=\frac{11}{2}$ $\therefore\ z=2$.

46. Proceed as in ex 12. 47. Proceed as in ex 12.

48. $xy+xz=5$.........(1), $xy+yz=8$...(2), $xz+yz=9$.........(3) by adding (1) and (2) we have $2xy+xz+yz=13$ or $2xy+9=13$ $\therefore$ $2xy=3$ $\therefore$ $xy=2$ $\therefore$ from (1) $xz=3$ and $\therefore$ from (3) $yz=6$ $\therefore$ $xy\times xz\times yz=2\times3\times6$ or $x^2y^2z^2=36$ $\therefore$ $xyz=6$ $\therefore x=\frac{6}{yz}=\frac{6}{6}=1$; $y=\frac{6}{xz}=\frac{6}{3}=2$ and $z=\frac{6}{xy}=\frac{6}{2}=3$.

49. Multiply the 1st, and 2nd equations then $\frac{z^3}{x^3}=\frac{5^3}{2^3}$ or $\frac{z}{x}=\frac{5}{2}$ $\therefore$ $x=\frac{2}{5}z$ (1) substitute this in the 3rd eqn then $\frac{2}{5}zy^2z^3=2250$ $\therefore$ $y^2z^4=5625$ or $yz^2=75$ $\therefore$ from the 2nd eqn $\frac{75}{x}=\frac{75}{z}$ $\therefore$ $x=2$ $\therefore$ from (1) $z=\frac{5x}{2}=5$ and from (2) $y=\frac{75}{z^2}=\frac{75}{25}=3$.

50. By addition $8=9-1=x^2(y-z)+y^2(z-x)+z^2(x-y)$ or $-2=x^2y-xy^2-(x^2-y^2)z+(x-y)z^2=\{xy-(x+y)z+z^2\}(x-y)=-$ $-(x-y)(z-x)(y-z)$ or $(x-y)(z-x)(y-z)=2$: again multiplying 3 the eqns we have, $8\times-9\times-1=x^2y^2z^2(y-z)(z-x)(x-y)$ $\therefore$ $x^2y^2z^2=\frac{72}{(y-z)(z-x)(x-y)}=\frac{72}{2}=36$ $\therefore$ $xyz=6$.

Now $xyzx^2(y-z)=6+8=14$ or $x\{yz+x(y-z)\}=14$......(1)

$xyz+y^2(z-x)=6-9=-3$ or $y\{xz+y(z-x)\}=-3$ ... (2)

$xyz+z^2(x-y)=6-1=5$ or $z\{xy+z(x-y)\}=5$.........(3)

$xyz-x^2(y-z)=6-8=-2$ or $x\{yz+x(z-y)\}=-2$ ... (4)

$xyz-y^2(z-x)=6+9=15$ or $y\{xz+y(x-z)\}=15$......(5)

$xyz-z^2(x-y)=6+1=7$ or $z\{xy+z(y-x)\}=7$ .....(6)

dividing (1) by (6) we have $\frac{x}{z}=2$ ... ... (7)

dividing (2) by (4) we have $\frac{y}{x}=\frac{3}{2}$ ... ... (8)

dividing (3) by (5) we have $\frac{z}{y}=\frac{1}{3}$ ... ... (9)

again dividing (7) by (8) we have $\frac{x^2}{yz}=\frac{4}{3}$ ∴ $\frac{x^3}{xyz}=\frac{4}{3}$ or $\frac{x^3}{6}=\frac{4}{3}$ ∴ $x^3=8$ ∴ $x=2$ ∴ from (7) $2z=x=2$ ∴ $z=1$ and from (9) $y=3z=3$

51. Multiplying the 1st and 2nd we have $x^2y^2\{z^2+(x+y)z+xy\}=80$ or $x^2y^2z^2+x^2y^2(yz+xz+xy)=80$ ∴ $x^2y^2(yz+xz+xy)=80-x^2y^2z^2=80-36=44$ ...(1), again by adding the 1st and 3rd equs we have $x(yz+xz+xy)=11$ ... (2) and by adding the 2nd and 3rd equs we have $y(yz+xz+xy)=22$ ... (3) divide (1) by (2) we have $xy^2=4$ ... (4); divide (1) by (3) then $x^2y=2$ ... (5); divide (2) by (3) we have $\frac{x}{y}=\frac{1}{2}$......(6); multiply (5) by (6) then $x^3=1$ ∴ $x=1$; divide (4) by (6) $y^3=8$ ∴ $y=2$; then from the 3rd eqn $z=\frac{6}{xy}=\frac{6}{2}=3$.

52. From the given eqns we have $\frac{4+z}{xyz}=\frac{3}{4}$ or $\frac{1}{xz}+\frac{1}{xy}=\frac{3}{4}$; $\frac{x+z}{xyz}=\frac{5}{8}$ or $\frac{1}{yz}+\frac{1}{xy}=\frac{5}{8}$; $\frac{x+y}{xyz}=\frac{3}{8}$ or $\frac{1}{yz}+\frac{1}{xz}=\frac{3}{8}$ then proceed as in ex 47.

53. Multiply the 3 eqns $x^9y^9z^9=324\times72\times432$ ∴ $x^3y^3z^3=216$ ......(1) and $xyz=6$......(2) Dividing 3rd and 2nd of the given eqns by (1) we have $\frac{x}{y}=2$ or $x=2y$; $\frac{y}{z}=\frac{1}{3}$ or $z=3y$; substitute these in (2) ∴ $2y\cdot y\cdot 3y=6$ or $6y^3=6$ ∴ $y=1$ ∴ $x=2$ and $z=3$.

54. Square the 2nd eqn we have $x^2+y^2+z^2+2xy+2xz+2yz=36$ ∴ from the 1st eqn $14+2x(y+z)+2yz=36$ or $14+2\times5+2yz=36$ ∴ $2yz=36-24=12$ ∴ $yz=6$; from the 2nd, $y+z=6-x$ and from the 3rd $y+z=\frac{5}{x}$ ∴ $6-x=\frac{5}{x}$ or $6x-x^2=5$ or $x^2-6x=-5$ or $x^2-6x+9=9-5=4$ ∴ $x-3=-2$ ∴ $x=1$ ∴ $y+z=5$ ∴ $y^2+2yz+z^2=25$ and $4yz=24$ ∴ $y^2-2yz+z^2=1$ ∴ $y-z=1$.

55. Proceed as in ex 18. 56. Proceed as in ex 29.

57. Adding the 3 eqns we have $2\left(\frac{1}{x}+\frac{1}{z}-\frac{1}{y}\right)=\frac{11}{36}$ ∴ $\frac{1}{x}+\frac{1}{z}-\frac{1}{y}=\frac{11}{72}$ from the 1st eqn $\frac{1}{8}+\frac{1}{z}=\frac{11}{72}$ hence z is found and similarly others.

58. The eqns can be written thus $mx+py+nz=px+ny+mz=nx+my+pz=m^3+n^3+p^3-3mnp$, subtracting the 2nd eqn from the 1st and the 3rd from the 2nd we have $(m-p)x+(p-n)y+(n-m)z=0$ ..... (1) and $(p-n)x+(n-m)y+(m-p)z=0$... .....(2) ; multiply (1) by $p-n$ then $(m-p)\ (p-n)x+(p-n)^2y+(p-n)\ (n-m)z=0$ also multiply (2) by $m-p$, then $(m-p)\ (p-n)+(m-p)\ (n-m)y+(m-p)^2z=0$; subtracting the 2nd result from the 1st we have $\{(p-n)^2-(m-p)(n-m)\}y+\{(p-n)(n-m)-(m-p)^2\}z=0$ ∴ $\{(p-n)^2-(m-p)(n-m)\}y=-\{(p-n)(n-m)-(m-p)^2\}z=\{(m-p)^2-(p-n)\ (n-m)\}z$ ∴

$$\frac{y}{(m-p)^2-(p-n)(n-m)}=\frac{z}{(p-n)^2-(m-p)(n-m)}\text{ and similarly}$$

$$\frac{x}{(n-m)^2-(p-n)(m-p)}=\frac{y}{(m-p)^2-(p-n)(n-m)}.$$

$$\therefore\ \frac{x}{(n-m)^2-(p-n)(m-p)}$$

$$=\frac{y}{(m-p)^2-(p-n)(n-m)}=\frac{z}{(p-n)^2-(m-p)\ (n-m)}\text{ ; now each of}$$

these denominators $=m^2+n^2+p^2-mn-mp-np$ ∴ $x=y=z$ hence substituting in one of the given eqns we have $(m+n+p)x=m^3+n^3+p^3-3mnp=(m+n+p)(m^2+n^2+p^2-mn-mp-np)$ ∴ $x=m^2+n^2+p^2-mn-mp-np$.

59. Multiply (1) by m then $mx+my+mz=m^2+mn+mp$

from (2) $mx+ny+pz=np+mp+mn$

Subtract then $(m-n)y+(m-p)z=m^2-np$...(4) multiply (1) by $n-p$ then $(n-p)x+(n-p)y+(n-p)z=(n-p)(m+n+p)$, from (3) $(m-p)x+(p-m)y+(m-n)z=0$, subtract then $(n-2p+m)y+(2n-m-p)z$, $=mn-mp+n^2-p^2$...(5), multiply (4) by $(n-2p+m)$ then $(n-2p$

$+ m)(m - n)y + (n - 2p + m)(m - p)z = (n - 2p + m)(m^2 - np)\ldots\ldots(6)$ multiply (5) by $m - n$ then $(m - n)(n - 2p + m)y + (m - n)(2n - m - p)z = (m - n)(mn - mp + n^2 - p^2)\ldots(7)$, subtract (7) from (6) then $\{(n - 2p + m)(m - p) - (m - n)(2n - m - p)\}z = (n - 2p + m)(m^2 - np) - (m - n)(mn - mp + n^2 - p^2)$ or $2(m^2 + n^2 + p^2 - mn - mp - np)z = (m + n)(m^2 + n^2 + p^2 - mn - mp - np)$ $\therefore\ z = \dfrac{m+n}{2}$; similarly x and y can be found.

60. By transposition $x + y + z = m + n + p\ldots(1)$; $nx + py + mz = px + my + nz$, multiply the 1st equation by m and from the result subtract the 2nd

i.e., $mx + my + mz = m^2 + mn + mp$

subtract $nx + py + mz = mn + np + mp$

$\therefore\ (m - n)x + (m - p)y = m^2 - np\ldots(3)$

again multiply the 1st eqn by n and from the result subtract the 2nd eqn

i.e., $nx + ny + nz = mn + np + n^2$

Subtract $px + my + nz = mn + np + mp$

$\therefore\ (n - p)x + (n - m)y = n^2 - mp\ldots\ldots(4)$

multiply (3) by $m - n$ and (4) by $m - p$ and add then $x\{(m - n)^2 + (m - p)(n - p)\} = (m - n)(m^2 - np) + (m - p)(n^2 - mp)$ i.e., $x(m^2 + n^2 + p^2 - mn - mp - np) = m(m^2 + n^2 + p^2 - mn - mp - np)$ $\therefore\ x = m$ and similarly y and z can be found 61. Proceed as in ex 59

Ex. 24. PROBLEMS.

1. Let x = numerator and y = denominator then $\dfrac{x+1}{y} = \frac{2}{3}$ and $\dfrac{x}{y+1} = \frac{1}{2}$ $\therefore\ x = 5,\ y = 9$.

2. Let x = numerator and y = denominator then $\dfrac{x+1}{y-2} = 1$ and $\dfrac{x+y}{y-x} = 3\frac{2}{3}$ $\therefore\ x = 4,\ y = 7$.

3. Let x = length of the floor and y = breadth then $(x+3)(y+2) = xy+40$, also $(x-3)(y+4) = xy+12$ from these $x=8$, $y=6$.

4. Let x = digit in the unit's place and y = digit in the ten's place then the no itself $= 10y+x$ and this by the question $= 3(x+y)$ also $10y+x+45 = 10x+y$ $\therefore$ $x=2$, $y=7$.

6. Let x = distance between Chandernagore and Calcutta, then rate of sailing from Chandernagore to Calcutta is $\frac{x}{2}$ miles; rate of sailing of the 1st half of the distance from Calcutta to Chandernagore $= \frac{x}{2}-6$ miles and the rate of the latter half $= \frac{x}{2}-4$ ($\because$ it proceeds 2 miles faster) $\therefore \frac{\frac{x}{2}}{\frac{x}{2}-6} + \frac{\frac{x}{2}}{\frac{x}{2}-4} : \frac{x}{\frac{x}{2}-6} :: 6 : 7$ or $\frac{x}{x-12} + \frac{x}{x-8} : \frac{2x}{x-12} :: 6 : 7$ or $\frac{1}{x-12} + \frac{1}{x-8} : \frac{2}{x-12} :: 6 : 7$ $\therefore$ $x = 22$ $\therefore$ rate of sailing in the 1st case $= \frac{x}{2} - 6 = 5$ and in the 2nd case $= \frac{x}{2} - 4 = 7$.

8. Let x = rate of the faster train in miles and y = rate in miles per hour of the slow train $\therefore$ when they move in opposite directions they pass $x+y$ miles in one hour $\therefore \frac{80+50}{5280(x+y)} = \frac{1\frac{2}{7}}{60 \times 60}$ and again when they pass in the same direction they pass $x-y$ miles in one hour $\therefore \frac{80+50}{5280(x+y)} = \frac{9}{60 \times 60}$ $\therefore$ $x = 40$, $y = 30$.

10. Let x = no of gallons issued from the 3rd then $x+8$ = no of gallons issued from the 1st and $x-7$ = no of gallons issued from the 2nd in every 3 minutes $\therefore$ in each minute each of the pipes issues $\frac{x}{3}, \frac{x+8}{3}, \frac{x-7}{3}$ gallons respectively $\therefore$ in each minute they conjointly issue $\frac{1}{3}(x + x + 8 + x - 7) = \frac{3x+1}{3}$ gallons $\therefore$ in 24 minutes they will issue $\frac{24(3x+1)}{3} = 8(3x+1)$ gallons and this by the question $= 1050$ gallons $\therefore 8(3x+1) = 1050$ $\therefore$ $x = 43\frac{5}{12}$ $\therefore$ in one minute it will issue $\frac{1}{3}$ of $43\frac{5}{12} = 14 = \frac{17}{36}$ gallons.

11. Let $x =$ no of sheep in A's flock and $y =$ no of sheep in B's then $x + 5 = y - 5$ and $x - 10 = \frac{y+10}{2}$ $\therefore$ $x = 40$, $y = 50$.

12. Let $x =$ digit in the unit's place, $y =$ digit in $=$ the ten's place then $10y + x = 6(x+y) + 7$ also $10y + x - 27 = 10x + y$ $\therefore$ $x = 5, y = 8$.

13. Let A have before the play x Rs. B, y Rs. C, z Rs. D, u Rs. and E, w Rs. ; after losing, A had $x - (y + z + u + w) = m$(suppose)...(1) Before losing, B got from A y, then he gave A, m ; C, $2y$ for C had got y from A and so on $\therefore$ after losing, B had $2y - m - - 2z - 2u - 2w = n$ (suppose)...(2) ; similarly C had $4z - 2m - n - 4u - 4w = p$(suppose) ...(3) ; D had $8u - 4m - 2n - p - 8w = q$ (suppose).. (4) and E had $16w - 8m - 4n - 2p - q = 32$...(5). From B, A got m ; from C, $2m$; from D, $4m$, and from E, $8m$ then by the question $m + m + 2m + 4m + 8m = 32$ $\therefore$ $m = 2$; after losing B got from C, D, E $\therefore$ $n + n + 2n + 4n = 32$ $\therefore$ $n = 4$ similarly $p + p + 2p = 32$ $\therefore$ $p = 8$ and $q + q = 32$ $\therefore$ $q = 16$; Substituting the values of m, n, p, q, in (5) $16w - 16 - 16 - 15 - 16 = 32$ $\therefore$ $w = 6$ Rs ; from (4) $u = 11$; from (3) $z = 21$, from (2), $y = 41$ and from (1), $x = 81$ ~~Rs~~.

14. Let $x =$ no of days required by A, $y =$ no of days required by B and $z =$ no of days required by C then $\frac{1}{x}$, $\frac{1}{y}$ and $\frac{1}{z}$ are their works in one day $\therefore$ $\frac{a}{x} + \frac{a}{y} = 1$, $\frac{b}{x} + \frac{b}{z} = 1$, $\frac{c}{y} + \frac{c}{z} = 1$ $\therefore$ $x = \frac{2abc}{ac + bc - ab}$

15. Let $x =$ no of days required by A

$y =$ .. .. B

$z =$ C

$1 =$ work, then $\frac{1}{x} =$ A's daily work, $\frac{1}{y} =$ B's daily work and $\frac{1}{z} =$ C's daily work since B and C working together can do m times as much per day as A $\frac{m}{x} = \frac{1}{y} + \frac{1}{z}$......... (1)

and in the same manner $\frac{n}{y}=\frac{1}{x}+\frac{1}{z}$..............(2)

and $\frac{p}{z}=\frac{1}{x}+\frac{1}{y}$......(3) from (1) and (2)

$\frac{m}{x}-\frac{n}{y}=\frac{1}{y}-\frac{1}{x}$ $\therefore$ $(m+1)\frac{1}{x}=(n+1)\frac{1}{y}$ or $\frac{x}{y}=\frac{m+1}{n+1}=\frac{\text{A's time}}{\text{B's time}}$ Similarly $\frac{x}{z}=\frac{m+1}{p+1}=\frac{\text{A's time}}{\text{C's time}}$ and $\frac{y}{z}=\frac{n+1}{p+1}=\frac{\text{B's time}}{\text{C's time}}$. From the 1st equ $m=\frac{x}{y}+\frac{x}{z}=\frac{m+1}{n+1}+\frac{m+1}{p+1}$ $\therefore$ $\frac{1}{n+1}+\frac{1}{p+1}=\frac{m}{m+1}=1-\frac{1}{m+1}$ $\therefore$ $\frac{1}{n+1}+\frac{1}{m+1}=1-\frac{1}{p+1}=\frac{p}{p+1}$.

16. Let $x=$ rate of the stream in miles per hour and $y=$ rate of the boat in still water then $\frac{12}{x+y}=$ no of hours required to come down the stream and this by the question $=2$ hours (for $\frac{1}{4}$ of the whole time viz 8 hours was required to come down) $\therefore$ $\frac{12}{x+y}=2$ and $\frac{12}{x-y}=6$ whence $x=2$, $y=4$. 17. Let $x=$ rate of the ebbtide in miles per hour and $y=$ rate of the flood tide in miles per hour then since the crew can pull at the rate of 6 miles in still water, the time required to pass the distance of 4 miles is $\frac{4}{6+x}$; and the time for passing the remaining 8 miles against the flood tide is $\frac{8}{6-y}$ $\therefore$ $\frac{4}{6+x}:\frac{8}{6-y}::3:16$ or $\frac{64}{6+x}=\frac{24}{6-y}$ or $\frac{8}{6+x}=\frac{3}{6-y}$ also the rate of the ebbtide $=\frac{2}{3}$ of the flood tide $\therefore$ $x=\frac{2}{3}y$ or $3x-2y=0$ and from 1st eqn $48-8y=18+3x$ $\therefore$ $3x+8y=30$ $\therefore$ $10y=30$ $\therefore$ $y=3$ $\therefore$ $x=2$.

18. Let $x=$ portion drawn from the 1st vessel end $y=$ portion drawn from the 2nd vessel, then when x portion is drawn, the quantity of wine is $\frac{2}{3}x$ end when y portion is drawn, the quantity of wine is

$\frac{1}{4}y$ ∴ $\frac{2}{3}x+\frac{1}{4}y=\frac{1}{2}$ (∵ the sum of the two portions will make half the tumbler) ; similarly $\frac{1}{3}x+\frac{3}{4}y=\frac{1}{2}$; clearing these two eqns of fractions, we have $8x+3y=6$ and $4x+9y=6$ ∴ $y=\frac{2}{5}$ and $x=\frac{3}{5}$.

19. Let $x=$ no of gallons B contained, then $\frac{2}{5}$ of this was wine but the quantity of wine in cask A was $\frac{4}{7}$ of $70=40$ gallons ∴ $40+\frac{2}{5}x$ $=\frac{70+x}{2}$ ∴ $x=50$ ∴ quantity of wine $=\frac{2}{5}$ of $50=20$.

20 Let n be put for Nogender, j for Jotish, h for Hem, s for Surrut, p for Preo then, $n=\frac{2}{5}$...(1) ; $j=2(n-p)$ ..(2) ; $h=3(j-s)$ ..(3) ; $s=\frac{1}{2}(n+j+h)$ ... (4) and $p=\frac{4}{3}(n+j+h-s)$...(5) ; from (2) and (3), $h=3(2n-2h-s)$ or $h=6n-6h-3s=6n-6h-3\times\frac{1}{2}(n+j+h)=6n-6n-\frac{3n}{2}-\frac{3j}{2}-\frac{3h}{2}$ or $2h=12n-12h-3n-3j-3h$ or $17h=9n-3(2n-2h)$ $=9n-6n+6h$ or $11h=3n=\frac{6}{5}$ ∴ $h=\frac{6}{55}$ ∴ $j=2(\frac{2}{5}-\frac{6}{55})=\frac{32}{55}$ ∴ $s=\frac{1}{2}(\frac{2}{5}+\frac{32}{55}+\frac{6}{55})=\frac{30}{55}$ and $p=\frac{4}{3}\times\frac{30}{55}=\frac{40}{55}$ ∴ Preo gets the highest mark and then Jotish.

21. Let $x=$rate of A in miles per hour and $y=$rate of B then

$$\left.\begin{array}{l} \frac{1}{x}-\frac{1}{y}=\frac{1}{60} \\ \frac{1}{y-2}-\frac{1}{x+2}=\frac{1}{60} \end{array}\right\} \begin{array}{l} \text{or } 60y-60x=xy\ldots\ldots(1) \\ 60x+120-60y+120=xy-2x+2y-4 \\ \text{or } 62x-62y+244=xy\ldots\ldots(2) \end{array}$$

Subtracting (1) from (2)

$122(x-y+2)=0$ or $x-y+2=0$ ∴ $x=y-2$

substitute the value of x in (1) we get $60y-60y+120=y^2-2y$ or $y^2-2y=120$ ∴ $y^2-2y+1=121$ ∴ $(y-1)^2=(11)^2$ ∴ $y-1=11$ ∴ $y=12$.

22. Let $x=$rate in miles per hour of the pulling of the boat and $y=$rate in miles of the stream then $\frac{20}{x+y}+\frac{20}{x-y}=10$......(1) also $\frac{2}{x-y}=\frac{3}{x+y}$.........(2) from (2) $2x+2y=3x-3y$ ∴ $x=5y$, substitute this in

(1) then $\frac{20}{5y+y}+\frac{20}{5y-y}=10$ or $\frac{10}{3y}+\frac{10}{2y}=10$ $\therefore$ $\frac{1}{3y}+\frac{1}{2y}=1$ $\therefore$ $5=6y$ $\therefore$ $y=\frac{5}{6}$ $\therefore$ $x=5y=\frac{25}{6}=4\frac{1}{6}$ $\therefore$ time of going $=\frac{20}{x+y}=\frac{20}{4\frac{1}{6}+\frac{5}{6}}=4$ and time of returning $=\frac{20}{4\frac{1}{6}-\frac{5}{6}}=6$ hours.

23 Procced as in ex 13.

24. Let $x=$ digit in the uuit's place and y that of the ten's place then $10y+x=$ no $\therefore$ by the question $\frac{10y+x}{x+y}=4$ also $\frac{10x+y}{2+y-x}=14$ whence $x=8$ and $y=4$.

25. Let x and $y=$ digits in order then $10x+y=$ number and $\frac{10x+y}{9}=x+\frac{8}{9}$ $\therefore$ $10x+y=9x+8$ $\therefore$ $x+y=8$ also $\frac{10x+y}{4}=\frac{3}{4}+17y$ $\therefore$ $10x+y=3+68y$, whence $x=7$, $y=1$.

26. Let $x=$ no of lbs of gold in the mixture of 106 lbs then $106-x$ $=$ no of lbs of silver $\therefore$ $\frac{18x}{19}+\frac{9}{10}(106-x)=99$ $\therefore$ $x=76$.

27 Let $x=$ no cut off by A $\therefore$ $52-x=$ no left by him. Let $y=$ no cut off by B $\therefore$ $52-y=$ no left by him then $x+y=$ whole no cut off and $104-(x+y)=$ whole no left, whence $104-2(x+y)=64$ $\therefore$ $2(x+y)=40$ $\therefore$ $x+y=20$, now $52-x+y=50$ $\therefore$ $x-y=2$ but $x+y=20$ $\therefore$ $2x=22$ $\therefore$ $x=11$ and by subtraction $2y=18$ $\therefore$ $y=9$.

28. Let $x=$ no of games A won, and $y=$ no B won $\therefore$ $y+x=36$ and $3y+3x=108$; but $5y-3x=116$ $\therefore$ $8y=224$ $\therefore$ $y=28$ $\therefore$ $x=8$.

29. Let $x=$ digit in the unit's place, then $x+5=$ digit in the ten's place, then $10(x+5)+x=$ no required ; when the digits are inverted the no $=10x+x+5=11x+5$ and this by the question $=\frac{3}{8}\{10(x+5)+x\}$ whence $x=2$ $\therefore$ other digit $=x+5=7$.

30. Let $x=$ digit in the unit's place and $y=$ digit in the ten's place then $10x+y$ and $10y+x$ are the numbers then $9(10x+y)=2(10y+x)$ ...(1) aud $x+y=2$...(2), whence $x=1$, $y=8$

31. Let x = digit in the tens place, y = digit in the hundreds place then $100y+10x$ = no required ∴ $100x+10y=180+(100y+10x)$ ·· (1) or $90x-90y=180$ ∴ $x-y=2$ also $(100\times\frac{y}{2}+x)+136=100y+10x$ or $50y+x+136=100y+10x$ or $50y+9x=136$ also $50x-50y=100$ ∴ $59y=236$ ∴ $x=4$ and $y=2$ ∴ no $=240$.

32. Let x = no. of yds. travelled by A in 1 min. and y = same by B in 1 min. then $\frac{1760}{x}$ = no. of min. required by Henry to travel the course and $\frac{1740}{y}$ = no. of min. required by Roberts to travel 1740 yds then by the question $\frac{1760}{x}+\frac{1}{2}=\frac{1740}{y}$ ... . (1) and for the same reason $\frac{1760}{x}+\frac{32}{60}=\frac{1750\frac{6}{11}}{y}$ or $\frac{1760}{x}+\frac{8}{15}=\frac{19256}{11y}$ ....(2) ; from (1) $\frac{1760}{x}=\frac{1740}{y}-\frac{1}{2}$ and from (2) $\frac{1760}{x}=\frac{19256}{11y}-\frac{8}{15}$ ∴ $\frac{1740}{y}-\frac{1}{2}=\frac{19256}{11y}-\frac{8}{15}$ or $\frac{1740}{y}-\frac{19256}{11y}=\frac{1}{2}-\frac{8}{15}=-\frac{1}{30}$ or $\frac{116}{11y}=\frac{1}{30}$ ∴ $y=\frac{116\times 30}{11}$ ∴ from (1) $\frac{1760}{x}=\frac{1740}{y}-\frac{1}{2}=\frac{1740\times 11}{116\times 30}-\frac{1}{2}=5$ ∴ $x=352$ ∴ in one hour distance travelled by Henry $=60\times 352$ yds. $=\frac{60\times 352}{1760}$ miles $=12$ miles

EXPONENTIAL EQUATIONS.

1. $5^x=125=5^3$ ∴ $x=3$. 2 $2^x=256=2^8$ ∴ $x=8$

3 $(\frac{3}{2})^x=\frac{81}{16}=(\frac{3}{2})^4$ ∴ $x=4$

4. $3^{2x+3}=243=3^5$ ∴ $2x+3=5$ ∴ $2x=2$ ∴ $x=1$

5. $25^x=5$ or $5^{2x}=5$ ∴ $2x=1$ ∴ $x=\frac{1}{2}$

6. $4^{\frac{x}{2}}+2^{x+1}=24$ or $(2^2)^{\frac{x}{2}}+2.2^x=24$ or $2^x+2\,2^x=24$
∴ $2^x(1+2)=24$ ∴ $2^x=8=2^3$ ∴ $x=3$
$(2^2)^x+2.2^x=80$ or $2^{2x}+2.2^x+1=81$ or $2^x+1=9$
∴ $2^x=8=2^3$ ∴ $x=3$

8. $a^{-x}(a^x+b^{-x})=a^{-x}.\,a^x+a^{-x}b^{-x}=1+\frac{1}{a^xb^x}$ also $\frac{a^2b^2+1}{a^2b^2}$

$=1+\frac{1}{a^2b^2}$ ∴ $\frac{1}{a^xb^x}=\frac{1}{a^2b^2}$ or $(ab)^x=(ab)^2$ ∴ $x=2$

9. $4^{x+1}+\frac{1}{(4^2)^{-\frac{x}{2}}}=4^{x+1}+4^x=4^x(4+1)=4^x.5$ ∴ $4^x.5=320$

∴ $4^x=64=4^3$ ∴ $x=3$

10. $2^{2x}+\{(2^3)^{2x}\}^2=2^{2x}+2^{12x}=2^{2x}(1+2^{10x})$ also $16+16\times2^{10x}$ $=16(1+2^{10x})$ ∴ $2^{2x}(1+2^{10x})=16(1+2^{10x})$ ∴ $2^{2x}=16=2^4$ ∴ $2x=4$ ∴ $x=2$.

11. $7^{\frac{x}{2}+\frac{y}{3}}=2401=7^4$ ∴ $\frac{x}{2}+\frac{y}{3}=4$...(1) also $6^{\frac{x}{4}+\frac{y}{2}}=1296=6^4$ ∴ $\frac{x}{4}+\frac{y}{2}=4$...(2) ∴ from (1) and (2) $x=4$, $y=6$

12. $x=y^{\frac{x}{y}}$ also $x=y^{\frac{2}{4}}$ ∴ $y^{\frac{x}{y}}=y^{\frac{1}{2}}$ ∴ $\frac{x}{y}=\frac{1}{2}$ ∴ $2x=y$ substituting this in the 2nd eqn. we have $x^4=(2x)^2=4x^2$ or $x^2=4$ ∴ $x=2$ and ∴ $y=4$

13. $y=\frac{6^x}{54}$ also $y=\frac{4^x}{16}$ ∴ $\frac{6^x}{54}=\frac{4^x}{16}$ ∴ $\frac{6^x}{4^x}=\frac{54}{16}$ or $(\frac{6}{4})^x=\frac{54}{16}$ or $(\frac{3}{2})^x$

$=\frac{27}{8}=\left(\frac{3}{2}\right)^3$ ∴ $x=3$ ∴ $y=\frac{4^3}{16}=\frac{64}{16}=4$.

14. $(3^2)^x-(2^2)^y=3^{2x}-2^{2y}$ ∴ $\frac{3^{2x}-2^{2y}}{3^x+2^y}=\frac{77}{11}=7$ or $3^x-2^y=7$...(1)

∴ adding the 1st given eqn and (1) we have $2.3^x=18$ ∴ $3^x=9=3^2$ ∴ $x=2$ and again subtracting (1) from the 1st given equation we have $2.2^y=4$ ∴ $2^y=2$ ∴ $y=1$.

15. $2^x.2^{2y}=2^{x+2y}$ ∴ $2^{x+2y}=32=2^5$ ∴ $x+2y=5$.......... (1)

also $3^x\div9^y=\frac{3^x}{(3^2)^y}=\frac{3^x}{3^{2y}}=3^{x-2y}$ ∴ $3^{x-2y}=3$ ∴ $x-2y=1$.........(2)

from 1) and (2) $x=3$, $y=1$.

16. Multiplying the whole eqn. by $2^{\frac{1}{x}}-1$ we have $2^{\frac{1}{2}}.2^{\frac{1}{x}}+2^{\frac{1}{x}}-2^{\frac{1}{2}}-2=0$ or $\left(2^{\frac{1}{2}}+1\right)2^{\frac{1}{x}}=2^{\frac{1}{2}}+2=2^{\frac{1}{2}}\left(1+2^{\frac{1}{2}}\right)$ $\therefore\ 2^{\frac{1}{x}}=2^{\frac{1}{2}}$ $\therefore\ \frac{1}{x}=\frac{1}{2}$ $\therefore x=2$.

17. $\sqrt{a^x}.\sqrt{a^y}=a^3$......(1) $\therefore a^{\frac{x}{2}}.a^{\frac{y}{2}}=a^3$

$(b^x)^3=b^6\,(b^y)$......(2) $\therefore\ a^{\frac{x}{2}+\frac{y}{2}}=a^3$ $\therefore\ a^{x+y}=a^6$

$\therefore\ x+y=6$....... (3)

Again $(b^x)^3=b^6(b^y)^3$ $\therefore b^x=b^2.b^y \therefore b^x=b^{2+y} \therefore x=2+y \therefore x-y=2$.. (4)

$$\begin{aligned} x+y&=6 \text{ ....(3)}\\ \underline{x-y}&\underline{=2} \text{ ....(4)}\\ 2x&=8\ \therefore\ x=4\ \therefore\ x+y \text{ or } 4+y=6\ \therefore\ y=2. \end{aligned}$$

18 $a^{2x}a^{3y+1}=a^{3m+5}$...(1) a^{4y}, $a^{2x+3}=a^{4m+y}$ .....(2) $\therefore\ a^{2x+3y+1}=a^{3m+5}$ or $2x+3y+1=3m+5$ $\therefore\ a^{4y+2x+3}=a^{4m+7}$ or $2x+4y+3=4m+7$. $\therefore\ 2x+3y=3m+4$. $\therefore\ 2x+4y=4m+4$ $\therefore y=m$ $\therefore\ 2x+3y=2x+3m=3m+4$. $\therefore\ 2x=4 \therefore x=2$.

19. The left hand side of the eqn. $=5^{x+1}+25^{\frac{x}{2}}=5^{x+1}+\{(25)^{\frac{1}{2}}\}^x$ $=5.5^x+5^x=5^x(5+1)=5^x.6$ $\therefore\ 5^x.6=150$ $\therefore\ 5^x=25=5^2 \therefore\ x=2$.

20. From (1) $y=x^{\frac{y}{x}}$ and from (2) $y=x^{\frac{m}{n}}$ $\therefore\ x^{\frac{y}{x}}=x^{\frac{m}{n}} \therefore\ \frac{y}{x}=\frac{m}{n}$ $\therefore \frac{x^{\frac{m}{n}}}{x}=\frac{m}{n}$ or $x^{\frac{m}{n}-1}=\frac{m}{n}$ or $x^{\frac{m-n}{n}}=\frac{m}{n}$ $\therefore\ x=\left(\frac{m}{n}\right)^{\frac{n}{m-n}}$ and $y=\frac{m}{n}x=\frac{m}{n}$ $\left(\frac{m}{n}\right)^{\frac{n}{m-n}}=\left(\frac{m}{n}\right)^{\frac{n}{m-n}+1}=\left(\frac{m}{n}\right)^{\frac{m}{m-n}}$.

21. From the 2*d* eqn. $x=3y$ substitute in the 1st eqn. then $4^{3y}=2^{4y}\times 2^3$ $\therefore\ (2^2)^{3y}$ or $2^{6y}=2^{4y+3}$ $\therefore\ 6y=4y+3 \therefore 2y=3$ $\therefore\ y=\frac{3}{2}$ and $x=3y=\frac{9}{2}$

22. $\frac{(8^2)^x}{8} = 8^{2x-1}$ and $4096^x = (8^4)^x = 8^{4x}$ ∴ $\frac{1}{4096^x} = \frac{1}{8^{4x}} = 8^{-4x}$

∴ $8^{2x-1} = 8^{-4x}$ or $2x - 1 = -4x$ ∴ $6x = 1$ ∴ $x = \frac{1}{6}$

23. $6\frac{1}{4} \times (\frac{2}{5})^x = 1$ ∴ $(\frac{2}{5})^x = \frac{4}{25} = (\frac{2}{5})^2$ ∴ $x = 2$.

24. $m^{2^{-x^2}} = . m^{2^{-2x}} . m^{2^{-2x}} = m^{2\cdot 2^{-2x}} = m^{2^{1-2x}}$ ∴ $2^{-x^2} = 2^{1-2x}$

$-x^2 = 1 - 2x$ ∴ $x^2 = 2x - 1$ ∴ $x^2 - 2x = -1$ ∴ $x^2 - 2x + 1 =$

$(x-1)^2 = 0$ ∴ $x = 1$

25. $m^{x^2-x} . m^{x^2+2x} = m^{2x^2+x}$ ∴ $m^{2x^2+x} = m^{5x}$ ∴ $2x^2 + x = 5x$

∴ $2x^2 = 4x$ ∴ $x = 2$

26. From (1) $3^x . 3^{2y} = 3^3$ or $3^{x+2y} = 3^3$ ∴ $x + 2y = 3$...(3) ; from (2) $2^{2x} . 2^{3y} = 2^5$ ∴ $2^{2x+3y} = 2^5$. $2x + 3y = 5$...(4) ; multipl (3) by 2 and subtract (4) from it, ∴ $y = 1$ and $x = 1$

RATIO.

1, $\frac{12\text{ as.}}{3\text{ Rs.}} = \frac{12\text{as.}}{3 \times 16\text{ as.}} = \frac{1}{4}$ ∴ ratio = 1 : 4.

2. Arrange the fractions $\frac{5}{6}$, $\frac{6}{9}$, $\frac{8}{10}$, $\frac{5}{8}$ in order of their magnitudes.

3. $\frac{2}{13} \times \frac{26}{30} = \frac{2}{15}$.

4. $\frac{p}{n} \times \frac{x}{y} \times \frac{y}{p} = \frac{x}{n}$.

5. $y^2 = 2ax - x^2 = x(2a - x)$ ∴ $\frac{x}{y} = \frac{y}{2a - x}$.

6. Let m be the required quantity then $a+m . b+m : x+m$ $y+m$ or $(a+m)(y+m) = (b+m)(x+m)$ or $ay + (a+y)m + m^2 = ba$ $+ (b+x)m + m^2$ or $ay - ba = (b + x - a - y)m$ ∴ $m = \frac{ay - bx}{b + x - a - y}$.

7. $\frac{a^2 - x^2}{a^2 + x^2} > = < \frac{a - x}{a + x}$ if $\frac{a + x}{a^2 + x^2} > = < \frac{1}{a + x}$ if $(a + x)^2 > = < a$

$+x^2$ if $a^2+2ax+x^2 > = < a^2+x^2$ if $2ax > = < 0$ but $2ax > 0$ $\therefore$ the first ratio is greater than the 2nd.

8 $\frac{x^3+y^3}{x^2+y^2} > = < \frac{x^2+y^2}{x+y}$ if $(x+y)(x^3+y^3) > = < (x^2+y^2)^2$ if $x^4+y^4+x^3y+xy^3 > = < x^4+2x^2y^2+y^4$ if $xy(x^2+y^2) > = < 2x^2y^2$ if $xy(x^2-2xy+y^2) > = < 0$ if $xy(x-y)^2 > = < 0$ but $(x-y)^2$ being positive is >0 $\therefore$ $xy(x-y)^2 > 0$ or the first ratio is $>$ the 2nd.

9. Put the 1st ratio in the fractional form and reduce it to its lowest term.

10. Let $x=$ quantity added then $a+x : b+x :: m : n$ or $n(a+x) = m(b+x)$ or $an+nx=bm+mx$ $\therefore$ $(m-n)x=an-bm$

$$\therefore\ x=\frac{an-bm}{m-n}$$

11. $\frac{a^2+b^2}{a^4-b^2}\times\frac{(a+b)^2}{(a-b)^2}=\frac{(a+b)(a^2+b^2)}{(a-b)^3}$

12. $\frac{x}{y}=\frac{a^2}{b^2}$ also $\frac{a}{b}=\frac{\sqrt{(a+x)}}{\sqrt{(a-y)}}$ $\therefore$ $\frac{a^2}{b^2}=\frac{a+x}{a-y}$ $\therefore$ $\frac{x}{y}=\frac{a+x}{a-y}$ or $ax-xy$

$=ay+xy$ $\therefore$ $ax-ay=2xy$ or $a(x-y)=2xy$ or $\frac{x-y}{y}=\frac{2x}{a}$

12*a*. Proceed as in Ex. 7. 13. $\frac{41}{6}\times\frac{6}{11}\times\frac{11}{4}=\frac{41}{4}$.

14 $\frac{(x^2)^{\frac{1}{2}}}{(y^2)^{\frac{1}{2}}}\times\frac{(x^{\frac{1}{3}})^3}{(y^{\frac{1}{3}})^3}=\frac{x^2}{y^2}$

15. $\frac{x^3+y^3}{x-y} > =$ or $< \frac{x^3-y^3}{x+y}$ if $(x^3+y^3)(x+y) > = < (x^3-y^3)(x-y)$ if $x^4+xy^3+x^3y+y^4 > = < x^4-xy^3-x^3y+y^4$ if $2(xy^3+x^3y) > = < 0$ or $2xy(y^2+x^2) > = < 0$ but $2xy(x^2+y^2)$ is positive and $\therefore > 0$ $\therefore$ the 1st is the greater.

16. Let $x=$ mean proportional then $\frac{a+b}{a-b}\ x : x : \frac{a-b}{a+b}$

$$\therefore\ x^2=\frac{a+b}{a-b}\times\frac{a-b}{a+b}=1\ \therefore\ x=1.$$

17. $\frac{a}{a-2b}$ is a ratio of greater inequality and by Ex 39 it can be shewn that the value will be diminished.

18. $\frac{16}{5}\times\left(\frac{8}{27}\right)^{\frac{1}{3}}\times\left(\frac{16}{25}\right)^{\frac{1}{2}}=\frac{16}{5}\times\frac{2}{3}\times\frac{4}{5}=\frac{128}{75}$.

19. $\frac{a-x}{a+x}\times\frac{a+x}{2+y}\times\frac{2+y}{a-x}=\frac{a+x}{a+x}=1$ i.e. the antecedent and consequent are equal.

20. Let $4x$ and $7x$ be the nos. then $4x+4 : 7x+4 :: 2 : 3 \therefore 3(4x+4)=2(7x+4)$ or $12x+12=14x+8$ or $2x=4 \therefore x=2$ the nos. are 8 and 14.

21. Let $9x$ and $11x$ be nos. then $9x-6 : 11x-6 :: 3 : 4 \therefore 4(9x-6)=3(11x-6)$ or $36x-24=33x-18 \therefore 3x=6 \therefore x=2 \therefore$ nos. are 18 and 22.

22. Let $2x$ and $3x$ be the nos then $2x+6 : 3x-3 :: 10 : 11$
$\therefore 11(2x+6)=10(3x-3)$ or $22x+66=30x-30 \therefore 8x=96 \therefore x=12$
$\therefore$ nos. are 24 and 36.

23. $x=$required no. then $\frac{9+x}{7+x}=\frac{4}{5}\cdot\frac{9-x}{7-x}=\frac{4(9-x)}{5(7-x)} \therefore (9+x)5(7-x)=(7+x)4(2-x)$ or $315-10x-5x^2=252+8x-4x^2 \therefore x^2+18x=63 \therefore x^2+18x+81=81+63=144 \therefore x+9=12 \therefore x=3$.

24. $\frac{3a+2}{6a+1}\times\frac{2a+3}{a+2}=\frac{6a^2+13a+6}{6a^2+13a+2}$ Here the numerator or the antecedent of the ratio is greater than the denominator or the consequent of the ratio $\therefore$ it is a ratio of greater inequality.

25. The 3 parts shall be as the nos. $1 : 3 : 5 \therefore$ let x, $3x$, and $5x$ be the parts then $x+3x+5x=18$, $\therefore 9x=18 \therefore x=2 \therefore$ nos. are 2, 6 and 10.

26. $\frac{x^3+a^3}{a}=\frac{x}{1}$ or $\frac{(x+a)(x^2-ax+a^2)}{x \qquad a}=1$ or $\frac{x^2-ax+a^2}{a}$ $=\frac{x}{x+a}$.

27. $\frac{a^3+b^3}{a^2+b^2} >$ or $< \frac{a^2+b^2}{a+b}$ if $a^4+ab^3+a^3b+b^4 >$ or $< a^4+2a^2b^2$ $+ b^4$ according as $ab^3+a^3b >$ or $< 2a^2b^2$, or as $a^2+b^2 >$ or $< 2ab$ or as $a^2-2ab+b^2 >$ or < 0 or as $(a-b)^2 > 0$; but $(a-b)^2$ is always positive and $\therefore > 0$ $\therefore$ $a^3+b^3 : a^2+b^2$ is the greater.

28. $\because \frac{c}{d} < \frac{a}{b} \therefore \frac{c}{a} < \frac{d}{b} \therefore \frac{c+a}{a} < \frac{b+d}{b}$ or $\frac{a+c}{b+d} < \frac{a}{b}$. Again $\because \frac{a}{b} > \frac{c}{d} \therefore \frac{a}{c} > \frac{b}{d}$ or $\frac{a+c}{c} > \frac{b+d}{d}$ or $\frac{a+c}{b+d} > \frac{c}{d}$

29. Let $4x$ and $5x$ be the nos. then $5x-4x : (5x^2-(4x)^2 :: 1 : 18$ or $1 : 5x+4x :: 1 \quad 18$ or $9x=18$ $\therefore x=2 \therefore$ nos are 8 and 10.

30. Let $5x$ and $7x$ be the nos then $5x+7x : 25x^2+49x^2 :: 3 : 37$ or $12x : 74x^2 :: 3 : 37$ or $37\times 12x = 3\times 74x^2$ or $37\times 4 = 74x$ $\therefore x=2$ $\therefore$ 10 and 14 are the nos.

31. Let $3x$ and $4x$ be the nos then $3x+4x : (4x)^2-(3x)^2 :: 1 : 3$ or $1 : 4x-3x :: 1 . 3$ or $x=3$ $\therefore$ nos. are 9 and 12

32 $x : 3 :: 3^2 : x^2$ or $x^3 = 3^3$ $\therefore x=3$

33 $6-x : 12-x :: 6^2 : 12^2$ or $144(6-x)=36(12-x)$ or $864-144x=432-36x$ $\therefore 108x=432$ $\therefore x=4$

34. Let each of the ratios be equal to m then $x=m(a-b)$, $y=m(b-c)$, $z=m(c-a)$ $\therefore x+y+z=m(a-b)+m(b-c)$ $+m(c-a)=m(a-b+b-c+c-a)=m\times 0=0$

35. $$\frac{x^6-y^6}{(x^2+y^2)^2}\times\frac{x^2+y^2}{x^2-xy+y^2}\times\frac{x+y}{(x-y)(x^2+xy+y^2)}$$
$$=\frac{(x^3+y^3)(x^3-y^3)(x+y)}{(x^2+y^2)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}$$
$$=\frac{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)(x+y)}{(x^2+y^2)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}=\frac{(x+y)^2}{x^2+y^2}$$

36. Let x and y be the nos then $x+y=7$ and $x-y=3$ $\therefore 2x=10$ $\therefore x=5$ and $2y=4$ $\therefore y=2 \therefore \frac{5+\frac{1}{2}\cdot 2}{2+\frac{5}{2}}=\frac{6}{4\frac{1}{2}}=\frac{12}{9}=\frac{4}{3}$.

37. $\frac{a^2}{b^2}=\frac{c^2}{d^2}=\frac{e^2}{f^2}\therefore\frac{pa^2}{pb^2}=\frac{qc^2}{qd^2}=\frac{re^2}{rf^2}=x^2$ when $x=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\therefore pa^2$ $=pb^2x^2, qc^2=qd^2x^2, re^2=rf^2x^2 \therefore pa^2+qc^2+re^2=pb^2x^2+qd^2x^2+rf^2x^2$ $=(pb^2+qd^2+rf^2)x^2 \therefore x^2=\frac{pa^2+qc^2+re^2}{pb^2+qd^2+rf^2}\therefore x=\sqrt{\frac{pa^2+qc^2+re^2}{pb^2+qd^2+rf^2}}$ by a process similar to this, the truth of the other two parts of this question can be shewn.

38. Let each of the ratios be $=x$ then $a=bx$, $c=dx$, $e=fx$ $\therefore a+c+e=bx+dx+fx=(b+d+f)x \therefore x=\frac{a+c+e}{b+d+f}$

39. Let the ratio be $\frac{a}{b}$, and let a new ratio be formed by adding x to both terms of the original ratio; then $\frac{a+x}{b+x}>$ or $<\frac{a}{b}$, if $b(a+x)$ $>$ or $<a(b+x)$ or as $bx>$ or $<ax$ or as $b>$ or $<a$.

40. Let the ratio be $\frac{a}{b}$, and let a new ratio be formed by taking x from both terms of the original ratio; then $\frac{a-x}{b-x}>$ or $<\frac{a}{b}$ if $b(a-x)$ $>$ or $<a(b-x)$ or if $bx>$ or $<ax$ or as $b>$ or $<a$.

41. $\frac{p(a-m)}{p(m-n)}=\frac{m(b-p)}{m(p-q)}\therefore\frac{pa-pm-mb+pm}{pm-pn-mp+mq}=$ each of the ratios or $\frac{ap-mb}{mq-np}=$ each of the ratios; $\frac{r(b-p)}{r(p-q)}=\frac{p(c-r)}{p(r-s)}\therefore\frac{r(b-p)-p(c-r)}{r(p-q)-p(r-s)}$ $=$ each of the ratios or $\frac{br-pc}{ps-qr}=$ each of the ratios again $\frac{m(c-r)}{m(r-s)}$ $=\frac{r(a-m)}{r(m-n)}\therefore\frac{m(c-r)-r(a-m)}{m(r-s)-r(m-n)}=$ each of the ratios or $\frac{cm-ar}{nr-ms}=$ each of the ratios; lastly as it has been proved in Ex. 37 that $\frac{\text{the sum of all the antecedents}}{\text{the sum of all the consequents}}=$ each of the ratios,

$\therefore \frac{(a-m)+(b-p)+(c-r)}{(m-n)+(p-q)+(r-s)}=$ each of the ratios.

42. Let x = no of seers of the mixture that will be drawn from the cask A then $17-x$ = no of seers of the mixture that will be drawn from the cask B, now when one seer of the mixture is drawn from A, $\frac{3}{7}$ seer would be water, and from B $\frac{2}{5}$ seer would be water ∴ $\frac{3}{7}x+\frac{2}{5}(17-x)=7$ whence $x=7$ and $17-x=10$.

Ex. 27. PROPORTIONS.

1. $x=\frac{16\times 24}{12}=32$.

2. $8x=12\times 6$ (∵ product of the extremes = product of the means)

$\therefore\ x=\frac{12\times 6}{8}=9$ 3. $x^2=3\times 27=9^2$ ∴ $x=9$.

4. $x^2=4\times 36=12^2$ ∴ $x=12$.

5. $(x+4)(x+5)=(x+2)(x+8)$ Product of the extremes = product of the means ∴ $x^2+9x+20=x^2+10x+16$ ∴ $x=4$.

6. $(4x-4)(2x+6)=2x(3x+7)$ from which $x=3$.

7. $63(x^2+x+1)(x-1)=62(x+1)(x^2-x+1)$ or $\frac{x^3-1}{x^3+1}=\frac{62}{63}$ or

$$\frac{x^3+1}{x^3-1}=\frac{63}{62}\ \therefore \frac{x^3}{1}=\frac{63+62}{63-62}=125\ \therefore\ x=5$$

8. Divide the 1st eqn. by the 2nd then $\frac{x}{z}=\frac{m}{p}$ or $\frac{x}{m}=\frac{z}{p}$ (alternendo)

(a) $32 : x :: 4 : 1$ ∴ $4x=32$ ∴ $x=8$

(b) Let m be the 4th proportional then $2x : 3x :: 8x : m$

$$\therefore\ 2mx=24x^2\ \therefore\ m=\frac{24x^2}{2x}=12x$$

(c) Let x = mean proportional then $3 : x :: x : 27$

$$\therefore\ x^2=81=9^2\ \therefore\ x=9$$

(d) Let m = mean proportional then $\frac{x+y}{x-y} : m :: m : \frac{x^2-y^2}{x^2y^2}$

$$\therefore\ m^2=\frac{(x+y)^2}{x^2y^2}\ \text{or}\ m=\frac{x+y}{xy}$$

10. $a : c :: b : d$ (alternendo) $\therefore a+c : c :: b+d : d$ (componend

12. $\frac{a}{b}=\frac{c}{d}=x \quad \therefore \quad \frac{a^2}{b^2}=\frac{c^2}{d^2}=x^2 \quad \therefore \quad a^2=b^2x^2,\ c^2=d^2x^2 \quad \therefore \quad a^2+c^2$

$$=b^2x^2+d^2x^2=(b^2+d^2)x^2 \quad \therefore x^2=\frac{a^2+c^2}{b^2+d^2} \text{ or } \frac{a^2}{b^2}=\frac{a^2+c^2}{b^2+d^2}$$

13. $\frac{m}{n}\cdot\frac{a}{b}=\frac{m}{n}\cdot\frac{c}{d}$ or $\frac{ma}{nb}=\frac{mc}{nd}$ or $\frac{ma+nb}{nb}=\frac{mc+nd}{nd}$ (componendo)

or $\frac{ma+nb}{b}=\frac{mc+nd}{b}$

14. $a : c = b : d$ (alternendo) $\therefore a : \frac{c}{m} = b : \frac{d}{m}$

$\therefore a \pm \frac{c}{m} : a = b \pm \frac{d}{m} : b$

15. Square the terms of the proportion and then apply com nendo and dividendo.

16. $\frac{a}{c}=\frac{b}{d} \therefore \frac{ma}{nc}=\frac{mb}{nd} \therefore \frac{ma+nc}{nc}=\frac{mb+nd}{nd}$ (componendo)

$\therefore \frac{ma+nc}{mb+nd}=\frac{nc}{nd}$ (alternendo) $=\frac{c}{d}$ Similarly $\frac{pa+qc}{pb+qd}=\frac{c}{d}$

$\therefore \frac{ma+nc}{mb+nd}=\frac{pa+qc}{pb+qd}$ or $\frac{ma+nc}{pa+qc}=\frac{mb+nd}{pb+qd}$ (alternendo)

17. $\frac{a}{c}=\frac{b}{d} \therefore \frac{a}{mc}=\frac{b}{md} \therefore \frac{a+mc}{a}=\frac{b+md}{b}$

or $\frac{a+mc}{b+md}=\frac{a}{b}$ (alternendo)

18. $\frac{a}{b}=\frac{c}{d} \therefore \frac{ma}{b}=\frac{mc}{d} \therefore \frac{ma+b}{b}=\frac{mc+d}{d} \therefore \frac{ma+b}{mc+d}=\frac{b}{d}$ also $\frac{b}{a}=\frac{d}{c}$

$\therefore \frac{mb}{a}=\frac{md}{c} \therefore \frac{mb+a}{a}=\frac{md+c}{c}$ or $\frac{mb+a}{md+c}=\frac{a}{c}$ or $=\frac{b}{d} \therefore \frac{ma+b}{mc+d}=\frac{mb+a}{md+c}$ or

$\frac{ma+b}{mb+a}=\frac{mc+d}{md+c}$ (alternendo).

19. Let $\frac{a}{b}=\frac{c}{d}=x$ then $a=bx$, $c=dx$ $\therefore$ $a+c=(b+d)x$ $\therefore$ x $=\frac{a+c}{b+d}=$ or $\frac{c}{d}=\frac{a+c}{b+d}$ $\therefore$ $\frac{c+d}{d}=\frac{a+c+b+d}{b+d}$ (componendo).

20. $\frac{a}{b}=\frac{c}{d}\therefore$ $\frac{a+b}{b}=\frac{c+d}{d}$ (componendo) and $\frac{a-b}{b}=\frac{c-d}{d}$ (dividendo) $\therefore\frac{a\pm b}{b}=\frac{c\pm d}{d}\therefore\left(\frac{a\pm b}{c\pm d}\right)^2=\frac{b^2}{d^2}=\frac{ab}{cd}\Big(\therefore\frac{a}{c}=\frac{b}{d}$ and $\therefore$ $\frac{a}{c}\times\frac{b}{d}=\frac{b}{d}\times\frac{b}{d}$ or $\frac{ab}{cd}=\frac{b^2}{d^2}\Big)$

21. $\frac{a}{c}=\frac{b}{d}\therefore\frac{a+c}{c}=\frac{b+d}{d}$ or $\frac{a+c}{b+d}=\frac{c}{d}$ or $\frac{a}{b}\cdot\frac{a+c}{b+d}=\frac{c}{d}\cdot\frac{c}{d},\left(\text{for }\frac{a}{b}=\frac{c}{d}\right)$ or $\frac{a(a+c)}{b(b+d)}=\frac{c^2}{d^2}$ or $\frac{a(a+c)}{c^2}=\frac{b(b+d)}{d^2}$

22. $\frac{a}{b}=\frac{c}{d}\therefore\frac{a}{c}=\frac{b}{d}=x\therefore\frac{a^3}{c^3}=\frac{b^3}{d^3}=x^3$ $\therefore$ $a^3=c^3x^3$, $b^3=d^3x^3\therefore a^3+b^3$ $=(c^3+d^3)x^3$ or $x^3=\frac{a^3+b^3}{c^3+d^3}$ $\therefore$ x or $\frac{a}{c}=\frac{\sqrt[3]{(a^3+b^3)}}{\sqrt[3]{(c^3+d^3)}}$

23. $\frac{a}{b}=\frac{c}{d}\therefore\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (componendo) $\therefore$ $\frac{a+b}{c+d}=\frac{a-b}{c-d}$ (alternendo) $\therefore\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$ (componendo) $\therefore$ $(a+b+c+d)(a-b-c+d)=(a+b-c-d)(a-b+c-d)$.

25. $\frac{x}{y}=\frac{4}{z}\therefore\frac{7}{9}\frac{x}{y}=\frac{7}{9}\frac{y}{z}$ or $\frac{7x}{9y}=\frac{7y}{9z}\therefore\frac{7x+9y}{7x-9y}=\frac{7y+9z}{7y-9z}$ (componendo and dividendo) $\therefore$ $\frac{7x+9y}{7y+9z}=\frac{7x-9y}{7y-9z}$ (alternendo).

26. $(y+z)^2=y^2+2yz+z^2=xz+2yz+z^2$ (for $y^2=xz$) $=z(x+2y+z)$ $\therefore$ $\frac{(y+z)^2}{z}=\frac{x+2y+z}{1}=\frac{x+2y+z}{x}$.

27. The right hand side expression $=(x^{2P}+z^{2P})^2-(y^{2P})^2=x^{4P}$ $+2(xz)^{2P}+z^{4P}-y^{4P}=x^{4P}+2(y^2)^{2P}+z^{4P}-y^{4P}=x^{4P}+y^{4P}+z^{4P}$.

28. $y^2 = xz \therefore y^4 = x^2z^2 \therefore y^4(x^{-2} - y^{-2} + z^{-2}) = y^4(x^{-2} + z^{-2}) - y^4 \cdot y^{-2}$ $= x^2z^2(x^{-2} + z^{-2}) - y^2 = z^2 + x^2 - y^2$; again $y^2 = xz \therefore \frac{1}{y^2} = \frac{1}{xz}$ or y^{-2} $= x^{-1}z^{-1} \therefore y^{-2}(x + y + z) = y^{-2}(x + z) + y^{-2} \cdot y = x^{-1}z^{-1}(x + z) + y^{-1} = z^{-1}$ $+ x^{-1} + y^{-1}$; also $y^4 = x^2z^2 \therefore \frac{1}{y^4} = \frac{1}{x^2z^2}$ or $y^{-4} = x^{-2}z^{-2} \therefore y^{-4}(x^2 + y^2$ $+ z^2) = y^{-4}(x^2 + z^2) + y^{-2} = x^{-2}z^{-2}(x^2 + z^2) + y^{-2} = x^{-2} + z^{-2} + y^{-2}$; lastly $y^2 = xz \therefore y^{2c} = x^cz^c \therefore \frac{1}{y^{2c}} = \frac{1}{x^cz^c}$ or $y^{-2c} = x^{-c}z^{-c} \therefore y^{-2c}(x^c + y^c + z^c)$ $= y^{-2c}(x^c + z^c) + y^{-2c} \cdot y^c = x^{-c}z^{-c}(x^c + z^c) + y^{-c} = z^{-c} + x^{-c} + y^{-c}$.

29. $(x^2 + y^2)(y^2 + z^2) = y^4 + x^2y^2 + y^2z^2 + x^2z^2 = y^4 + x^2y^2 + y^2z^2 + y^4$ $(\because y^2 = xz) = x^2y^2 + 2y^4 + y^2z^2 = y^2(x^2 + 2y^2 + z^2) = y^2(x^2 + 2xz + z^2)$ $= y^2(x + z)^2 = (xy + yz)^2$.

30. $\frac{x}{y} = \frac{z}{w},\ \frac{m}{n} = \frac{p}{q} \therefore \frac{x}{y} \times \frac{m}{n} = \frac{z}{w} \times \frac{p}{q}$ or $\frac{mx}{ny} = \frac{pz}{qw}$

31. $\frac{x}{y} = \frac{z}{w}$ and $\frac{n}{m} = \frac{q}{p}$ then $\frac{nx}{my} = \frac{qz}{pw}$.

32. $\frac{xw}{qw} = \frac{yz}{py}$ or $\frac{x}{q} = \frac{z}{p}$.

34. $\frac{x}{y} = \frac{z}{w} = \frac{r}{s} \because \frac{z}{w} = \frac{r}{s} \therefore \frac{z}{w} \times \frac{r}{s} = \frac{r}{s} \times \frac{r}{s} = \frac{r^2}{s^2} = \frac{x^2}{y^2}$

35. Let $\frac{x}{y} = \frac{z}{w} = \frac{r}{s} = m \therefore \frac{x^2}{y^2} = \frac{z^2}{w^2} = \frac{r^2}{s^2} = m^2$ then proceed as in Ex. 33 p. 110.

37. $\frac{x}{y} = \frac{r}{s} \therefore \frac{x + r}{y + s} =$ each ratio $= \frac{z}{w} = \frac{mz}{mw} \therefore \frac{x - mz + r}{y - mw + s} =$ each ratio : again $\frac{mx}{my} = \frac{qr}{qs} \therefore \frac{mx + qr}{my + qs} =$ each ratio $= \frac{z}{w} = \frac{nz}{nw} \therefore \frac{mx + qr - nz}{my + qs - nw}$ $=$ each ratio $\therefore \frac{x - mz + r}{y - mw + s} = \frac{mx + qr - nz}{my + qs - nw} = \frac{x}{y}$.

38. $\frac{m^2}{n^2} = \frac{a}{b}$ but from the 2nd proportion $\frac{m^2}{n^2} = \frac{r^2 + a^2}{r^2 - b^2} \therefore \frac{a}{b} = \frac{r^2 + a^2}{r^2 - b^2}$

by cross multiplication $ar^2-ab^2=br^2+ba^2$ $\therefore$ $ar^2-br^2=ab^2+a^2b$ or $r^2(a-b)=ab(a+b)$.

39 $\left(\frac{a_1}{a_2}\right)^4=\frac{a_1}{a_2}\times\frac{a_1}{a_2}\times\frac{a_1}{a_2}\times\frac{a_1}{a_2}=\frac{a_1}{a_2}\times\frac{a_2}{a_3}\times\frac{a_3}{a_4}\times\frac{a_4}{a_5}=\frac{a_1}{a_5}$.

40. Equate the products of the extremes and means, then $8ax+12bx+10ay+15\ b2=8ax+10bx+12ay+15by$ or $2bx=2ay$ or $bx=ay$ or $\frac{a}{b}=\frac{x}{y}$.

40* Equate the products of the extremes and means, then $8ax+12bx+10ay+\ +15by=8ax+10bx+12ay+15by$ or $2bx=2ay$ or $bx=ay$ or $\frac{a}{b}=\frac{x}{y}$.

41. Let x and y be the extreme numbers then $\sqrt{(xy)}=$middle number then $x+y=20$, $\sqrt{(xy)}=8$ $\therefore$ $x^2+2xy+y^2=400$.

$$4xy \quad =256$$

$$\therefore x^2-2xy+y^2=144$$

$\therefore$ $x-y=12$ and $x+y=20$ $\therefore$ combining the 2 eqns we have $x=4, y=16$ and $\sqrt{(xy)}=8$.

43. $\frac{x}{y}=\frac{z}{w}\therefore\frac{x}{z}=\frac{y}{w}\therefore\frac{x}{mz}=\frac{y}{mw}\therefore\frac{x-mz}{mz}=\frac{y-mw}{mw}$ (Dividendo) $\therefore$ $\frac{x-mz}{y-mw}=\frac{mz}{mw}$ (alternendo) $=\frac{z}{w}\therefore\left(\frac{x-mz}{y-mw}\right)^2=\frac{z^2}{w^2}$ also $\frac{x^2}{z^2}=\frac{y^2}{w^2}\therefore\frac{x^2\pm z^2}{z^2}=\frac{y^2\pm w^2}{w^2}\therefore\frac{x^2\pm z^2}{y^2\pm w^2}=\frac{z^2}{w^2}\therefore\left(\frac{x-mz}{y-mw}\right)^2=\frac{x^2\pm z^2}{y^2\pm w^2}$.

44. $\frac{c}{a}=\frac{(a+c\ +c+a}{b+a+c-b}=\frac{2(a+c)}{a+c}=2$ $\therefore$ $c=2a$; substitute this in the 3rd then $\frac{a}{2a-b}=2$ or $a=4a-2b$ or $3a-2b=0$ $\therefore 3a=2b$ $\therefore\frac{a}{b}=\frac{2}{3}$ $\because\frac{a}{c}=\frac{1}{2}=\frac{2}{4}$ $\therefore$ a, b and c are as 2, 3 and 4.

45. $\frac{x}{y}=\frac{z}{w}$ $\therefore\frac{x}{z}=\frac{y}{w}$ or $\frac{x-z}{z}=\frac{y-w}{w}$ or $\frac{x-z}{y-w}=\frac{z}{w}=\frac{x}{y}$ $\therefore$ $\frac{(x-z)-(y-w)}{x-z}$

$= \frac{x-y}{x}$ (convertendo) $\therefore x-z-(y-w) = \frac{(x-z)(x-y)}{x}$ Again $\therefore \frac{x}{y} = \frac{z}{w}$

$\therefore \frac{x+y}{y} = \frac{z+w}{w}$ or $\frac{x+y}{z+w} = \frac{y}{w}$ (alternendo) or $\frac{(x+y)-(z+w)}{x+y} = \frac{y-w}{y}$

(convertendo) $\therefore (x+y) - (z+w) = \frac{(x+y)(y-w)}{y}$.

46. From the method followed in Ex 36 $\frac{x^2+y^2}{z^2+w^2} = \frac{xy}{zw} \therefore \frac{z^2+y^2}{xy}$

zw (alternendo) $\therefore \frac{x^2+y^2-xy}{xy} = \frac{z^2+w^2-zw}{zw}$ (Dividendo) $\therefore$

$\frac{5xy}{x^2-xy+y^2} = \frac{5zw}{z^2-zw+w^2}$ or $1 - \frac{5xy(x+y)}{(x+y)(x^2-xy+y^2)} = 1 -$

$\frac{5zw(z+w)}{(z+w)(z^2-zw+w^2)}$ or $1 - \frac{5x^2y+5xy^2}{x^3+y^3} = 1 - \frac{5z^2w+5zw^2}{z^3+w^3}$ or

$\frac{x^3-5x^2y-5xy^2+y^3}{x^3+y^3} = \frac{z^3-5z^2w-5zw^2+w^3}{z^3+w^3}$

47. $\frac{x+w}{xw} - \left(\frac{1}{y}+\frac{1}{z}\right) = \frac{x+w}{xw} - \frac{y+z}{yz} = \frac{x+w}{yz} - \frac{y+z}{yz} = \frac{x+w-y-z}{yz}$

$= \frac{x^2+xw-xy-xz}{xyz} = \frac{x^2+yz-xy-xz}{xyz} = \frac{(x-y)(x-z)}{xyz}$.

48. $\frac{mx}{ny} = \frac{mz}{nw} \therefore \frac{mx \pm ny}{ny} = \frac{mz \pm nw}{nw} \therefore \frac{mx \pm ny}{mz \pm nw} = \frac{ny}{nw} = \frac{y}{w}$

also $\frac{px}{qy} = \frac{pz}{qw} \therefore \frac{px \pm qy}{qy} = \frac{pz \pm qw}{qw} \therefore \frac{px \pm qy}{pz \pm qw} = \frac{qy}{qw} = \frac{y}{w}$

$\therefore \frac{mx \pm ny}{mz \pm nw} = \frac{px \pm qy}{pz \pm qw}$

49. $\frac{x^2}{y^2} = \frac{z^2}{w^2} \therefore \frac{x^2-y^2}{y^2} = \frac{z^2-w^2}{w^2}$ (dividendo) $\therefore \frac{x^2-y^2}{z^2-w^2} = \frac{y^2}{w^2}$

also $\frac{x^2}{z^2} = \frac{y^2}{w^2} \therefore \frac{x^2-z^2}{z^2} = \frac{y^2-w^2}{w^2} \therefore \frac{x^2-z^2}{y^2-w^2} = \frac{z^2}{w^2}$

$$\therefore\ \frac{x^2-z^2}{y^2-w^2}\times\frac{z^2-w^2}{x^2-y^2}=\frac{z^2}{w^2}\times\frac{w^2}{y^2}=\frac{z^2}{y^2}\ \therefore\ \frac{(x^2-z^2)(z^2-w^2)}{(y^2-w^2)(x^2-y^2)}=\frac{z^2}{y^2}.$$

$x(x+y+z+w)=x^2+xy+xz+xw=x^2+xy+xz+yz$ (for $xw=yz$) $=x(x+y)+z(x+y)=(x+z)(x+y)$.

50. $\dfrac{x^2+y^2}{y^2}=\dfrac{z^2+w^2}{w^2}$ or $\dfrac{x^2+y^2}{z^2+w^2}=\dfrac{y^2}{w^2}=\dfrac{y}{w}\times\dfrac{x}{z}=\dfrac{xy}{zw}\ \therefore\ \dfrac{x^2+y^2}{xy}=\dfrac{z^2+w^2}{zw}$

or $\dfrac{x^2+y^2-xy}{xy}=\dfrac{z^2-zw+w^2}{zw}$ or $\dfrac{x^2+y^2-xy}{z^2-zw+w^2}=\dfrac{xy}{zw}=\dfrac{x^2}{z^2}.$

51. By componendo $\dfrac{x+y}{y}=\dfrac{z+w}{w}\ \therefore\ \left(\dfrac{x+y}{y}+\dfrac{z+w}{w}\right)^2$

$$=\left\{\frac{2(x+y)}{y}\right\}^2=\frac{4(x+y)^2}{y^2}=\frac{4(x+y)}{y}\,\frac{(z+w)}{y}=\frac{4(x+y)(z+w)}{yw}$$

$$\therefore\ 4(x+y)(z+w)=yw\left\{\frac{x+y}{y}+\frac{z+w}{w}\right\}^2$$

52 $ad=bc\ \therefore\ c=\dfrac{bc}{d},\ ma=bc\dfrac{m}{d}\ \therefore\ \dfrac{1}{ma}=\dfrac{1}{bc}\cdot\dfrac{d}{m}$ again $\dfrac{1}{nb}=\dfrac{1}{bc}\cdot\dfrac{c}{n}.$

Also $\dfrac{1}{pc}=\dfrac{1}{bc}\cdot\dfrac{b}{p}$ and $\dfrac{1}{qd}=\dfrac{1}{ad}\cdot\dfrac{a}{q}=\dfrac{1}{bc}\cdot\dfrac{a}{q}\ \therefore\ \dfrac{1}{ma}+\dfrac{1}{nb}+\dfrac{1}{pc}+\dfrac{1}{qd}=\dfrac{1}{bc}$

$$\left\{\frac{a}{q}+\frac{b}{p}+\frac{c}{n}+\frac{d}{m}\right\}$$

53 $\dfrac{x^2}{ax}=\dfrac{y^2}{by}=\dfrac{z^2}{cz}\ \therefore\ \dfrac{x^2+y^2+z^2}{ax+by+cz}=$ each of the ratios.

54 We have to shew that $\dfrac{x^2+z^2+m^2}{y^2+w^2+n^2}=\left(\dfrac{xz+mz+mx}{yw+xw+ny}\right)\ \because\ \dfrac{x}{y}=\dfrac{z}{w}$
$=\dfrac{m}{n}\ \therefore\ \dfrac{x^2}{y^2}=\dfrac{z^2}{w^2}=\dfrac{m^2}{n^2}\ \therefore\ \dfrac{x^2+z^2+m^2}{y^2+w^2+n^2}=\dfrac{x^2}{y^2}$ also $\dfrac{x^2}{y^2}=\dfrac{x}{y}\times\dfrac{x}{y}=\dfrac{x}{y}\times\dfrac{z}{w}=\dfrac{xz}{yw}$

Similarly $\dfrac{x^2}{y^2}=\dfrac{mz}{nw}$ and also $=\dfrac{mx}{ny}\ \therefore\ \dfrac{xz}{yw}=\dfrac{mz}{nw}=\dfrac{mx}{ny}\ \therefore\ \dfrac{xz+mz+mx}{yw+nw+ny}=$ each

fraction $=\dfrac{x^2}{y^2}\ \therefore\ \dfrac{x^2+z^2+m^2}{y^2+w^2+n^2}=\dfrac{xz+mz+mx}{yw+nw+ny}.$

55. $\frac{x}{m}=\frac{y}{n}$ (alternendo) $\therefore \frac{x+m}{x-m}=\frac{y+n}{y-n}$ (comp & div), also $\frac{x^2}{m^2}=\frac{y^2}{n^2}$ $\therefore \frac{x^2+m^2}{x^4-m^2}=\frac{y^2+n^2}{y^2-n^2}$ (comp & div) $\therefore \frac{x+m}{x-m}\times\frac{x^2+m^2}{x^2-m^2}=\frac{y+n}{y-n}\times\frac{y^2+n^2}{y^2-n^2}$.

56. $\frac{x}{m}=\frac{y}{n} \therefore \frac{x^2}{m^2}=\frac{y^2}{n^2} \therefore \frac{px^2}{pm^2}=\frac{ty^2}{tn^2}$ also $\frac{x^2}{m^2}=\frac{x}{m}\times\frac{x}{m}=\frac{x}{m}\times\frac{y}{n}=\frac{xy}{mn}$ $\therefore \frac{px^2}{pm^2}=\frac{qxy}{qmn} \therefore \frac{px^2}{pm^2}=\frac{ty^2}{tn^2}=\frac{qxy}{qmn} \therefore \frac{pn^2+qxy+ty^2}{pm^2+qmn+tn^2}=\frac{x^2}{m^2}$, in the same manner $\frac{lx^2+sxy+ty^2}{lm^2+smn+tn^2}=\frac{x^2}{m^2} \therefore \frac{px^2+qxy+ty^2}{pm^2+qmn+tn^2}=\frac{lx^2+sxy+ty^2}{lm^2+smn+tn^2}$.

57. $nx=my \therefore$ the left hand expression $=\frac{nx}{x}-\frac{nx}{2y}-\frac{nx}{3m}+\frac{nx}{4n}=n-\frac{my}{2y}$ $-\frac{my}{3m}+\frac{x}{4}=n-\frac{m}{2}-\frac{y}{3}+\frac{x}{4}$.

58. $\frac{x}{m}=\frac{y}{n} \therefore \frac{x+m}{m}=\frac{y+n}{n} \therefore \frac{x+m}{y+n}=\frac{m}{n} \therefore \frac{mx(x+m)}{ny(y+n)}=\frac{xm^2}{yn^2}=\frac{x^3}{y^3}$ (for $\frac{x}{y}=\frac{m}{n}$) also $\frac{x+m}{x}=\frac{y+n}{y} \therefore \left(\frac{x+m}{y+n}\right)^3=\frac{x^3}{y^3} \therefore \frac{(x+m)^3}{(y+n)^3}=\frac{mx(x+m)}{ny(y+n)}$

59. $y : x :: n : m$ (Invertendo) $\therefore y+x : y-x :: n+m : n-m$ or $y+x :$ $n+m :: y-x : n-m$ (alternendo) $\frac{y+x}{xy} : \frac{n+m}{mn} :: \frac{y-x}{xy} : \frac{n-m}{mn}$.

60. $\frac{b+(a+c-b)}{(a+b)+(b+c-a)}=$ each ratio or $\frac{a+c}{2b+c}=\frac{a+b+c}{2a+b+2c}$ $\therefore \frac{(a+b+c)-(a+c)}{(2a+b+2c)-(2b+c)}=$ each ratio or $\frac{b}{2a+c-b}=$ each ratio $=\frac{b}{a+b}$ $\therefore 2a+c-b=a+b$ or $a+c=2b$ $\therefore \frac{b}{a+b}=\frac{(a+c)+b}{2(a+c)+b}=\frac{2b+b}{4b+b}=\frac{3b}{5b}=\frac{3}{5}$ $\therefore 5b=3a+3b$ or $2b=3a$ $\therefore \frac{a}{b}=\frac{2}{3}$; again $\because a=\frac{2}{3}b \therefore 2a=\frac{4}{3}b$ $\therefore \frac{a+b+c}{2a+b+2c}=\frac{3b}{\frac{4}{3}b+b+2c} \therefore \frac{3b}{\frac{4}{3}b+b+2c}=$ each ratio $=\frac{3}{5} \therefore 15b=4b$ $+3b+6c$ or $8b=6c$ $\therefore 4b=3c \therefore \frac{b}{c}=\frac{3}{4}$.

61. $\frac{a^3}{b^3}=\frac{ace}{bdf} \therefore \frac{a^{3n}}{b^{3n}}=\frac{a^nc^ne^n}{b^nd^nf^n} \therefore \frac{c^{3n}}{d^{3n}}=\frac{a^nc^ne^n}{b^nd^nf^n} \therefore \frac{a^{3n}-c^{3n}}{b^{3n}-d^{3n}}$

$=\frac{a^nc^ne^n}{b^nd^nf^n}$ also $\frac{a^n-c^n+e^n}{b^n-d^n+f^n}=\frac{a^n}{b^n} \therefore \left(\frac{a^n-c^n+e^n}{b^n-d^n+f^n}\right)^3=\frac{a^{3n}}{b^{3n}}$

$\therefore \frac{a^nc^ne^n}{b^nd^nf^n}=\left(\frac{a^n-c^n+e^n}{b^n-d^n+f^n}\right)^3 \therefore \frac{a^nc^ne^n-(a^n-c^n+e^n)^3}{b^nd^nf^n-(b^n-d^n+f^n)^3}=\frac{a^nc^ne^n}{b^nd^nf^n}.$

62. Let x and y be the extreme terms then x : mean term :: mean term : y ∴ mean term $=\sqrt(xy)$ then $x+\sqrt xy+y=14$, $x^2+xy+y^2=84$ dividing 2nd eqn by the 1st we have $x-\sqrt xy+y=6$ subtract this from the first then $2\sqrt(xy)=8$ ∴ $4xy=64$ ∴ $3xy=48$, subtract this from the 2nd, then $x^2-2xy+y^2=36$ ∴ $x-y=6$ and from the 1st eqn $x+y=14-\sqrt xy=14-4=10$ ∴ $2x=16$ ∴ $x=8$ ∴ $y=10-x=2$ ∴ $\sqrt(xy)=4$.

63. $a(a-2b+c)=a^2-2ab+ac=a^2-2ab+b^2=(a-b)^2$. Similarly the other case can be shewn.

64. $a^2+b^2+c^2=a^2+2b^2+c^2-b^2=a^2+2ac+c^2-b^2=(a+c)^2-b^2$ $=(a+c+b)(a+c-b) \therefore \frac{a+b+c}{a^2+b^2+c^2}=\frac{1}{a+c-b} \therefore \frac{(a+b+c)^2}{a^2+b^2+c^2}$ $=\frac{a+b+c}{a+c-b}.$

65. $\frac{a}{c}=\frac{4a^2}{4ac}=\frac{4a^2}{b^2}$ ($\because b^2=4ac$).

66. $\frac{a}{x}=\frac{11+7}{11-7}$ (comp. & divi.) $=\frac{18}{4}=\frac{9}{2}$

67 Let $2x$ and $3x$ be the nos. then their sum $=5x$ and product $=6x^2$ ∴ $5x:6x^2::5:12$ ∴ $x=2$ ∴ required nos. are 4 and 6.

68. $\frac{a+b}{b}=\frac{b+c}{c}$ (comp) $\therefore \frac{a+b}{b+c}=\frac{b}{c}$ also from the 2nd proportion

$$\frac{b+c}{b}=\frac{c+d}{c} \therefore \frac{b+c}{c+d}=\frac{b}{c} \therefore \frac{a+b}{b+c}=\frac{b+c}{c+d}.$$

69. $\frac{a+c}{b+c}=\frac{a+\sqrt{ab}}{b+\sqrt{ab}}=\frac{\sqrt a(\sqrt a+\sqrt b)}{\sqrt b(\sqrt a+\sqrt b)}=\frac{\sqrt a}{\sqrt b} \therefore \frac{a}{b}$ is duplicate of $\frac{a+c}{b+c}$.

70. Let x = one part then $n-x$ is the other part

$$\therefore\ x : n-x :: n : 1$$

$$x = n^2 - nx \ \therefore\ (n+1)x = n^2 \ \therefore\ x = \frac{n^2}{n+1}.$$

71. Let x = required no, then $x+1 : x+3 :: 2 : 7 \ \therefore\ 7(x+1) = 2(x+3) \ \therefore\ x = -\frac{1}{5}$.

72. Let a, b, c and d be the 4 nos which are proportional then $\frac{a}{b} = \frac{c}{d}$ also by the question $\frac{c}{b} = \frac{b}{d}$ or $\frac{b}{c} = \frac{d}{b} \ \therefore\ \frac{a}{b} \times \frac{b}{c} = \frac{c}{d} \times \frac{d}{b}$ or $\frac{a}{c} = \frac{c}{b}$ $\therefore$ c is a mean proportional between a and b.

73. $\frac{a}{b} = \frac{e}{g} \ \therefore\ \frac{a}{e} = \frac{b}{g} \ \therefore\ \frac{a-e}{e} = \frac{b-g}{g} \ \therefore\ \frac{a-e}{b-g} = \frac{e}{g} = \frac{c}{d}$

74. $\frac{a}{b} = \frac{c}{d} = \frac{e}{g} \ \therefore\ \frac{ma}{mb} = \frac{nc}{nd} = \frac{re}{rg} \ \therefore\ \frac{ma+nc+re}{mb+nd+rg} = \frac{ma}{mb} = \frac{a}{b}$ Similarly

$$\frac{pa+qc+le}{pb+qd+lg} = \frac{a}{b}.$$

75. $\frac{a^2}{b^2} = \frac{c^2}{d^2} \ \therefore\ \frac{a^2-b^2}{b^2} = \frac{c^2-d^2}{d^2} \ \therefore\ \frac{a^2-b^2}{c^2-d^2} = \frac{b^2}{d^2}$ also $\frac{a}{mb} = \frac{c}{md}$

$$\therefore\ \frac{a+mb}{mb} = \frac{c+md}{md} \ \therefore\ \frac{a+mb}{c+md} = \frac{mb}{md} = \frac{b}{d} \ \therefore\ \left(\frac{a+mb}{c+md}\right)^2 = \frac{b^2}{d^2}.$$

76. Product of the extremes = product of the means.

$$\therefore\ (a^2+c^2)(b^2+d^2) = (ab+cd)^2 \text{ whence } ad = bc \ \therefore\ \frac{a}{b} = \frac{c}{d}.$$

77. $\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$ or $\frac{a^3}{b^3} = \frac{a}{d}$.

78. $np = mq \ \therefore$ the left hand expression $= \frac{m^2 - mn - pm + np}{m}$

$$= m - n - p + \frac{mq}{m} = m - n - p + q.$$

79. By the equation $\frac{a}{b} = \frac{(a+c)^2}{(b+c)^2}$, and by cross multiplication

$$ab^2+2abc+ac^2=a^2b+2abc+bc^2 \quad \therefore \quad ab^2-a^2b=bc^2-ac^2$$

or $ab(b-a)=c^2(b-a) \quad \therefore \quad ab=c^2$ or $\frac{a}{c}=\frac{c}{b}$.

80. $\frac{x^2}{y^2}=\frac{m}{n}$ or $\frac{x^2}{m}=\frac{y^2}{n} \therefore \frac{x^2+y^2}{y^2}=\frac{m+n}{n} \therefore \frac{x^2+y^2}{m+n}=\frac{y^2}{n}=\frac{x^2}{m}$

also $\frac{x^4}{y^4}=\frac{m^2}{n^2} \therefore \frac{x^4+y^4}{y^4}=\frac{m^2+n^2}{n^2}$ or $\frac{x^4+y^4}{m^2+n^2}=\frac{y^4}{n^2}=\frac{x^4}{m^2}$

$$\therefore \frac{x^2+y^2}{m+n}\,\frac{x^4+y^4}{m^2+n^2}=\frac{x^6}{m^3}$$

81. Each ratio $=\frac{\text{sum of all the antecedents}}{\text{sum of all the consequents}}$

$=\frac{7x-6y+7y-6z+7z-6x}{8a+9b+8b+9c+8c+9a}=\frac{x+y+z}{17(a+b+c)}$ now if the question were put in the form $\frac{x+y+z}{17(a+b+c)}=\frac{95x+59y-110z}{172a+337b+239c}$ it is evident that the right hand side of the equation would be equal to the sum of different multiples of the given ratios; let $\frac{m(7x-6y)}{m(8a+9b)}=\frac{n(7y-6z)}{n(8b+9c)}=\frac{p(7z-6x)}{p(8c+9a)}$ then $m(7x-6y)+n(7y-6z)+p(7z-6x)=95x+59y-110z$, equate the coefficients of x, y and z from both sides $\therefore 7m-6p=95$, $-6m+7n=59$, $7p-6n=-110$ whence $m=17$, $n=23$, $p=4 \therefore$ each of the given

$$\text{ratios}=\frac{17(7x-6y)}{17(8a+9b)}=\frac{23(7y-6z)}{23(8b+9c)}=\frac{4(7z-6x)}{4(8c+9a)}$$

$$=\frac{17(7x-6y)+23(7y-6z)+4(7z-6x)}{17(8a+9b)+23(8b+9c)+4(8c+9a)}=\frac{95x+59y-110z}{172a+337b+239c}$$

$$\therefore \frac{x+y+z}{17(a+b+c)}=\frac{95x+59y-110z}{172a+337b+239c)}$$

82 Each of the given ratios $=\frac{\text{sum of all the antecedents}}{\text{sum of all the consequents}}$

$=\frac{5(x+y+z)}{5(a+b+c)}=\frac{x+y+z}{a+b+c}$ also as we do not know yet what multiples of the ratios have been taken, let us assume $\frac{m(3x+3y-z)}{m(5a+b-c)}=\frac{n(3z+3x-y)}{n(5c+a-b)}$ $=\frac{p(3y+3z-x)}{p(5b+c-a)}$ ∴ each of the given ratios $=\frac{m(3x+3y-z)+n(3z+3x-y)+p(3y+3z-x)}{m(5a+b-c)+n(5c+a-b)+p(5b+c-a)}$ ∴ $m(3x+3y-z)$ $+n(3z+3x-y)+p(3y+3z-x)=6x+10y+14z$ ∴ equating the coefficients of x, y and z we have $m=1$, $n=2$, $p=3$ ∴ proceeding as in the previous example we will arrive at the required result.

83. Subtract the 2nd eqn from 1st we have $z-y=2(y-z)=2y-2z$ ∴ $3z=3y$ ∴ $z=y$.; again subtract the 3rd eqn from 2nd then $x-z$ $=2(z-x)=2z-2x$ ∴ $3x=3z$ ∴ $x=z$ ∴ $x=y=z$.

84. $\frac{\text{Sum of all the numerators}}{\text{Sum of all the denominators}}=$ each given fraction

∴ $\frac{x+y+z}{3(m+n+p)}=$ each fraction; hence we infer that we have to shew $\frac{x+y+z}{3(m+n+p)}=\frac{7(x+2y+3z)}{41m+38n+47p}$. the 2nd side of this eqn must be formed by taking the sum of certain multiples of the numerators for the numerator and the sum of the same multiples of the denominators for the denominator, let us assume $\frac{a(2x-y)}{a(2m+n)}=\frac{b(2y-z)}{b(2n+p)}=\frac{c(2z-x)}{c(2p+m)}$

∴ $a(2x-y)+b(2y-z)+c(2z-x)=7(x+2y+3z)=7x+14y+21z$ ∴ equating the coefficients of x, y and z from both sides we have $2a-c$ $=7$, $2b-a=14$ and $2c-b=21$ ∴ $4c-2b=42$ and $2b-a=14$ adding these we have $4c-a=56$ or $8c-2a=112$ and $2a-c=7$ ∴ $7c=119$ ∴ $c=17$ but $2a-c=7$ ∴ $2a=7+c=24$ ∴ $a=12$ and ∴ $b=13$.

85. By cross multiplication $w+wxy-z-xyz=x-y+xwz-wyz$

$\therefore \ y - z - xwz + wxy = x + xyz - w - wyz \ \therefore \ (y - z)(1 + xw) = (x - w)(1 + yz) \ \therefore \frac{x - w}{y - z} = \frac{1 + xw}{1 + yz}$

86. Proceed as in Ex. 81.

87. $\frac{a^2}{b^2} = \frac{c^2}{d^2} \ \therefore \frac{a^2 + b^2}{b^2} = \frac{c^2 + d^2}{d^2} \ \therefore \frac{a^2 + b^2}{c^2 + d^2} = \frac{b^2}{d^2} = \frac{b}{d} \cdot \ \frac{b}{d} = \frac{b}{d}$

$\left(\text{for } \frac{b}{d} = \frac{a}{c}\right) = \frac{ab}{cd} \ \therefore \ \frac{}{ab} = \frac{}{cd} \ \therefore \ \frac{}{ab} = \frac{}{cd}$

$\frac{a^2 + b^2 - ab}{c^2 + d^2 - cd} = \frac{ab}{cd}$ also $\frac{ab}{cd} = \frac{ab + ad - bc}{cd - ad + bc}$ (in the numerator $ad - bc = 0$ $ad = bc$ and so in the denominator) $\therefore \ \frac{a^2 + b^2 - ab}{c^2 + d^2 - cd}$ $= \frac{ab + ad - bc}{cd - ad + bc}$, then use alternendo.

88. $\frac{a}{b} = \frac{a - c}{b - d} \ \therefore \frac{a - c}{b - d} = \frac{e}{f} \ \therefore \ \frac{a - c - e}{b - d - f} = \frac{e}{f} = \frac{a}{b}$ also $\frac{ma}{mb} = \frac{nc}{nd}$

$\therefore \ \frac{ma - nc}{mb - nd} = \frac{ma}{mb} = \frac{a}{b} = \frac{e}{f} = \frac{pe}{pf} \ \therefore \frac{ma - nc - pe}{mb - nd - pf}$ = each of the ratios $= \frac{pe}{pf} = \frac{e}{f} = \frac{a}{b} \therefore \ \frac{a - c - e}{b - d - f} = \frac{ma - nc - pe}{mb - nd - pf} = \frac{a}{b}$.

89. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ \therefore \ \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{c}}{\sqrt{d}} = \frac{\sqrt{e}}{\sqrt{f}} \ \therefore \ \frac{\sqrt{a} + \sqrt{c} + \sqrt{e}}{\sqrt{b} + \sqrt{d} + \sqrt{f}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\therefore \left(\frac{\sqrt{a} + \sqrt{c} + \sqrt{e}}{\sqrt{b} + \sqrt{d} + \sqrt{f}}\right)^3 = \frac{a^{\frac{3}{2}}}{b^{\frac{3}{2}}}$ again $\frac{a^3}{b^3} = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{ace}{bdf}$

$\therefore \ \frac{a^{\frac{3}{2}}}{b^{\frac{3}{2}}} = \frac{(ace)^{\frac{1}{2}}}{(bdf)^{\frac{1}{2}}} \ \therefore \ \frac{a^{\frac{3}{2}} - (ace)^{\frac{1}{2}}}{b^{\frac{3}{2}} - (bdf)^{\frac{1}{2}}} = \frac{a^{\frac{3}{2}}}{b^{\frac{3}{2}}} = \frac{e^{\frac{3}{2}}}{f^{\frac{3}{2}}} \ \therefore \ \frac{a^{\frac{3}{2}} - (ace)^{\frac{1}{2}} + e^{\frac{3}{2}}}{b^{\frac{3}{2}} - (bdf)^{\frac{1}{2}} + f^{\frac{3}{2}}}$

$\frac{e^{\frac{3}{2}}}{f^{\frac{3}{2}}} = \frac{a^{\frac{3}{2}}}{b^{\frac{3}{2}}}$

90. $\frac{a+b}{b}=\frac{c+d}{d}$ $\therefore\frac{a+b}{c+d}=\frac{b}{d}=\frac{a}{c}$, also $\frac{a-b}{b}=\frac{}{d}$ $\therefore\frac{}{a-b}=\frac{}{b}$

$$\therefore\frac{a^2(c-d)}{c^2(a-b)}=\frac{a^2d}{c^2b}=\frac{a.\ ad}{c.\ bc}=\frac{a.\ bc}{c.\ bc}\ (\because\ ad=bc\ )=\frac{a}{c}.$$

91. $\frac{a}{b}=\frac{c}{d}=\frac{m}{n}\therefore\frac{a^2}{b^2}=\frac{c^2}{d^2}=\frac{m^2}{n^2}\ \therefore\frac{a^2+c^2+m^2}{b^2+d^2+n^2}=\frac{a^2}{b^2}$

$$\therefore a^2+c^2+m^2=\frac{a^2}{b^2}\ (b^2+d^2+n^2)\therefore(a^2+c^2+m^2)(b^2+d^2+n^2)=\frac{a^2}{b^2}$$

$$(b^2+d^2+n^2)^2=\left\{\frac{a}{b}(\ b^2+d^2+n^2\ )\right\}^2=\left\{\frac{a}{b}.\ b^2+\frac{c}{d}.\ d^2+\frac{m}{n}.\ n^2\right\}^2$$

$=(ab+cd+mn)^2$. The first part of the question may be deduced from the 2nd by putting $\sqrt{a}$ for a, $\sqrt{b}$ for b, $\sqrt{c}$ for c &c. then

$$(a+c+m)(b+d+n)=(\sqrt{(ab)}+\sqrt{(cd)}+\sqrt{(mn)})^2 \text{ or}$$

$$\sqrt{(a+c+m)\ (b+d+n)}=\sqrt{ab}+\sqrt{cd}+\sqrt{mn}.$$

92. $\frac{(ax+by)+(cz+ax)+(by+cz)}{cz+by+ax}=$ each of the given ratios

$$=x+y+z\ \therefore\ \frac{2(ax+by+cz)}{ax+by+cz}=x+y+z \text{ or } x+y+z=2\cdots(1)$$

$$\therefore\ \frac{ax+by}{cz}=2\ \therefore\frac{ax+by}{cz}+1=2+1=3 \text{ or}\frac{ax+by+cz}{cz}=3$$

Similarly $\frac{ax+by+cz}{by}=3$ and $\frac{ax+by+cz}{ax}=3$

$$\therefore\ \frac{ax+by+cz}{cz}=\frac{ax+by+cz}{by}=\frac{ax+by+cz}{ax} \text{ or } cz=by=ax$$

$\therefore\ x=\frac{by}{a}$ and $z=\frac{by}{c}$; substitute these values in (1) then

$$\frac{by}{a}+y+\frac{by}{c}=2\ \therefore\ y=\frac{2ac}{ab+ac+bc}\ \therefore x=\frac{by}{a}=\frac{b}{a}.\frac{2ac}{ab+ac+bc}$$

$$=\frac{2bc}{ab+ac+bc} \text{ and } z=\frac{by}{c}=\frac{b}{c}.\frac{2ac}{ab+ac+bc}=\frac{2ac}{ab+ac+bc}.$$

93. Let $x =$ the time the clock pointed to ; then

$$x : x + 6\tfrac{1}{3} :: 29 : 105 ;$$
$$\therefore x + 6\tfrac{1}{3} : x :: 105 : 29$$
$$\therefore 6\tfrac{1}{3} : x :: 76 : 29 \quad \text{(dividendo)}$$
$\therefore x = 2$ hrs. 25 min. ; $\therefore$ true time $= 6$ hrs. 20 min. $+$ 2 hrs. 25 min. $= 8$ hrs. 45 min.

94. Let $x =$ speed of the slow train in miles per hour then $\frac{x}{60 \times 60} =$ rate of the same in miles per second then in 2 seconds on the 1st supposition this train passes over $\frac{2x}{60 \times 60}$ and the other $\frac{2 \times 24}{60 \times 60} \cdot \frac{2r}{60 \times 60} + \frac{2 \times 24}{60 \times 60}$ $=$ length of the quicker train which passes before the observers eye. Again on the 2nd supposition $30\left(\frac{24}{60 \times 60} - \frac{x}{60 \times 60}\right) =$ length of the quicker train $\therefore 30\left(\frac{24}{60 \times 60} - \frac{x}{60 \times 60}\right) = \frac{2x}{60 \times 60} + \frac{2 \times 24}{60 \times 60}$ $\therefore 30(24 - x) = 2(x + 24)$ $\therefore x = 21$ and the length of the quicker train $= \frac{2(21 + 24)}{60 \times 60} = \frac{1}{40}$ of a mile $= \frac{5280}{40}$ ft $= 132$ ft.

95. Let $2x =$ distance from A to B ; $\therefore 3x =$ distance from C to D and $\frac{x}{2} + \frac{3x}{2} = 2x) = 3$ B C ; $\therefore$ B C $= \frac{2}{3}x$ and $(2x + 3x + \frac{2}{3}x = \frac{17x}{3}) = 34$ $\therefore x - 6$ $\therefore$ A B $= 12$, B C $= 4$ and C D $= 18$.

96. Let $x =$ rate of A in miles per hour and $y =$ rate of B in miles per hour, then when both were proceeding in the same direction their rate of approach was $x - y$ miles in an hour and when B turned his course and was moving in the opposite direction, the rate of approach was $x + y$ miles in an hour and by the question $x + y = 3(x - y)$ or $4y = 2x \therefore \frac{x}{y} = \frac{2}{1}$.

97. Let $x =$ no of miles A travelled $\therefore x - 18 =$ no B travelled and $x - 18 \cdot x :: 15\tfrac{3}{4}$: no of hours A travelled $= \frac{63x}{4(x - 18)}$ also x :

$x - 18 :: 28 :$ no of hours B travelled $= \frac{28(x-18)}{x} \therefore \frac{28(x-18)}{x}$

$$= \frac{63x}{4(x-18)} \therefore x = 72.$$

98. Let $x =$ no of Rs. that each yd cost then $\frac{5}{x} =$ length in yds of the least and $\frac{6\frac{1}{2}}{x} =$ length of the longest also $\frac{5}{x} + 10 : \frac{6\frac{1}{2}}{x} + 10 :: 5 : 6$ $\therefore 6(\frac{5}{x} + 10) = 5(\frac{13}{2x} + 10)$ whence $x = \frac{1}{4}$ Rs. $= 4$ as.

Ex. 28. IDENTITIES.

1. The left hand expression $= a^2x^2 + b^2x^2 + a^2y^2 + b^2y^2 = (a^2x^2 + 2abxy + b^2y^2) + (b^2x^2 - 2abxy + a^2y^2) = (ax + by)^2 + (bx - ay)^2$.

2. $x^2 + 2xy + y^2 + y^2 + 2yz + z^2 + x^2 + 2xz + z^2 = (x^2 + y^2 + z^2 + 2xy + 2xz + 2yz) + x^2 + y^2 + z^2 = (x + y + z)^2 + x^2 + y^2 + z^2$.

3. The left hand expression $= x\{1 - y^2 - z^2 + y^2z^2\} + y\{1 - z^2 - x^2 + x^2z^2\} + z\{1 - x^2 - y^2 + x^2y^2\} - 4xyz = x - xy^2 - xz^2 + xy^2z^2 + y - yz^2 - x^2y + x^2yz^2 + z - x^2z - y^2z + x^2y^2z - 4xyz = (x + y + z - xyz) - (x^2y + xy^2 + xyz - x^2y^2z) - (xyz + y^2z + yz^2 - xy^2z^2) - x^2z + xyz + xz^2 - x^2yz^2) = (x + y + z - xyz) - xy(x + y + z - xyz) - yz(x + y + z - xyz) - xz(x + y + z - xyz) = (x + y + z - xyz)(1 - xy - yz - xz)$.

4. $(x + y + z)^3 = (x + y)^3 + z^3 + 3(x + y)z(x + y + z) = x^3 + y^3 + 3xy(x + y) + z^3 + 3(x + y)z(x + y + z) = x^3 + y^3 + z^3 + 3(x + y)\{xy + z(x + y + z)\}$ $= x^3 + y^3 + z^3 + 3(x + y)\{x(y + z) + z(y + z)\} = x^3 + y^3 + z^3 + 3(x + y)(x + z)(y + z)$. Again $(x + y + z)^3 = x^3 + 3x^2(y + z) + 3x(y + z)^2 + (y + z)^3 = x^3 + 3x^2(y + z) + 3x(y^2 + 2yz + z^2) + y^3 + 3y^2z + 3yz^2 + z^3 = x^3 + y^3 + z^3 + 3x^2(y + z) + 3y^2(x + z) + 3z^2(x + y) + 6xyz$.

5. $$\frac{x^4(y^2 - z^2) + y^4(z^2 - x^2) - z^4\{(y^2 - z^2) + (z^2 - x^2)\}}{x^2(y - z) + y^2(z - x) - z^2\{(y - z) + (z - x)\}}$$

$$= \frac{(y^2 - z^2)(x^4 - z^4) + (z^2 - x^2)(y^4 - z^4)}{(y - z)(x^2 - z^2) + (z - x)(y^2 - z^2)}$$

$$= \frac{(y^2 - z^2)(x^4 - z^4) - (x^2 - z^2)(y^4 - z^4)}{(y - z)(x^2 - z^2) - (x - z)(y^2 - z^2)}$$

$$= \frac{(y^2 - z^2)(x^2 - z^2)\{x^2 + z^2 - y^2 - z^2\}}{(y - z)(x - z)\{x + z - y - z\}}$$

$$= \frac{(y^2 - z^2)(x^2 - z^2)(x^2 - y^2)}{(y - z)(x - z)(x - y)} = (y + z)(x + z)(x + y).$$

6. The left side expression $= (a - b)\{x^2 - (a + b)x + ab\} + (b - c)$ $\{x^2 - (b + c)x + bc\} + (c - a)\{x^2 - (c + a)x + ac\} = (a - b + b - c + c - a)$ $x^2 - (a^2 - b^2 + b^2 - c^2 + c^2 - a^2)x + (a - b)ab + (b - c)bc + (c - a)ac$ $= (a - b)ab + (b - c)bc + (c - a)ac = (a - b)ab + b^2c - bc^2 + ac^2 - a^2c$ $= (a - b\ ab + (a - b)c^2 - (a^2 - b^2)c = (a - b)\{ab + c^2 - ac - bc\} = (a - b)$ $\{b(a - c) - c(a - c)\} = (a - b)(b - c)(a - c).$

7. Let $x + 1$ and x be two consecutive nos, then $(x + 1)^2 - x^2 = x^2$ $+ 2x + 1 - x^2 = 2x + 1 = (x + 1) + x.$

8. The left hand expression $= \dfrac{(x + y)(y + z)(x + z)}{xyz} + 1$

$$= \frac{xy^2 + x^2y + xyz + x^2z + y^2z + xyz + yz^2 + xz^2}{xyz} + 1$$

$$= \frac{xy^2 + xy^2 + xyz + x^2z + y^2z + xyz + yz^2 + xz^2 + xyz}{xyz}$$

$$= \frac{x(xy + yz + xz) + y(xy + yz + xz) + z(xy + yz + xz)}{xyz}$$

$$= (x + y + z)\left\{\frac{xy + yz + xz}{xyz}\right\} = (x + y + z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

9. $$\frac{x^2\left(\frac{1}{y} - \frac{1}{z}\right) + y^2\left(\frac{1}{z} - \frac{1}{x}\right) - z^2\left\{\left(\frac{1}{y} - \frac{1}{z}\right) + \left(\frac{1}{z} - \frac{1}{x}\right)\right\}}{x\left(\frac{1}{y} - \frac{1}{z}\right) + y\left(\frac{1}{z} - \frac{1}{x}\right) - z\left\{\left(\frac{1}{y} - \frac{1}{z}\right) + \left(\frac{1}{z} - \frac{1}{x}\right)\right\}}$$

$$= \frac{x^2\left(\frac{1}{y} - \frac{1}{z}\right) + y^2\left(\frac{1}{z} - \frac{1}{x}\right) - z^2\left(\frac{1}{y} - \frac{1}{z}\right) - z^2\left(\frac{1}{z} - \frac{1}{x}\right)}{x\left(\frac{1}{y} - \frac{1}{z}\right) + y\left(\frac{1}{z} - \frac{1}{x}\right) - z\left(\frac{1}{y} - \frac{1}{z}\right) - z\left(\frac{1}{z} - \frac{1}{x}\right)}$$

$$\frac{\left(\frac{1}{y}-\frac{1}{z}\right)(x^2-z^2)+\left(\frac{1}{z}-\frac{1}{x}\right)(y^2-z^2)}{\left(\frac{1}{y}-\frac{1}{z}\right)(x-y)+\left(\frac{1}{z}-\frac{1}{x}\right)(y-z)}=\frac{\frac{(z-y)(x^2-z^2)}{yz}+\frac{(x-z)(y^2-z^2)}{xz}}{\frac{(z-x)(x-z)}{yz}+\frac{(x-z)(y-z)}{xz}}$$

$$=\frac{x(z-y)(x^2-z^2)-y(x-z)(z^2-y^2)}{x(z-y)(x-z)-y(x-z)(z-y)}=\frac{(z-y)(x-z)\{x^2+xz-yz-x^2\}}{(z-y)(x-z)\{x-y\}}=$$

$$=\frac{x^2+xz-yz-y^2}{x-y}=\frac{x^2+xz+xy-xy-yz-y^2}{x-y}$$

$$=\frac{(x^2+xy+xz)-(xy+y^2+yz)}{x-y}=\frac{x(x+y+z)-y(x+y+z)}{x-y}$$

$$=\frac{(x+y+z)(x-y)}{(x-y)}=x+y+z.$$

10. $a^2+b^2+c^2+2ab+2ac+2bc+\{a^2-(b+c)^2\}-2ab-2ac=2a^2$

11. Cube $a+b+c$, $b+c-a$, &c. and then add.

12. Proceed as in the foregoing.

13. Let $x-y=a$, $y-z=b$, $z-x=c$ then $a+b+c=0$ $\therefore a+b=-c$ $\therefore a^3+b^3+3ab(a+b)=-c^3$ but $a+b=-c$ $\therefore a^3+b^3+3ab(-c)=-c^3$ or $a^3+b^3+c^3=3abc$ or $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$.

14. $(m^2-np)^3+(n^2-mp)^3+(p^2-mn)^3-3(m^2-np)(n^2-mp)$
$(p^2-mn)=(m^2-np)^3-(m^2-np)(n^2-mp)(p^2-mn)+(n^2-mp)^3$
$-(m^2-np)(n^2-mp)(p^2-mn)+(p^2-mn)^3-(m^2-np)(n^2-mp)$
$(p^2-mn)=(m^2-np)\{(m^2-np)^2-(n^2-mp)(p^2-mn)\}$
$+(n^2-mp)\{(n^2-mp)^2-(m^2-np)(p^2-mn)\}+(p^2-mn)\{(p^2-mn)^2$
$-(m^2-np)(n^2-mp)\}=(m^2-np)\{(m^4-2m^2np+n^2p^2)-(n^2p^2-mp^3$
$-mn^3+m^2np)\}+(n^2-mp)\{(n^4-2n^2mp+m^2p^2)-(m^2p^2-np^3-m^3n$
$+n^2mp)\}+(p^2-mn)\{(p^4-2mnp^2+m^2n^2)-(m^2n^2-n^2p-m^3p+mnp^2)\}$
$=(m^2-np)m(m^3+n^3+p^3-3mnp)+(n^2-mp)n(m^3+n^3$
$+p^3-3mnp)+(p^2-mn)p(m^3+n^3+p^3-3mnp)$
$=(m^3+n^3+p^3-3mnp)(m^3+n^3+p^3-3mnp)$

15. Square $x+y$, $x+z$, &c. and arrange the results.

16. The left hand expression $= \{(16x^2+24xy+9y^2)+(36x^2-24xy+4y^2)\}\{16x^2+24xy+9y^2-(36x^2-24xy+4y^2)\}$
$=13(4x^2+y^2)(5y^2-20x^2+48xy)$

19. Let x, $x+1$, $x+2$, $x+3$ be 4 consecutive nos then $x(x+1)(x+2)(x+3)+1=\{(x^2+3x)+2\}(x^2+3x)+1=(x^2+3x)^2+2(x^2+3x)+1=(x^2+3x+1)^2$.

20. Let $m+n+p=x$ then $n+p=x-m$, $m+p=x-n$, $m+n=x-p$
$\therefore x^4-(x-m)^4-(x-n)^4-(x-p)^4+m^4+n^4+p^4=x^4-(x^4-4mx^3+6m^2x^2-4m^3x+m^4)-(x^4-4nx^3+6n^2x^2-4n^3x+n^4)-(x^4-4px^3+6p^2x^2-4p^3x+p^4)+m^4+n^4+p^4=x^4-3x^4+4x^3(m+n+p)-6x^2(m^2+n^2+p^2)+4x(m^3+n^3+p^3)=x^4-3x^4+4x^4-6x^2(m^2+n^2+p^2)+4x\{(m^2+n^2+p^2-mn-mp-np)(m+n+p)+3mnp\}=2x^4-2x^2(3m^2+3n^2+3p^2)+4x^2(m^2+n^2+p^2-mn-mp-np)+12mnpx=2x^4-2x^2(3m^2+3n^2+3p^2-2m^2-2n^2-2p^2+2mn+2mp+2np)+12mnpx=2x^4-2x^2(m^2+n^2+p^2+2mn+2mp+2np)+12mnpx=2x^4-2x^2(m^2+n^2+p^2+2mn+2mp+2np)+12mnpx=2x^4-2x^2(m+n+p)^2+12mnpx=2x^4-2x^4+12mnpx=12mnpx$.

21. The left hand expression $=\left(\frac{a^2}{b^2}+\frac{b^2}{a^4}+2\right)+\left(\frac{a^2}{c^2}+\frac{c^2}{a^2}+2\right)+\left(\frac{b}{c}+\frac{c}{b}\right)^2=4+\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)+\left(\frac{a^2}{c^2}+\frac{c^2}{a^2}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)^2$
$=4+\left(\frac{b^2+c^2}{a^2}\right)+a^2\left(\frac{1}{b^2}+\frac{1}{c^2}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)^2=4+\frac{b^2+c^2}{a^2}+\frac{a^2(b^2+c^2)}{b^2c^2}+\frac{(b^2+c^2)^2}{b^2c^2}=4+(b^2+c^2)\left\{\frac{1}{a^2}+\frac{a^2}{b^2c^2}+\frac{b^2+c^2}{b^2c^2}\right\}$
$=4+(b^2+c^2)\left\{\frac{b^2c^2+a^4+a^2b^2+a^2c^2}{a^2b^2c^2}\right\}=4+\frac{(b^2+c^2)(a^2+b^2)(a^2+c^2)}{bc.\ ab.\ ac}$.

22. In ex. 13 put x^2 for x, y^2 for y, z^2 for z then the numerator $=3(x^2-y^2)(y^2-z^2)(z^2-x^2)$ and the denominater $=3(x-y)(y-z)(z-x)$ $\therefore$ quotient $=(x+y)(y+z)(x+z)$.

23. Let $x-y=a$, $z-x=b$, $y-z=c$ $\therefore a+b+c=0$ $\therefore a+b=-c$ $\therefore a^2+2ab+b^2=c^2$ $\therefore a^2+b^2-c^2=-2ab$, again squaring we have $a^4+b^4+c^4+2a^2b^2-2a^2c^2-2b^2c^2=4a^2b^2$ $\therefore a^4+b^4+c^4=2a^2b^2+2a^2c^2+2b^2c^2$ or $(x-y)^4+(z-x)^4+(y-z)^4$ $=2\{(x-y)^2(z-x)^2+(x-y)^2(y-z)^2+(z-x)^2(y-z)^2\}$.

24. Let $m-n=x$, $n-p=y$, $p-m=z$ then from the foregoing ex. $x^4+y^4+z^4=2x^2y^2+2x^2z^2+2y^2z^2$ add $x^4+y^4+z^4$ to both sides then $2(x^4+y^4+z^4)=x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2$ $=(x^2+y^2+z^2)^2$ or $\{(m-n)^2+(n-p)^2+(p-m)^2\}^2$ $=2\{(m-n)^4+(n-p)^4+(p-m)^4\}$.

25. The left side expression $=x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6+y^7-x^7-y^7=7xy(x^5+3x^4y+5x^3y^2+5x^2y^3+3xy^4+y^5)=7xy(x+y)(x^4+2x^3y+3x^2y^2+2xy^3+y^4)$ $=7xy(x+y)(x^2+xy+y^2)^2$.

26. $$\frac{1}{1-\frac{z}{x}}\cdot\frac{1}{1-\frac{y}{x}}+\frac{1}{1-\frac{x}{y}}\cdot\frac{1}{1-\frac{z}{y}}+\frac{1}{1-\frac{y}{z}}\cdot\frac{1}{1-\frac{x}{z}}=\frac{x}{x-z}\cdot\frac{x}{x-y}$$
$$+\frac{y}{y-x}\cdot\frac{y}{y-z}+\frac{z}{z-y}\cdot\frac{z}{z-x}=\frac{x^2}{(x-z)(x-y)}+\frac{y^2}{(y-x)(y-z)}$$
$$\frac{z^2}{(z-y)(z-x)}=1$$ (Ex. 67 p. 43)

28. Numerator $=2^{3x+1}+2.\ 2^{3x}-(2^4)^{\frac{3x}{4}}=2^{3x+1}+2^{3x+1}-2^{3x}$ $=2\cdot2^{3x+1}-2^{3x}=2^{3x+2}-2^{3x}=2^{3x}(2^2-1)=3\cdot2^{3x}$; denominator $=(2^2)^{\frac{3x}{4}}$ $+(2^5)^{\frac{3x}{5}}+2^{3x}=2^{3x}+2^{3x}+2^{3x}=3\cdot2^{3x}$ $\therefore$ fraction $=1$.

29. $(x+y+z)(x^3+y^3+z^3+xyz)=x^4+y^4+z^4+(x^3y+xy^3+xyz^2)+(x^2yz+y^3z+yz^3)+(x^3z+xy^2z+xz^3)$ $=x^4+y^4+z^4+xy(x^2+y^2+z^2)+yz(x^2+y^2+z^2)+xz(x^2+y^2+z^2)=x^4+y^4+z^4+(xy+yz+xz)(x^2+y^2+z^2)$.

30. $(m^2y^2-2mnxy+n^2x^2)+(p^2x^2-2mpxz+m^2z^2)+(n^2z^2-2npyz+p^2y^2)+m^2x^2+n^2y^2+p^2z^2+2mnxy+2mpxz+2npyz=(m^2x^2+m^2y^2$

$+m^2z^2)+(n^2x^2+n^2y^2+n^2z^2)+(p^2x^2+p^2y^2+p^2z^2=(m^2+n^2+p^2)$ $(x^2+y^2+z^2)$.

31. $$\left(\frac{1}{y-z}+\frac{1}{z-x}+\frac{1}{x-y}\right)^2=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}$$

$$+2\left\{\frac{1}{(y-z)(z-x)}+\frac{1}{(y-z)(x-y)}+\frac{1}{(z-x)(x-y)}\right\}=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}$$

$$+\frac{1}{(x-y)^2}+2\left\{\frac{x-y+z-x+y-z}{(y-z)(z-x)(x-y)}\right\}=\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}.$$

32. Let $y-z=a$, $z-x=b$ then $a+b=y-x=-(x-y)$ or $x-y$ $=-(a+b)$ $\therefore$ $25\{(y-z)^7+(z-x)^7+(x-y)^7\}\{(y-z)^3+(z-x)^3$ $+(x-y)^3\}=25\{a^7+b^7-(a+b)^7\}\{a^3+b^3-(a+b)^3\}=25\{(a+b)^7$ $-a^7-b^7\}\{(a+b)^3-a^3-b^3\}=25\{(a^2+ab+b^2)^2 7ab(a+b)\}$ $\{3ab(a+b)\}=25\times 21\times(a^2+ab+b^2)^2a^2b^2(a+b)^2$

also $21\{(y-z)^5+(z-x)^5+(x-y)^5\}^2=21\{a^5+b^5-(a+b)^5\}^2$ $=21\{(a+b)^5-a^5-b^5\}^2=21\{5(a+b)(a^2+ab+b^2)ab\}^2$ $=21\times 25(a+b)^2(a^2+ab+b^2)a^2b^2$.

$\therefore$ $25\{(y-z)^7+(z-x)^7+(x-y)^7\}\{(y-z)^3+(z-x)^3+(x-y)^3\}$ $=21\{(y-z)^5+(z-x)^5+(x-y)^5\}^2$.

33 $(1-x)(1+x)=1-x^2$. … . …product of 2 terms
$(1-x^2)(1+x^2)=1-x^4$…. …… ……… 3 terms
$(1-x^4)(1+x^4)=1-x^8$ …. ………………4 terms
$(1-x^8)(1+x^8)=1-x^{16}$… …… … ….. ..5 terms

and so on to $n+1$ terms From the above result it is seen that the powers of x are $2, 2^2, 2^3, 2^4$ &c ; now the product of 5 terms $=1-x^{2^4}$ $=1-x^{2^{(5-1)}}$ $\therefore$ it is evident that the product of $n+1$ terms $=1-x^{2^{(n+1)-1}}=1-x^{2^n}$.

34. The left side expression $=(mp+nq)^2x^2+(mq-np)^2y^2+(mp$ $+nq)^2x^2+(mq-np)y^2=\{(mp+nq)^2+(mq-np)^2\}(x^2+y^2)=(m^2p^2$ $+n^2q^2+m^2q^2+n^2p^2)(x^2+y^2)=(m^2+n^2)(p^2+q^2)(x^2+y^2)$.

35. The left side expression $=\{x^2+(y^2-z^2)\}\{x^2-(y^2-z^2)\}+\{y^2+(x^2-z^2)\}\{y^2-(x^2-z^2)\}+z^4-(x^2-y^2)^2=2z^2(x^2+y^2-z^2)+z^4-(x^2-y^2)^2=2z^2(x^2+y^2)-z^4-(x^2-y^2)^2=z^2\{(x+y)^2+(x-y)^2\}-z^4-(x^2-y^2)^2=z^2(x+y)^2-z^4+z^2(x-y)^2-(x^2-y^2)^2=(x+y)^2\{z^2-(x-y)^2\}-z^2\{z^2-(x-y)^2\}=\{(x+y)^2-z^2\}\{z^2-(x-y)^2\}=(x+y+z)(x+y-z)(x+z-y)(y+z-x)$.

Ex. 29. CONDITIONAL IDENTITIES.

1. $x+y=-z$ $\therefore x^3+y^3+3xy(x+y)=-z^3$ or $x^3+y^3-3xyz=-z^3$ $\therefore$ $x^3+y^3+z^3=3xyz$.

2. Divide the result of ex 1 by xyz $\therefore$ $\frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}=3$.

3. $x(y^2+yz+z^2)^2+y(z^2+xz+x^2)+z(x^2+xy+y^2)=xy^2+xyz+xz^2+yz^2+xyz+x^2y+x^2z+xyz+y^2z=x(y^2+z^2)+y(x^2+z^2)+z(x^2+y^2)+3xyz$......(1) but $x+y=-z$ $\therefore$ $x^2+2xy+y^2=z^2$ $\therefore$ $x^2+y^2=z^2-2xy$ similarly $y^2+z^2=x^2-2yz$ and $x^2+z^2=y^2-2xz$ $\therefore$ $(1)=x(x^2-2yz)+y(y^2-2xz)+z(z^2-2xy)+3xyz=x^3+y^3+z^3-3xyz=3xyz-3xyz$ (Ex. 1.) $=0$.

4. $x^3-xyz+y^3-xyz+z^3-xyz=x^3+y^3+z^3-3xyz=3xyz-3xyz=0$.

5. $x+y=-z$ $\because$ $(x+y)(x-y)=-(x-y)z$ $\therefore$ $x^2-y^2=-xz+yz$ $\therefore$ $x^2-yz=y^2-xz$ similarly $y+z=-x$ $\because$ $(y+z)(y-z)=-(y-z)x$ $\therefore$ $y^2-z^2=-xy+xz$ $\therefore$ $y^2-xz=z^2-xy$ $\therefore$ $x^2-yz=y^2-xz=z^2-xy$.

6. From ex 1 $x+y+z=3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$ $\therefore$ $(x+y+z)^3=27xyz$.

7. $\frac{x}{y+z}+\frac{z}{x+y}=\frac{2y}{x+z}$ $\therefore$ $\frac{x(x+y)+z(y+z)}{(y+z)(x+y)}=\frac{2y}{x+z}$ $\therefore$ $\{x^2+y(x+z)+z^2\}(x+z)=2y(x+y)(y+z)$ $\therefore$ $(x^2+z^2)(x+z)+y(x+z)^2=2y(x+y)(y+z)$ or $(x^2+z^2)(x+z)+y(x^2+z^2)+2xyz=2y(xy+y^2+xz+yz)$

$=2xy^2+2y^3+2xyz+2y^2z \;\therefore\; (x^2+z^2)(x+y+z)=2y^2(x+y+z)$ $\therefore\; (x^2+z^2-2y^2)(x+y+z)=0$ or $x^2+z^2-2y^2=0 \;\therefore\; x^2+z^2=2y^2$ or $x+y+z=0 \;\therefore\; x+y=-z$.

8. $\because a+b=2c \;\therefore\; a^2+2ab+b^2=4c^2 \;\therefore\; a^2+b^2=4c^2-2ab$ $\therefore\; a^2(b+c)+b^2(a+c)+c^2(a+b)=a^2b+a^2c+ab^2+b^2c+ac^2+bc^2$ $=c(a^2+b^2)+ab(a+b)+c^2(a+b)=c(4c^2-2ab)+ab\times 2c+c^2\times 2c=4c^3$ $-2abc+2abc+2c^3=6c^3$.

9. $(x+\frac{1}{x})^3=\left(p+\frac{1}{p}\right)^3$ or $x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=p^3+\frac{1}{p^3}+3p$ $\times\frac{1}{p}\left(p+\frac{1}{p}\right)$, or $x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=p^3+\frac{1}{p^3}+3\left(p+\frac{1}{p}\right)$ or $x^3+\frac{1}{x^3}=p^3+\frac{1}{p^3}$

10. $\frac{x(y^2-z^2)}{y(x^2-z^2)}=\frac{x^3}{y^3} \;\therefore\; \frac{y^2-z^2}{x^2-z^2}=\frac{x^2}{y^2} \;\therefore\; y^4-y^2z^2=x^4-x^2z^2 \;\therefore\; x^4-y^4$ $=x^2z^2-y^2z^2=z^2(x^2-y^2)$ or $(x^2+y^2)(x^2-y^2)=z^2(x^2-y^2) \;\therefore\; x^2+y^2$ $=z^2 \;\therefore\; x^2=z^2-y^2=(z+y)(z-y)$.

11. $\frac{1}{y^3(x-z)}+\frac{1}{x^3(y-z)}=\frac{1}{xy(x-z)(y-z)}$ or $\frac{x^3(y-z)+y^3(x-z)}{y^3x^3(x-z)(y-z)}$ $=\frac{1}{xy(x-z)(y-z)}$ or $\frac{x^3y-x^3z+xy^3-y^3z}{x^2y^2}=1$ or x^3y-x^3z $+xy^3-y^3z=x^2y^2 \;\therefore\; x^3y+xy^3-x^2y^2=z(x^3+y^3)$ or $xy(y^2-xy$ $+y^2)=z(x^3+y^3)=z(x+y)(x^2-xy+y^2) \;\therefore\; x^2-xy+y^2=0$ or $z(x+y)$ $=xy$, now when $x^2-xy+y^2=0$ then $x^2+y^2=xy$.

12. Multiplying the whole eqn. by the *l c m* of the denominators we have $2x(y-z)+2z(y-x)=0$ or $2xy-4xz+2zy=0 \;\therefore\; 2xy+2yz$ $=4xz$ or $xy+yz=2xz \;\therefore\; \frac{xy}{xyz}+\frac{yz}{xyz}=\frac{2xz}{xyz}$ or $\frac{1}{z}+\frac{1}{x}=\frac{2}{y}$.

13 $\frac{x+2a}{x-b}+\frac{2(a+b)-x}{x-a}=\frac{x+2a}{x-b}-2+\frac{x+2b}{x-a}=2 \;\therefore\; \frac{x+2a}{x-b}+\frac{x+2b}{x-a}$

$=4\ \therefore\ x^2+2ax-ax-2a^2+x^2+2bx-bx-2b^2=4(x^2-bx-ax+ab)$
$=4x^2-4bx-4ax+4ab\ \ \therefore\ 2x^2-5ax+2a^2-5bx+4ab+2b^2=0$
$\therefore\ (2a+2b-x)(a+b-2x)=0\ \therefore\ 2a+2b-x=0$ and $a+b-2x=0$
$\therefore\ x=2a+2b$ and $2x=a+b$.

14. $x^2+z^2=2y^2\ \therefore\ (x^2+xy+xz+yz)+(z^2+xz+xy+yz)=2y^2+2xy+2xz+2yz$ or $(x+y)(x+z)+(y+z)(x+z)=2(x+y)(y+z)$
Divide the whole eqn by $(x+y)(x+y)(x+z)$ then $\frac{1}{y+z}+\frac{1}{x+y}=\frac{2}{x+z}$
or $(y+z)^{-1}+(x+y)^{-1}=2(x+z)^{-1}$.

15. $x^y=\frac{1}{y^x}=y^{-x}\ \therefore\ x=y^{-\frac{y}{x}}\ \therefore\ \frac{x}{y}=\frac{y^{-\frac{x}{y}}}{y}=y^{-\frac{x}{y}}=y^{-(\frac{x}{y}+1)}$

16. $x^2+y^2=1=z^2+w^2\ \therefore (x^2+y^2)w^2=w^2$ or $x^2w^2+y^2w^2=w^2\ ..\ (1)$
also $z^2+w^2=1\ \therefore\ (z^2+w^2)y^2=y^2$ or $y^2z^2+y^2w^2=y^2\ \ldots\ \ldots\ldots(2)$
subtracting (2) from (1) we have $x^2w^2-y^2z^2=w^2-y^2=x^2-z^2$ or
$(xw+yz)(xw-yz)=(x+z)(x-z)\ \therefore\ \frac{xw+yz}{x+z}=\frac{x-z}{xw-yz}$ again (x^2+y^2)
$(z^2+w^2)=1$ or $x^2z^2+x^2w^2+y^2z^2+y^2w^2=1$ or $(x^2z^2-2xyzw+y^2w^2)$
$+(x^2w^2+2xyzw+y^2z^2)=1$ or $(xz-yw)^2+(xw+yz)^2=1$.

17. $\frac{2}{9}(a+b+c)^3=\frac{2}{9}\left\{a+\frac{a+c}{2}+c\right\}^3=\frac{2}{9}\{\frac{3}{2}(a+c)\}^3=\frac{2}{9}\times\frac{27}{8}(a+c)^3$
$=\frac{3}{4}(a+c)^3$ again $a^2(b+c)+b^2(a+c)+c^2(a+b)=b^2(a+c)+b(a^2+c^2)$
$+ac(a+c)=\left(\frac{a+c}{2}\right)^2(a+c)+\frac{a+c}{2}(a^2+c^2)+ac(a+c)=\frac{(a+c)^3}{4}$
$+(a+c)\left\{\frac{a^2+c^2}{2}+ac\right\}=\frac{(a+c)^3}{4}+\frac{(a+c\ (a+c)^2}{2}=\frac{3}{4}(a+c)^3$.

18. Squaring we have $4x^2y^2=(x+y+z)(x+y-z)(x+z-y)(y+z-x)=(x^2+2xy+y^2-z^2)(z^2-x^2-y^2+2xy)=(2xy+\overline{x^2+y^2-z^2})$
$(2xy-\overline{x^2+y^2-z^2})=4x^2y^2-(x^2+y^2-z^2)^2\ \ \therefore\ (x^2+y^2-z^2)^2=0\ \therefore\ x^2+y^2-z^2=0\ \therefore\ x^2+y^2=z^2$.

19. $a(1-b^2)^{\frac{1}{2}}=c-b(1-a^2)^{\frac{1}{2}}\ \therefore\ a^2(1-b^2)=c^2-2bc(1-a^2)^{\frac{1}{2}}$

$+ b^2(1 - a^2) \therefore\ 2bc(1 - a^2)^{\frac{1}{2}} = c^2 - a^2 + a^2b^2 + b^2 - a^2b^2 = b^2 + c^2 - a^2$ $\therefore\ 4b^2c^2(1 - a^2) = (b^2 + c^2 - a^2)^2$ or $4b^2c^2 - (b^2 + c^2 - a^2)^2 = 4a^2b^2c^2$ or $(2bc + b^2 + c^2 - a^2)(2bc - b^2 - c^2 + a^2) = 4a^2b^2c^2$ or $\{(b + c)^2 - a^2\}\{a^2 - (b - c)^2\} = (b + c + a)(b + c - a)(a + b - c)(a - b + c) = 4a^2b^2c^2$

20. $ax^2 = a(a^2 - bc) = a^3 - abc$; $by^2 = b(b^2 - ac) = b^3 - abc$; $cz^2 = c(c^2 - ab) = c^3 - abc$ $\therefore ax^2 + by^2 + cz^2 = a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc) = (a + b + c)(a^2 - bc + b^2 - ac + c^2 - ab)$

$$= (a + b + c)(x^2 + y^2 + z^2)$$

21. $n^2p^2x - m^2y + m^2z = 0$, $m^2p^2y + n^2x - n^2z = 0$ and $m^2n^2z - p^2x + p^2y = 0$; multiply the 2nd eqn by m^2 and the 1st, by n^2 and then add ; also add m^2 times the 2nd to the 3rd and we get $n^2(m^2 + n^2p^2)x = m^2(n^2 - m^2p^2)y$...(1) ; $p^2(1 + m^4)y = x(p^2 - m^2n^2)$......(2) ; multiply (1) by (2) we get after expunging equal factors from both sides $n^4p^4 + m^4n^4p^4 = -m^4p^4 - m^4n^4$ or $m^4p^4 + m^4n^4 + n^4p^4 + m^4n^4p^4 = 0$.

22. $my + nx = m$, $\therefore\ m^2y^2 + 2mnxy + n^2x^2 = m^2$ and $ny - mx = n$ $\therefore\ n^2y^2 - 2mnxy + m^2x^2 = n^2$ adding we have $(m^2 + n^2)y^2 + (m^2 + n^2)x^2 = m^2 + n^2$ or $y^2 + x^2 = 1$.

23. $x^4 - 2m^2x^2 = (y^2 + z^2)^2$ $\therefore x^4 - (y^2 + z^2)^2 = 2m^2x^2$ or $(x^2 + y^2 + z^2)(x^2 - y^2 - z^2) = 2m^2x^2$...(1) also $y^4 - 2m^2y^2 = (z^2 + x^2)^2$ $\therefore\ y^4 - (z^2 + x^2)^2 = 2m^2y^2$ or $(x^2 + y^2 + z^2)(y^2 - z^2 - x^2) = 2m^2y^2$......(2) subtracting (2) from (1) we have $(x^2 + y^2 + z^2)\ (2x^2 - 2y^2) = m^2(2x^2 - 2y^2)$ or $x^2 + y^2 + z^2 = m^2$.

24. The 3 factors are each $= 0$ $\therefore\ m^2 - np = 0$ $\therefore\ m^2 = np$ for the same reason $n^2 = mp$ and $p^2 = mn$ $\therefore\ m^3 = mnp$, $n^3 = mnp$, $p^3 = mnp$

$$\therefore\ \frac{1}{m^3} + \frac{1}{n^3} + \frac{1}{p^3} = \frac{3}{mnp} = \frac{3mnp}{m^2n^2p^2} = \frac{mnp + mnp + mnp}{m^2x^2p^2} = \frac{m^3 + n^3 + p^3}{(mnp)^2}$$

$= (m^3 + n^3 + p^3)(mnp)^{-2}$.

25. The left hand expression $= (x - 2a + 2a)(x - 2b)(x - 2c) + x(x - 2c)(x - 2a) + x(x - 2a)(x - 2b) = (x - 2a)(x - 2b)(x - 2c) + 2a(x - 2b)(x - 2c) + x(x - 2c)(x - 2a) + x(x - 2a)(x - 2b)$

$=(x-2a)(x-2b)(x-2c)+2a\{x^2-2x(b+c)+4bc\}+x(x-2a)\{x-2c+x-2b\}=(x-2a)(x-2b)(x-2c)+2a\{x^2-2x(b+c)+4bc\}+x(x-2a)2a=(x-2a)(x-2b)(x-2c)+2a\{x^2-2x(b+c)+4bc+x^2-2a\}=(x-2a)(x-2b)(x-2c)+2a(2x^2-2x.x+4bc)=(x-2a)(x-2b)(x-2c)+8abc.$

26. The expression $=\dfrac{4a^2b^2-(a^2+b^2-c^2)^2}{4b^2}$

$=\dfrac{\{2ab+(a^2+b^2-c^2)\}\{2ab-(a^2+b^2-c^2)\}}{4b^2}=\dfrac{\{(a+b)^2-c^2\}\{c^2-(a-b)^2\}}{4b^2}$

$=\dfrac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{4b^2}=\dfrac{x(x-2c)(x-2b)(x-2a)}{4b^2}$

27. The expression $=x(x-a)(2x-b-c)+(x+a)(x.\overline{x-c}+c\cdot\overline{x+b})$ $=x(x-a)(2x-\overline{x-a})+\overline{x+a}(x^2-cx+cx+bc)=x(x-a)(x+a)+(x+a)(x^2+bc)=(x+a)(x^2-ax+x^2+bc)=(x+a)(x^2+x.\overline{x-a}+bc)=(x+a)(x^2+x\cdot\overline{b+c}+bc)=(x+a)(x+b)(x+c)$

28. the expression $=(\overline{x-a}+\overline{x-b}+\overline{x-c})\{(x-a)^2+(x-b)^2+(x-c)^2-(x-a)(x-b)-(x-a)(x-c)-(x-b)(x-c)\}=\{3x-\overline{a+b+c}\}\{(x-a)^2-(x-a)(x-b)+(x-b)^2-(x-b)(x-c)+(x-c)^2-(x-a)(x-c)\}$ $=2(a+b+c)\{(x-a)(b-a)+(x-b)(c-b)+(x-c)(c-a)\}=2(a+b+c)\{(b+c)(b-a)+(a+c)(c-b)+(a+b)(c-a)\}=2(a+b+c)(b^2+bc-ab-ac+c^2+ac-bc-ab+a^2+ab-ac-bc)=2(a+b+c)(a^2+b^2+c^2-bc-ab-ac)=2(a^3+b^3+c^3-3abc)$

29. From Ex. 4 page 119, $(a+b+c)^3=a^3+b^3+c^3+3a^2(b+c)+3b^2(a+c)+3c^2(a+b)+6abc$ or $x^3=a^3+b^3+c^3+3a^2(x-a)+3b^2(x-b)+3c^2(x-c)+6abc.$

30. $(x-a)^2=x^2-2ax+a^2,(x-b)^2=x^2-2bx+b^2,(x-c)^2=x^2-2cx+c^2$ $\therefore$ the given expression $=4x^2-2x(a+b+c)+a^2+b^2+c^2=4x^2-4x^2+a^2+b^2+c^2=a^2+b^2+c^2.$

31. The expression $=x^3-(a+b+c)x^2+(ab+ac+bc)x-abc=x^3-\dfrac{(a+b+c)^2}{2}x+(2ab+2ac+2bc)\dfrac{x}{2}-abc=x^3-\dfrac{x}{2}\{(a+b+c)^2-(2ab+2ac+2bc)\}-abc=x^3-\dfrac{x}{2}(a^2+b^2+c^2)-abc.$

32. The left hand expression $=\left(\frac{1}{x-a}+\frac{1}{x-b}\right)+\left(\frac{1}{x-c}-\frac{1}{x}\right)$

$$=\frac{x-b+x-a}{(x-a)(x-b)}+\frac{x-x+c}{x(x-c)}=\frac{c}{(x-a)(x-b)}+\frac{c}{x(x-c)}=c\left(\frac{1}{(x-a)(x-b)}\right.$$

$$\left.+\frac{1}{x(x-c)}\right)=c\left(\frac{x^2-cx+x^2-ax-bx+ab}{x(x-a)(x-b)(x-c)}\right)=c\left(\frac{2x^2-(a+b+c)x+ab}{x(x-a)(x-b)(x-c)}\right)$$

$$=\frac{abc}{x(x-a)(x-b)(x-c)}.$$

33. The 1st expression $=2\{x^2-(a+b)x+ab\}+2\{x^2-(b+c)x+bc\}+2\{x^2-(a+c)x+ac\}+a^2+b^2+c^2=6x^2-4x(a+b+c)+2ab+2bc+2ac+a^2+b^2+c^2=6x^2-4x\,2x+(a+b+c)^2=-2x^2+(2x)^2=2x^2$.

34. The left hand expression $=\{2(x-a)(x-b)(x-c)+a(x-b)(x-c)\}+\{b(x-c)(x-a)+c(x-a)(x-b)\}-abc=(x-b)(x-c)(2x-2a+a)+(x-a)(bx-bc+cx-bc)-abc=(x-b)(x-c)(b+c)+(x-a)\{(b+c)x-2bc\}-abc=(x-b)(x-c)(b+c)+x(x-a)(b+c)-2bc(x-a)-abc=(b+c)\{x^2-(b+c)x+bc+x^2-ax\}-2bc(x-a)-abc=(b+c)\{2x^2-(a+b+c)x+bc\}-2bc(x-a)-abc=(b+c)bc-2bc(x-a)-abc=bc(b+c-2x+2a)-abc=abc-abc=0$.

35. The left side expression $=2x\cdot 2(x-a)\cdot 2(x-b)\cdot 2(x-c)=(a+b+c)(b+c-a)(a+c-b)(b+a-c)=\{(b+c)^2-a^2\}\{a^2-(c-b)^2\}=\{b^2+2bc+c^2-a^2\}\{a^2-c^2+2bc-b^2\}=\{2bc-(a^2-b^2-c^2)\}\{2bc+(a^2-b^2-c^2)\}=4b^2c^2-(a^2-b^2-c^2)^2=4b^2c^2-(a^4+b^4+c^4-2a^2b^2-2a^2c^2+2b^2c^2)=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$.

36. The left side expression $=6x^2-3(a+b+c)x+ab+ac+bc=6x^2-3x\cdot 2x+ab+ac+bc=ab+ac+bc$.

37. The expression $=\dfrac{rx-(a+b+c\ldots\ldots\text{to } r \text{ terms})}{x}=\dfrac{rx-x}{x}$

$$=\frac{x(r-1)}{x}=r-1.$$

38. $(x-p_1)^3=x^3-3p_1x^2+3p_1{}^2x-p_1{}^3, (x-p_2)^3=x^3-3p_2x^2+3p_2{}^2x-p_2{}^3$, &c=&c. $(x-p_r)^3=x^3-3p_rx^2+3p_r{}^2x-p_r{}^3$ $\therefore$ sum $=rx^3-3(p_1+p_2+p_3\dots p_r)x^2+3(p_1{}^2+p_2{}^2+\dots\dots p_r{}^2)x-(p_1{}^3+p_2{}^3\dots p_r{}^3)=rx^3-3\cdot\frac{rx}{3}x^2+3\frac{r}{3x}\cdot x=rx^3-rx^3+r=r$.

39. The left hand expression $=3s^4-2s^2(a^2+b^2+c^2)+a^2b^2+b^2c^2+a^2c^2=3s^4-4s^4+a^2b^2+b^2c^2+a^2c^2=-\left(\frac{a^2+b^2+c^2}{2}\right)^2+a^2b^2+b^2c^2+a^2c^2=\frac{1}{4}(2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4)=4x(x-a)(x-b)(x-c)$ by Ex. 35.

40. The eqn can be put into this form $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{x+y+z}$; here $\frac{1}{x}+\frac{1}{y}=\frac{1}{x+y+z}-\frac{1}{z}=\frac{z-x-y-z}{z(x+y+z)}=\frac{-(x+y)}{z(x+y+z)}$ or $\frac{x+y}{xy}=\frac{-(x+y)}{z(x+y+z)}$ or $\frac{1}{xy}=-\frac{1}{xz+yz+z^2}$ $\therefore$ $xz+yz+z^2=-xy$ or $xz+xy+yz+z^2=0$ or $x(y+z)+z(y+z)=0$ $\therefore$ $(x+z)(y+z)=0$ $\therefore$ $x+z=0$ and $y+z=0$ $\therefore$ $y=-z$ $\therefore$ $y^3=-z^3$ $\therefore$ $y^3+z^3=0$ $\therefore$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{x+y+z}=\frac{1}{x}$ $\therefore\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)^3=\frac{1}{x^3}=\frac{1}{x^3+y^3+z^2}$. In the remaining case if r be odd or even $2r+1$ is always odd and the case may be proved in like manner.

41. $x^2+y^2=1$ $\therefore$ $(x^2+y^2)^3=1$ $\therefore$ $x^6+y^6+3x^2y^2(x^2+y^2)=1$ or $x^6+y^6+3x^2y^2=1$ $\therefore$ $x^6+y^6=1-3x^2y^2$ $\therefore 2(x^6+y^6)=2-6x^2y^2$ .. (1); again squaring we have $x^4+2x^2y^2+y^4=1$ $\therefore$ $x^4+y^4=1-2x^2y^2$ $\therefore$ $3(x^4+y^4)=3-6x^2y^2$ ......(2) subtracting (2) from (1) we have the remainder $=-1$.

42. $m^3+n^3+p^3-3mnp=(m+n+p)\{m^2+n^2+p^2-mn-mp-np\}$ $=\{ax+cy+bz+cx+by+az+bx+ay+cz\}\{(ax+cy+bz)^2+(cx+by+az)^2+(bx+ay+cz)^2-(ax+cy+bz)(cx+by+az)-(ax+cy+bz)(bx+ay+cz)-(cx+by+az)(bx+ay+cz)\}=(a+b+c)(x+y+z)\{a^2x^2$

$+a^2y^2+a^2z^2-a^2xy-a^2xz-a^2yz+b^2x^2+b^2y^2+b^2z^2-b^2xy-b^2xz$ $-b^2yz+c^2x^2+c^2y^2+c^2z^2-c^2xy-c^2xz-c^2yz-abx^2-aby^2-abz^2$ $+abxy+abxz+abyz-acx^2-acy^2-acz^2+acxy+acxz+acyz-bcx^2$ $-bcy^2-bcz^2+bcxy+bcxz+bcyz\}=(a+b+c)(x+y+z)(a^2+b^2+c^2$ $-ab-ac-bc)(x^2+y^2+z^2-xy-xz-yz)=(a^3+b^3+c^3-3abc)(x^3+y^3$ $+z^3-3xyz)$.

43. $\dfrac{a+b-c}{a+b}=\dfrac{b+c-a}{b+c}$ $\therefore$ by cross multiplication $ab+ac+b^2-c^2$ $=ac+bc+b^2-a^2$ $\therefore$ $ab+ac+b^2-c^2=ac+bc+b^2-a^2$ $\therefore(ab+ac+a^2)$ $-(c^2+ac+bc)=0$ or $a(a+b+c)-c(a+b+c)=0$ $\therefore$ $(a+b+c)(a-c)$ $=0$ but by the question $a+b+c$ is not $=0$ $\therefore$ $a-c=0$ $\therefore$ $a=c$: similarly $\dfrac{a+b-c}{a+b}=\dfrac{a+c-b}{a+c}$ and by cross multiplication $ab+bc-c^2$ $=ac+bc-b^2$ or $(ab+bc+b^2)-(c^2+ac+bc)=0$ or $b(a+b+c)-c$ $(a+b+c)=0$ $\therefore$ $(b-c)(a+b+c)=0$ $\therefore$ $b-c=0$ $\therefore$ $b=c$ and it has been shewn that $a=c$ $\therefore$ $a=b=c$.

44. $z^0=1=m^0=x^m=y^{mn}=z^{mnp}$ $\therefore$ $mnp=0$.

45. $\dfrac{a^2+b^2}{c^3}+\dfrac{a^2+c^2}{b^3}+\dfrac{b^2+c^2}{a^3}=a^2-\dfrac{1}{a}+b^2-\dfrac{1}{b}+c^2-\dfrac{1}{c}$ $\therefore$ $\left(\dfrac{a^2+b^2}{c^3}+\dfrac{1}{c}\right)+\left(\dfrac{a^2+c^2}{b^3}+\dfrac{1}{b}\right)+\left(\dfrac{b^2+c^2}{a^3}+\dfrac{1}{a}\right)=a^2+b^2+c^2$ or $\dfrac{a^2+b^2+c^2}{c^3}$ $+\dfrac{a^2+b^2+c^2}{b^3}+\dfrac{a^2+b^2+c^2}{a^3}=a^2+b^2+c^2$ $\therefore$ $\dfrac{1}{c^3}+\dfrac{1}{b^3}+\dfrac{1}{a^3}=1$.

46. By cross multiplication $\{(a+c)+b(1-bc)\}(1-b^2)=2b(1-bc$ $-ac-ab)$ or $(a+c)(1-b^2)+(b-abc)=2(b-abc)-2b^2(a+c)$ $\therefore$ $(a+c)$ $(1-b^2+2b^2)=(2-1+b^2)(b-abc)$ $\therefore$ $a+c=b-abc=b(1-ac)$.

47. From the given eqn, $ab+ac=1-bc$ or $a(b+c)=1-bc$ $\therefore\dfrac{b+c}{1-bc}=\dfrac{1}{a}$; $ab+bc=1-ac$ or $b(a+c)=1-ac$ $\therefore$ $\dfrac{a+c}{1-ac}=\dfrac{1}{b}$ and simi-

larly $\frac{a+b}{1-ab}=\frac{1}{c}$ $\therefore$ sum $=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{ac+ab+bc}{abc}=\frac{1}{abc}=\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}$

$$=\frac{b+c}{1-bc}\cdot\frac{a+c}{1-ac}\cdot\frac{a+b}{1-ab}.$$

48. $\frac{a-b}{c}+\frac{b-c}{a}+\frac{a+c}{b}-1=0$ $\therefore$ $\frac{a^2b-ab^2+b^2c-bc^2+a^2c-abc}{abc}$

$=0$ $\therefore$ $\frac{(ab+ac-bc)(a-b+c)}{abc}=0$ $\therefore$ $ab+ac-bc=0$ $\therefore$ $a(b+c)=bc$

$$\therefore\ a=\frac{bc}{b+c}\text{ or }\frac{1}{a}=\frac{b+c}{bc}=\frac{1}{c}+\frac{1}{b}.$$

49. Here $\frac{x^2}{2x^2+yz}=\frac{x^2}{x^2+x^2+yz}=\frac{x^2}{x^2+(y+z)^2+(x+z)(x+y)}$

$$=\frac{x^2}{x^2+y^2+z^2+2yz+x^2+xz+yz+xy}$$

$=\frac{x^2}{(x^2+y^2+z^2+2yz+2xz+2xy)+x^2+yz-xz-xy}=\frac{x^2}{x^2+yz-xz-xy}$

$=\frac{x^2}{(x-y)(x-z)}$ similarly $\frac{y^2}{2y^2+xz}=\frac{y^2}{(y-x)(y-z)}$ and $\frac{z^2}{2z^2+xy}$

$=\frac{z^2}{(z-x)(z-y)}$ $\therefore$ sum $=\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-x)(y-z)}+\frac{z^2}{(z-x)(z-y)}$

$=1$, by ex 67 page 13.

50. $a=x(b+c)=bx+cx=bx+xz(a+b)=bx+axz+bxz$

$\therefore$ $(1-xz)a=x(1+z)b$; again $b=y(a+c)=ay+cy=ay+ayz+byz$ $\therefore$ $(1-yz)b=y(1+z)a$ multiplying cross wise we get $y(1+z)(1-xz)a^2$ $=x(1+z)(1-yz)b^2$ or $y(1-xz)a^2=x(1-yz)b^2$ $\therefore$ $\frac{a^2}{x(1-yz)}=\frac{b^2}{y(1-xz)}$.

Similarly $a=bx+cx=axy+cxy+cx$ $\therefore$ $(1-xy)a=x(1+y)c$ also $c=az+bz=az+ayz+cyz$ $\therefore$ $(1-yz)c=z(1+y)a$, multiplying crosswise the last two results we have $z(1+y)(1-xy)a^2=x(1+y)(1-yz)c^2$ or $z(1-xy)a^2=x(1-yz)c^2$ $\therefore$ $\frac{a^2}{x(1-yz)}=\frac{c^2}{z(1-xy)}$. Next $xy+xz$

$$+yz+2xyz=\frac{ab}{(b+c)(a+c)}+\frac{ac}{(b+c)(a+b)}+\frac{bc}{(a+c)(a+b)}$$

$$+\frac{2abc}{(b+c)(a+c)(a+b)}=\frac{ab(a+b)+bc(b+c)+ac(a+c)+2abc}{(a+b)(a+c)(b+c)}$$

$$=\frac{(a+b)(a+c)(b+c)}{(a+b)(a+c)(b+c)}=1$$

51. The left side expression $=(x-a)(x-b)(x-c)+b(x-a)(c-x)\}+\{c(x-a)(x-b)+a(x-b)(x-c)\}-2abc=(x-a)(x-c)\{x-b+b\}+(x-b)\{cx-ac+ax-ac\}-2abc=x(x-a)(x-c)+(x-b)\{(a+c)x-2ac\}-2abc=x\{x^2-(a+c)x+ac\}+(a+c)x^2-2acx-b(a+c)x+2abc-2abc=x^3-(a+c)x^2+acx+(a+c)x^2-2acx-abx-bcx=x^3-(ab+ac+bc)x=x^3-x^2\cdot x=0$

52. $a+b=2x-c$, $a+c=2x-b$, $b+c=2x-a$ ∴ the expression $=-6x^2-3x(a+b+c)+a^2+b^2+c^2=6x^2-6x^2+a^2+b^2+c^2=a^2+b^2+c^2$.

53. Each of the given factors must be zero ∴ $x^2+yz=0$ ∴ $x^2=-yz$ ∴ $x^3=-xyz$; $y^2+xz=0$ ∴ $y^2=-xz$ ∴ $y^3=-xyz$ and $z^2+xy=0$ ∴ $z^2=-xy$ ∴ $z^3=-xyz$ ∴ $\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{z^3}=\frac{-3}{xyz}=\frac{-3xyz}{x^2y^2z^2}$

$$=\frac{-xyz-xyz-xyz}{x^2y^2z^2}=\frac{x^3+y^3+z^3}{x^2y^2z^2}.$$

54. The left side expression $=\{s^3-(s-a)^3\}-\{(s-b)^3+(s-c)^3\}=\{s-(s-a)\}\{s^2+s(s-a)+(s-a)^2\}-\{(s-b)+(s-c)\}\{(s-b)^2-(s-b)(s-c)+(s-c)^2\}=a(3s^2-3as+a^2)-a(s^2+b^2+c^2-bs-cs-bc)=3as^2-3a^2s+a^3-as^2-ab^2-ac^2+abs+acs+abc=2as^2-3a^2s+abs+acs+a^3-ab^2-ac^2+abc=as(2s-3a+b+c)+a^3-ab^2-ac^2+abc=2as(b+c-a)+a^3-ab^2-ac^2+abc=ab^2+ac^2+2abc-a^3+a^3-ab^2-ac^2+abc=3abc.$

55. $x=cy+bz=cy+b(bx+ay)$ ∴ $x(1-b^2)=y(c+ab)$; again $y=az+cx=a(bx+ay)+cx$ ∴ $y(1-a^2)=x(c+ab)$, multiplying crosswise

we get $x^2(1-b^2)=y^2(1-a^2)$ ∴ $\frac{x^2}{1-a^2}=\frac{y^2}{1-b^2}$, by the same process of reasoning we get $\frac{x^2}{1-a^2}=\frac{z^2}{1-c^2}$.

56. Multiplying the 1st 3 equations together we have $x^2y^2z^2(y+z)(x+z)(x+y)=m^3n^3p^3$ but from the 4th eqn $x^2y^2z^2=m^2n^2p^2$ ∴ $m^2n^2p^2(y+z)(x+z)(x+y)=m^3n^3p^3$ or $(y+z)(x+z)(x+y)=mnp$ ∴ $(y+z)\{x^2+(y+z)x+yz\}=mnp$ ∴ $x^2(y+z)+y^2(x+z)+z^2(x+y)+2xyz=mnp$ or $m^3+n^3+p^3+2mnp=mnp$ or $m^3+n^3+p^3+mnp=0$.

57. $\frac{(x+y\ )^2}{(y+xz)^2}=\frac{1-y^2}{1-x^2}$ ∴ $\frac{x^2+2xyz+y^2z^2}{y^2+2xyz+x^2z^2}=\frac{1-y^2}{1-x^2}$

∴ $\frac{x^2+2xyz+y^2z^2}{y^2+2xyz+x^2z^2}-1=\frac{1-y^2}{1-x^2}-1$ or $\frac{x^2-y^2-z^2(x^2-y^2)}{y^2+x^2z^2+2xyz}=\frac{x^2-y^2}{1-x^2}$;

rejecting x^2-y^2 from both sides we have $\frac{1-z^2}{y^2+x^2z^2+2xyz}=\frac{1}{1-x^2}$

∴ $1-x^2-z^2+x^2z^2=y^2+x^2z^2+2xyz$ ∴ $x^2+y^2+z^2+2xyz=1$.

58. $ab+ac+bc=1$ ∴ $ab+bc=1-ac$ or $b(a+c)=1-ac$ similarly $a(b+c)=1-bc$ and $c(a+b)=1-ab$ ∴ the given expression

$$=\frac{a(1-b^2)(1-c^2)+b(1-a^2)(1-c^2)+c(1-a^2)(1-b^2)}{(1-a^2)(1-b^2)(1-c^2)}$$

$$=\frac{a\{1-(b^2+c^2)+b^2c^2\}+b\{1-(a^2+c^2)+a^2c^2\}+c\{1-(a^2+b^2)+a^2b^2\}}{(1-a^2)(1-b^2\ (1-c^2)}$$

$$=\frac{a-a(b^2+c^2)+ab^2c^2+b-b(a^2+c^2)+a^2bc^2+c-c(a^2+b^2)+a^2b^2c}{(1-a^2)(1-b^2)(1-c^2)}$$

$$=\frac{a+b+c-a^2(b+c)-b^2(a+c)-c^2(a+b)+abc(ab+ac+bc)}{(1-a^2)(1-b^2)(1-c^2)}$$

$$=\frac{a+b+c-a(1-bc)-b(1-ac)-c(1-ab)+abc}{(1-a^2)(1-b^2)(1-c^2)}$$

$$=\frac{4abc}{(1-a^2)(1-b^2)(1-c^2)}.$$

59. Subtract the 2nd eqn from the 1st and we get $x - y = ny - mx$ or $x(1+m) = y(1+n)$; Subtract the 3rd eqn from the 2nd and we get $y - z = pz - ny$ or $y(1+n) = z(1+p)$, subtract the 4th eqn from the 3rd and we get $z - u = qu - pz$ or $z(1+p) = u(1+q)$ $\therefore$ $x(1+m) = y(1+n)$ $= z(1+p) = u(1+q) = r$ suppose $\therefore x = \frac{r}{1+m}$, $y = \frac{r}{1+n}$, $z = \frac{r}{1+p}$, $u = \frac{r}{1+q}$, substitute these values of x, y, z and u in the 1st eqn and we get $\frac{r}{1+m} = n\frac{r}{1+n} + p\frac{r}{1+p} + q\frac{r}{1+q}$ or $\frac{1}{1+m} = \frac{n}{1+n} + \frac{p}{1+p} + \frac{q}{1+q}$

but $\frac{1}{1+m} = 1 - \frac{m}{1+m}$ $\therefore$ $1 - \frac{m}{1+m} = \frac{n}{1+n} + \frac{p}{1+p} + \frac{q}{1+q}$ or $\frac{m}{1+m} + \frac{n}{1+n}$

$$+ \frac{p}{1+p} + \frac{q}{1+q} = 1.$$

60. $xy + xz + yz = 1$ $\therefore$ the given expression $= \frac{1+x^2}{x^2 + yz + xz + xy}$ $+ \frac{1+y^2}{y^2 + xy + xz + yz} + \frac{1+z^2}{z^2 + xy + xz + yz} = \frac{1+x^2}{x^2+1} + \frac{1+y^2}{y^2+1} + \frac{1+z^2}{z^2+1} = 3.$

61. The left side expression

$$= \frac{(3x - x^3)(1 - 3y^2)(1 - 3z^2) + (3y - y^3)(1 - 3x^2)(1 - 3z^2) + (3z - z^3)(1 - 3x^2)}{(1 - 3x^2)(1 - 3y^2)(1 - 3z^2)}$$

$$\frac{(1 - 3y^2)}{(1 - 3x^2)(1 - 3y^2)(1 - 3z^2)}$$

Numerator $= 3x - 9xy^2 - 9xz^2 + 27xy^2z^2 - x^3 + 3y^2x^3 + 3x^3z^2 - 9x^3y^2z^2$ $+ 3y - 9x^2y - 9yz^2 + 27x^2yz^2 - y^3 + 3x^2y^3 + 3y^3z^2 - 9x^2y^3z^2 + 3z - 9x^2z$ $- 9y^2z + 27x^2y^2z - z^3 + 3x^2z^3 + 3y^2z^3 - 9x^2y^2z^3 = 3(x + y + z) - 9$ $(x + y + z)(xy + yz + xz) + 27xyz + 27xyz(xy + yz + xz) + 3(x + y + z)$ $(x^2y^2 + y^2z^2 + x^2z^2) - 3xyz(xy + yz + xz) - 9x^2y^2z^2(x + y + z) - (x^3 + y^3$ $+ z^3) = 27xyz + 3xyz(x^2y^2 + y^2z^2 + x^2z^2) + 18xyz(xy + yz + xz) - 10x^3$ $y^3z^3 = 27xyz + 3xyz(x^2y^2 + y^2z^2 + x^2z^2) - x^3y^3z^3 + 9xyz\{2(xy + yz$ $+ xz) - (x + y + z)^2\} = 27xyz + 3x^3y^3z + 3xy^3z^3 + 3x^3yz^3 - x^3y^3z^3 - 9xyz$ $(x^2 + y^2 + z^2) = (3x - x^3)(3y - y^3)(3z - z^3).$

62. $$\frac{1}{\frac{2ab}{a+b}-a}+\frac{1}{\frac{2ab}{a+b}-b}=\frac{a+b}{ab-a^2}+\frac{a+b}{ab-b^2}=\frac{a+b}{a(b-a)}+\frac{a+b}{b(a-b)}$$

$$=\frac{a+b}{a(b-a)}-\frac{a+b}{b(b-a)}=\frac{1}{a}+\frac{1}{b}.$$

63. $(a^2+bc)(b^2+ac)(c^2+ab)=\pm(a^2-bc)(b^2-ac)(c^2-ab)$

$\therefore a(a^2+bc)\ \ b(b^2+ac)\ \ c(c^2+ab)=\pm a(a^2-bc).\ b(b^2-ac).\ c(c^2-ab)$

or $(a^3+abc)(b^3+abc)(c^3+abc)=\pm(a^3-abc)(b^3-abc)(c^3-abc)$

or $$\frac{(a^3+abc)(b^3+abc)}{(a^3-abc)(b^3-abc)}=\pm\frac{c^3-abc}{c^3+abc}\text{ or }\frac{a^3b^3+(a^3+b^3)abc+a^2b^2c^2}{a^3b^3-(a^3+b^3,abc+a^2b^2c^2},$$

$$=\pm\frac{c^3-abc}{c^3+abc}\ \therefore\frac{a^3b^3+a^2b^2c^2}{(a^3+b^3)abc}=-\frac{c^3}{abc}\text{ or }=-\frac{abc}{c^3}$$

$\therefore\ a^3b^3+a^3c^3+b^3c^3+a^2b^2c^2=0$ or $(a^3+b^3+c^3)a^2b^2c^2+a^3b^3c^3=0$

$$\therefore\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}+\frac{a^2b^2c^2}{a^3b^3c^3}=0\text{ or }a^3+b^3+c^3+\frac{a^3b^3c^3}{a^2b^2c^2}=0$$

$\therefore\ a^{-3}+b^{-3}+c^{-3}+a^{-1}.\ b^{-1}.\ c^{-1}=0$ or $a^3+b^3+c^3+abc=0.$

64. The left side expression $=\{(a+b)^2-c^2\}\{c^2-(a-b)^2\}$
$=(a^2+2ab+b^2-c^2)(c^2-a^2+2ab-b^2)=2ab\times 2ab=4a^2b^2.$

65. $2y=b^x-b^{-x}\ \therefore\ 2b^xy=b^{2x}-1\ \therefore\ 1+y^2=b^{2x}-2b^xy+y^2$
$\therefore\ \sqrt{(1+y^2)}=b^x-y$

66. From the 1st, $z=2a-(x+y)$ squaring $z^2=4a^2+(x+y)^2$ $-4a(x+y)$ but $\because\ 2a(x+y)=x^2+xy+y^2+a^2\ \therefore\ 4a(x+y)$ $=2x^2+2xy+2y^2+2a^2\ \therefore\ z^2=4a^2+x^2+2xy+y^2-2x^2-2xy$ $-2y^2-2a^2=2a^2-x^2-y^2$ Now $(x-a)^2+(y-a)^2+(z-a)^2$ $=x^2+y^2+z^2-2a(x+y+z)+3a^2=x^2+y^2+z^2-2a\times 2a+3a^2$ $=x^2+y^2+(2a^2-x^2-y^2)-4a^2+3a^2=a^2.$

67. $\sqrt[4]{x}+\sqrt[4]{y}=-\sqrt[4]{z}\ \therefore\ \sqrt{x}+\sqrt{y}+2\sqrt[4]{(xy)}=\sqrt{z}\ \therefore\ \sqrt{x}+\sqrt{y}-\sqrt{z}$ $=-2\sqrt[4]{(xy)}$ or $x+y+z+2\sqrt{(xy)}-2\sqrt{(xz)}-2\sqrt{(yz)}=4\sqrt{(xy)}$ or $x+y+z$

$= 2(\sqrt{(xy)} + \sqrt{(xz)} + \sqrt{(yz)}$ $\therefore$ $(x+y+z)^2 = 4\{(xy+xz+yz) + 2x\sqrt{(yz)}$ $+ 2y\sqrt{(xz)} + 2z\sqrt{(xy)}\}$ $\therefore$ $x^2+y^2+z^2 - 2(xy+xz+yz) = 8\{x\sqrt{(yz)}$ $+ y\sqrt{(xz)} + z\sqrt{(xy)}\}$ $\therefore$ $\{x^2+y^2+z^2-2(xy+xz+yz)\}^2 = 64\{x^2yz + y^2xz$ $+ z^2xy + 2xyz\sqrt{(xy)} + 2xyz\sqrt{(xz)} + 2xyz\sqrt{(yz)}\} = 4xyz(x+y+z+2\sqrt{(xy)}$ $+ 2\sqrt{(xz)} + 2\sqrt{(yz)} = 64xyz(\overline{x+y+z} + x+y+z) = 128xyz.\ (x+y+z).$

68. (1) Let each of the fractions $= p$ then $x = p\ b + c - a)$, $y = p(a+c-b)$ and $z = p(a+b-c)$

$\therefore$ $x(b-c) = p(b-c)(b+c-a) = p\{b^2 - c^2 - ab + ac\}$;

$y(c-a) = p(c-a)(c+a-b) = p(c^2 - a^2 - bc + ab)$;

$z(a-b) = p(a-b)(a+b-c) = p(a^2 - b^2 - ac + bc)$

$\therefore$ $x(b-c) + y(c-a) + z(a-b)$

$= p\{b^2 - c^2 - ab + ac + c^2 - a^2 - bc + ab + a^2 - b^2 - ac + bc\}$

$= p \times 0 = 0$

(2). $$\frac{x(y+z)}{(b+c-a)(y+z)} = \frac{y(x+z)}{(a+c-b)(x+z)} = \frac{z(x+y)}{(a+b-c)(x+y)}$$

$$= \frac{x+y+z}{a+b+c} \therefore \frac{x+y+z}{a+b+c}$$

$$= \frac{x(y+z) + y(x+z) + z(x+y)}{x(a+c-b+a+b-c) + y(b+c-a+a+b-c) + z(b+c-a+a+c-b)}$$

$$= \frac{2(xy+yz+xz)}{2(ax+by+cz)}.$$

69. $$a^2b^2c^2\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right) = a^3c^3\left(\frac{b^3c^3 + a^3c^3 + a^3b^3}{a^3b^3c^3}\right)$$

$$= a^3 + c^3 + \frac{a^3c^3}{b^3} = a^3 + c^3 + \frac{b^6}{b^3}$$

(for $ac = b^2$ $\therefore$ $a^3c^3 = b^6$) $= a^3 + b^3 + c^3$

70. $$\frac{x+y+z}{a+b+c} = \frac{x}{a} \ldots(1) \quad \therefore \frac{(x+y+z) + (a+b+c)}{a+b+c} = \frac{x+a}{a} \ldots(2)$$

again $\because$ $$\frac{(x+y+z)^2}{(a+b+c)^2} = \frac{x^2}{a^2}$$

$$\therefore \frac{(x+y+z)^2+(a+b+c)^2}{(a+b+c)^2}=\frac{x^2+a^2}{a^2} \quad \ldots(3)$$ Divide (3) by (2)

$$\therefore \frac{(x+y+z)^2+(a+b+c)^2}{(a+b+c)\{(x+y+z)+(a+b+c)\}}=\frac{x^2+a^2}{(x+a)}$$

$$\therefore \frac{x^2+a^2}{x+a}=\frac{a}{a+b+c}\left\{\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}\right\}$$

Similarly $$\frac{y^2+b^2}{y+b}=\frac{b}{a+b+c}\left\{\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}\right\}$$

and $$\frac{z^2+c^2}{z+c}=\frac{c}{a+b+c}\left\{\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}\right\}$$

$$\therefore \text{Sum}=\frac{a+b+c}{a+b+c}\left\{\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}\right\}$$

$$=\frac{(x+y+z)^2+(a+b+c)^2}{(x+y+z)+(a+b+c)}$$

71. Let each of the fractions $=x$ then $a+b=x(a-b)$, $\frac{b+c}{2}=x(b-c)$, $\frac{a+c}{3}=x(c-a)$ $\therefore a+b+\frac{b+c}{2}+\frac{a+c}{3}=x(a-b+b-c+c-a)$ $=0$ $\therefore 6a+6b+3b+3c+2a+2c=0.$

72 $$\frac{x+y}{3a-b}=\frac{y+z}{3b-c}=\frac{x+z}{3c-a}=m \text{ suppose}$$

$$\therefore \frac{(x+y)-(y+z)}{(3a-b)-(3b-c)}=\frac{x-z}{3a-4b+c}=m=\frac{x+z}{3c-a} \therefore \frac{(x+z)-(x-z)}{(3c-a)-(3a-4b+c)}$$ $=m$ or $\frac{z}{2b+c-2a}=m$ $\therefore z=(2b+c-2a)m$ $\therefore cz=(2bc+c^2-2\,a)m$

also $$\frac{(y+z)-(x+y)}{(3b-c)-(3a-b)}=\frac{z-x}{4b-3a-c}=m=\frac{x+z}{3c-a}$$

$$\therefore \frac{(x+z)-(z-x)}{(3c-a)-(4b-3a-c)}=m \text{ or } \frac{x}{2c+a-2b}=m$$

$$\therefore\ x=(2c+a-2b)m \quad \therefore\ ax=(2ac+a^2-2ab)m$$

again $\dfrac{(x+z)-(x+y)}{(3c-a)-(3a-b)}=\dfrac{z-y}{3c-4a+b}=m=\dfrac{x+z}{3b-c}$

$$\therefore\ \frac{(y+z)-(z-y)}{(3b-c)-(3c-4a+b)}=m \text{ or } \frac{y}{b-2c+2a}=m$$

$$\therefore\ y=(b-2c+2a)m \quad \therefore\ by=(b^2-2bc+2ab)m$$

$$\therefore\ ax+by+cz=(2ac+a^2-2ab+b^2-2bc+2ab+2bc+c^2-2ac)m$$

$$=(a^2+b^2+c^2)m \quad \therefore\ \frac{ax+by+cz}{a^2+b^2+c^2}=m$$

Now $\dfrac{(x+y)+(y+z)+(x+z)}{(3a-b)+(3b-c)+(3c-a)}=\dfrac{x+y+z}{a+b+c}=m$

$$\therefore\ \frac{ax+by+cz}{a^2+b^2+c^2}=\frac{x+y+z}{a+b+c}$$

73. The left side expression $=\{(ad+bc)+x\}\{(ad+bc)-x\}$

$$=\left\{\frac{2(ad+bc)+a^2+d^2-(b^2+c^2)}{2}\right\}\left\{\frac{2(ad+bc)-(a^2+d^2)+(b^2+c^2)}{2}\right\}$$

$$=\tfrac{1}{4}\{(a+d)^2-(b-c)^2\}\{(b+c)^2-(a-d)^2\}=\tfrac{1}{4}(a+b+d-c)(a+c+d-b)$$
$$(a+b+c-d)(b+c+d-a)$$

74. $\dfrac{14x}{3}=\dfrac{7y}{2}$ or $\dfrac{2x}{3}=\dfrac{y}{2}$ or $4x=3y$ $\therefore\ y=\dfrac{4x}{3}$ $\therefore\ z=\dfrac{14x}{3}-y-x$

$$\frac{14x}{3}-\frac{4x}{3}-x=\frac{7x}{3}=\tfrac{1}{2}\cdot\frac{14x}{3}=\tfrac{1}{2}(x+y+z).$$

75. $7z+y=x$ from the 1st eqn, and $4y+z=x$ from the 2nd eqn

$$\therefore\ 7z+y=4y+z \quad \therefore\ 6z=3y \text{ or } 2z=y \quad \therefore\ y-z=z \quad \therefore\ \frac{y-z}{x}=\frac{z}{x}=\frac{z}{7z+y}$$

$$=\frac{z}{7z+2z}=\tfrac{1}{9} \quad \therefore\ 9(y-z)=x.$$

76. $x^2+y^2=123z$ and $x^2-y^2=27z$, adding the two we have $2x^2=150z$ $\therefore$ $x^2=75z$, again subtracting the 2nd eqn from the 1st, we have $2y^2=96z$ $\therefore$ $y^2=48z$ $\therefore$ $x^2y^2=75\times 48z^2=25\times 144z^2$ $\therefore$ xy $5\times 12z=60z$.

77. Subtracting (2) from (1), $ab-cd=\frac{1}{2}(a+b-c-d)(p+q)=0$.

$$\therefore\ p+q=\frac{2(ab-cd)}{a+b-c-d},\ \text{again}\ \frac{a+b}{c+d}=\frac{ab+pq}{cd+pq}\ \text{from (1) and (2)}$$

$$\therefore\ (a+b-c-d)pq=ab(c+d)+cd(a+b)$$

$\therefore$ $(a+b-c-d^2\{(p+q)^2-4pq\}=4(ab-cd)^2-4(a+b-c-d)\times\{ab(c+d)-cd(a+b)\}$ $\therefore$ $(a+b-c-d)^2$

$\left(\frac{p-q}{2}\right)^2=(ab-cd)^2-(a+b-c-d)(abc+abd-acd-bcd)$

$=(ab-cd)^2-(a-d+b-c)\{(a-d)bc+(b-c)ad\}$
$=a^2b^2+c^2d^2-2abcd-[bc(a-d)^2+bc(a-d)(b-c)+ad(a-d)(b-c)+ad(b-c)^2]=a^2b^2+c^2d^2+2abcd-a^2bc-bcd^2-ab^2d-ac^2d-(a-d)(b-b)\times(ad+bc)=a^2b(b-c)-cd^2(b-c)+(acd-abd)(b-c)-(a-d)(b-c)(ad+bc)=(b-c)\{a^2b-cd^2+acd-abd^2-(a-d)(ad+bc)\}=(b-c)\{ab(a-d)+cd(a-d)-(ad+bc)(a-d)\}=(a-d)(b-c)\{ab+cd-ad-bc\}=(a-d)(b-c)\{a(b-d)-c(b-d)\}$
$=(a-d)(b-c)\{a(b-d)-c(b-d)\}=(a-d)(b-c)(a-c)(b-d)$

$$\therefore\ \left(\frac{p-q}{2}\right)^2=\frac{(a-c)(a-d)(b-c)(b-d)}{(a+b-c-d)^2}.$$

78. Extract the cube roots of the 3 quantities and then

$$a^{\frac{1}{3}}x=b^{\frac{1}{3}}y=c^{\frac{1}{3}}z\qquad\therefore\ \frac{1}{x}=\frac{a^{\frac{1}{3}}}{c^{\frac{1}{3}}}\cdot\frac{1}{z},\ \frac{1}{y}=\frac{b^{\frac{1}{3}}}{c^{\frac{1}{3}}}\cdot\frac{1}{z},\ \frac{1}{z}=\frac{c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\cdot\frac{1}{z}$$

$$\therefore\ \frac{1}{d}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\left(\frac{a^{\frac{1}{3}}}{c^{\frac{1}{2}}}+\frac{b^{\frac{1}{3}}}{c^{\frac{1}{3}}}+\frac{c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)\cdot\frac{1}{z}=\left(\frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)\cdot\frac{1}{z}$$

$$\therefore\ z = d\,\frac{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}{c^{\frac{1}{3}}}$$; in the same manner

$$y = d\,\frac{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}{b^{\frac{1}{3}}},\ \text{and}\ x = d\,\frac{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}{a^{\frac{1}{3}}}$$

$$\therefore\ ax^2 = ad^2\,\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2}{a^{\frac{1}{3}}} = d^2(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2 a^{\frac{1}{3}}$$

$$by^2 = bd^2\,\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2}{b^{\frac{2}{3}}} = d^2\,(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2 b^{\frac{1}{3}}$$

$$cz^2 = cd^2\,\frac{(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2}{\text{[illegible]}} = d^2(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2 c^{\frac{1}{3}}$$

$$\therefore\ ax^2 + by^2 + cz^2 = d^2(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^2(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})$$
$$= d^2(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})^3$$

79. The expression $= (x - a + \overline{x - b} + x - c)\{(x - a)^2 + (x - b)^2$
$+ (x - c)^2 - (x - a)(x - b) - (x - a)(x - c) - (x - b)(x - c)\}$
$= x\{x^2 - 2ax + a^2 + x^2 - 2bx + b^2 + x^2 - 2cx + c^2 - (x^2 - ax - bx + ab)$
$- (x^2 - ax - cx + ac) - (x^2 - bx - cx + bc)\} = x(a^2 + b^2 + c^2 - ab - ac - bc)$
$= \frac{1}{2}(a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc) = \frac{1}{2}(a^3 + b^3 + c^3 - 3abc)$

80. Subtracting (2) from (1) we have $z - y = 2(y - z) = 2y - 2z$
$\therefore\ 3z = 3y\ \therefore\ z = y$, again subtracting (3) from (2), $x - z = 2(z - x)$
$= 2z - 2x\ \therefore\ 3x = 3z\ \therefore\ x = y = z$.

81. $$\frac{2x}{1 - x^2} + \frac{2y}{1 - y^2} + \frac{2z}{1 - z^2} = 2\left\{\frac{x - xy^2 + y - x^2y}{(1 - x^2)(1 - y^2)} + \frac{z}{1 - z^2}\right\}$$

$$= 2\left\{\frac{(x + y)(1 - xy)}{(1 - x^2)(1 - y^2)} + \frac{z}{1 - z^2}\right\} = 2\left\{\frac{z}{1 - z^2} - \frac{(x + y)(x + y)}{z(1 - x^2)(1 - y^2)}\right\}$$

But since $x+y+z=xyz$ $\quad\therefore\ z-xyz=-(x+y)$

$$\therefore\ z(1-xy)=-(x+y) \qquad \therefore\ 1-xy=-\frac{x+y}{z}$$

$$\therefore \text{ the expression, } =2\left\{\frac{z^2(1-x^2)(1-y^2)-(x+y)^2(1-z^2)}{z(1-x^2)(1-y^2)(1-z^2)}\right\}$$

$$=2\left\{\frac{z^2(1-x^2-y^2+x^2y^2)-(x^2+2xy+y^2-x^2z^2-2xyz^2-y^2z^2)}{z(1-x^2)(1-y^2)(1-z^2)}\right\}$$

$$=2\left\{\frac{z^2+x^2y^2z^2+2xyz^2-(x+y)^2}{z(1-x^2)(1-y^2)(1-z^2)}\right\} \quad \text{But } x+y=xyz-z \quad \therefore\ (x+y)^2=(xyz-z)^2$$

$$=2\left\{\frac{z^2+x^2y^2z^2+2xyz^2-(xyz-z)^2}{z(1-x^2)(1-y^2)(1-z^2)}\right\}=\frac{2\cdot 4xyz^2}{z(1-x^2)(1-y^2)(1-z^2)}$$

$$=\frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}$$

82. $\dfrac{x^2-yz}{x-xyz}=\dfrac{y^2-xz}{y-xyz}=\dfrac{xz-y^2}{xyz-y}$ $\therefore$ each of the given ratios

$$=\frac{(x^2-yz)+(xz-y^2)}{(x-xyz)+(xyz-y)}=\frac{(x^2-y^2)+z(x-y)}{x-y}=\frac{(x-y)(x+y+z)}{x-y}=x+y+z$$

$\therefore\ \dfrac{x^2-yz}{x-xyz}=x+y+z$ $\ \therefore\ x^2-yz=x(x+y+z)-xyz(x+y+z)=x^2+xy$ $+xz-xyz(x+y+z)$ $\ \therefore z^2-xy=z^2+yz+xz-xyz(x+y+z)=z(1-xy)$ $(x+y+z)$ $\ \therefore\ \dfrac{z^2-xy}{z(1-xy)}=x+y+z$, also $y^2-xz=y^2+yz+xy-xyz$ $(x+y+z)$ $\ \therefore\ (x^2-yz)+(z^2-xy)+(y^2-xz)=x^2+y^2+z^2+2(xy+xz$ $+yz)-3xyz(x+y+z)$ or $3xyz(x+y+z)=3(xy+xz+yz)$

$$\therefore\ x+y+z=\frac{xy+xz+yz}{xyz}=\frac{1}{z}+\frac{1}{y}+\frac{1}{x}$$

83. From the 1st, $ax^2+abc=a^3$ $\quad\therefore\ ax^2=a^3-abc$ from the 2nd, $by^2=b^3-abc$ and from the 3rd, $cz^2=c^3-abc$

$$\therefore\ ax^2+by^2+cz^2=a^3+b^3+c^3-3abc$$

$$=(a+b+c)(a^2+b^2+c^2-ab-ac-bc)$$

$$=(a+b+c)(a^2-bc+b^2-ac+c^2-ab)=(a+b+c)(x^2+y^2+z^2)$$

84. Let each of the fractions $= m$ then $cy + bz = m(b - c)$, $az + cx = m(c - a)$, $bx + ay = m(a - b)$ $\therefore$ $cy + bz + az + cx + bx + ay$ $= m(b - c + c - a + a - b) = 0$, adding to each side $ax + by + cz$ we have $ax + bx + cz + ay + by + cy + az + bz + cz = ax + by + cz$ or $x(a + b + c) + y(a + b + c) + z(a + b + c) = ax + by + cz$ or $(a + b + c)(x + y + z) = ax + by + cz$

85. $a + b + c = 0$ $\therefore$ $a + b = -c$ $\therefore$ $a^2 + 2ab + b^2 = c^2$ or $a^2 + b^2 - c^2 = -2ab$ $\therefore$ $c(a^2 + b^2 - c^2) = -2abc$ Similarly $b(a^2 + c^2 - b^2) = -2abc$ and $a(b^2 + c^2 - a^2) = -2abc$

86. $a + x = a + \frac{2ab}{b^2 + 1} = \frac{a(b^2 + 1 + 2b)}{b^2 + 1} = \frac{a(b + 1)^2}{b^2 + 1}$

$\therefore$ $\sqrt{a + x} = \frac{\sqrt{a}(b + 1)}{\sqrt{b^2 + 1}}$; $\sqrt{a - x} = \frac{\sqrt{a}\ b - 1)}{\sqrt{b^2 + 1}}$ $\therefore$ the expression $= b$

87. $a + b + c = 0$ $\therefore$ $a^2 + b^2 + 2ab = c^2$ or $(a^2 + b^2 - c^2)^2 = 4a^2b^2$ or $a^4 + b^4 + c^4 = 2a^2b^2 + 2b^2c^2 + 2c^2a^2$, adding $a^4 + b^4 + c^4$ to both sides we have $2(a^4 + b^4 + c^4) = a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$ $= a^2 + b^2 + c^2)^2$ again $a^4 + b^4 + c^4 = 2(a^2b^2 + b^2c^2 + a^2c^2)$ $\therefore$ $2(a^4 + b^4 + c^4) = 4(a^2b^2 + b^2c^2 + c^2a^2)$

88. $a = b^x = (c^y)^x = c^{xy}) = (a\ )^{xy} = a^{xyz}$ $\therefore$ $xyz = 1$

89. multiply the 3equs. we have $x \times y \times z = yz \times xz \times xy$ or $xyz = 1$.

90. By cross multiplication $b^2x^2 + a^2by = a^2y^2 + ab^2x$ or $b^2x^2 - a^2y^2$ $= ab^2x - ab^2x - a^2by$ or $(bx + ay)(bx - ay) = ab(bx - ay)$ $\therefore$ $bx + ay = ab$ and $bx - ay = 0$ $\therefore$ $bx = ay$.

INEQUALITIES.

Ex. 30. 1. $x - y$ may be either positive or negative and $\because$ the square of any quantity either positive or negative is positive $\therefore (x - y)^2$ is positive and $\therefore$ > 0; i.e. $x^2 - 2xy + y^2 > 0$ or $x^2 + y^2 > 2xy$.

2. If $x+y>z$, $x+z>y$, $y+z>x$ then $z(x+y)>z^2$, $y(x+z)>y^2$ and $x(y+z)>x^2$ or $xz+yz>z^2$, $xy+yz>y^2$ and $xy+xz>x^2$ or $2(xy+xz+yz)>x^2+y^2+z^2$ or $2(xy+xz+yz)-x^2-y^2-z^2>0$.

3. From ex. 2 $2xy+2xz+2yz>x^2+y^2+z^2$ $\therefore$ $4xy+4xz+4yz$ $>x^2+y^2+z^2+2xy+2xz+2yz$ or $2x(y+z)+2y(x+z)$ $+2z(x+y)>(x+y+z)^2$.

4. Let $y=x+a$, $z=x+b$ then $(x+y+z)^3=(3x+a+b)^3$ $=27x^3+27x^2(a+b)+9x(a+b)^2+(a+b)^3$ .....(1) again $27xyz$ $=27\{x(x+a)(x+b)\}=27x^3+27x^2(a+b)+27abx$...(2) $\therefore$ (1) − (2) $=9x(a+b)^2-27abx+(a+b)^3=9x(a-b)^2+9abx+(a+b)^3$ a positive quantity $\therefore$ $(x+y+z)^3>27xyz$

5. Since every quantity, when squared, is positive and therefore >0 we have $(a-b)^2>0$, unless $a=b$ or $a^2-2ab+b^2>0$, or a^2+b^2 $>2ab$. Similarly $a^2+c^2>2ac$ unless $a=c$, and $b^2+c^2>2bc$, unless $b=c$ $\therefore$ by addition $2a^2+2b^2+2c^2>2ab+2ac+2bc$ or $a^2+b^2+c^2$ $>ab+ac+bc$.

6. Let $y=x+a$ and $z=x+b$ then $9(x^3+y^3+z^3)=9(x^3+x^3+3ax^2$ $+3a^2x+a^3+x^3+3bx^2+3b^2x+b^3)=27x^3+27x^2(a+b)+27x(a^2+b^2)$ $+9(a^3+b^2)$.. ......(1 and it has been shewn in ex 4 that $(x+y+z)^3$ $=27x^3+27x^2(a+b)+9x(a+b)^2+(a+b)^3$...(2) $\therefore$ (1) − (2) $=27x(a^2+b^2)$ $-9x(a+b)^2+9(a^3+b^3)-(a+b)^3=9x(2a^2+2b^2-2ab)+(a+b)$ $(8a^2+8b^2-7ab)=18x(a-b)^2+18ax+(a+b)\{8(a-b)^2+9ab\}$ $=$ a positive quantity $\therefore$ $9(x^3+y^3+z^3)>(x+y+z)^3$.

7. $x^4-(x+1)^3=x^4-x^3-3x^2-3x-1=x^3(x-3)+2x^2(x-3)$ $+2x(x-3)+6(x-3)+17$ which is always positive if x is not less than 3 $\therefore$ $x^4>(x+1)^3$ $\therefore$ $x^{\frac{4}{12}}>(x+1)^{\frac{3}{12}}$ or $\sqrt[3]{x}>\sqrt[4]{(x+1)}$.

8. From ex. 5 $x^4+y^4+z^4>x^2y^2+x^2z^2+y^2z^2$ again $\because$ x^2+y^2 $>2xy$ $\therefore$ $z^2(x^2+y^2)>2xyz^2$, similarly $y^2(x^2+z^2)>2xzy^2$ and $x^2(y^2+z^2)>2yzx^2$ $\therefore$ $z^2(x^2+y^2)+y^2(x^2+z^2)+x^2(y^2+z^2)>2xyz^2$

$+ 2xzy^2 + 2yzx^2$ or $2(x^2z^2 + y^2z^2 + x^2y^2) > 2xyz(x + y + z)$ $\therefore$ $x^2z^2 + y^2z^2 + x^2y^2 > xyz(x + y + z)$ but it has been shewn that $x^4 + y^4 + z^4 > x^2y^2 + x^2z^2 + y^2z^2$ $\therefore$ $x^4 + y^4 + z^2 > xyz(x + y + z)$.

9. $x^3 + y^3 - (x^2y + y^2x) = (x + y)(x^2 - xy + y^2) - xy(x + y)$ $=(x + y)(x^2 - 2xy + y^2) = (x + y)(x - y)^2$ a positive quantity $\therefore$ $x^3 + y^3 > x^2y + y^2x$

10, Multiply by 12 the L. C. M. of the denominators then $3x + 6 + 4x < 6x + 12$ and $> 6x + 10$ or $7x + 6 < 6x + 12$ and $> 6x + 10$ or $x < 6$ and > 4 ; hence $x=5$

11. $2(y+1)^3 >$ or $< y+2$ if $2(y^3 + 3y^2 + 3y + 1) >$ or $< y+2$ if $y^3 + 6y^2 + 6y + 2 >$ or $< y+2$ if $2y^3 + 6y^2 + 5y >$ or < 0 but $2y^3 + 6y^2 + 5y$ is positive and $\therefore$ > 0 $\therefore$ $2(y + 1)^3 > y + 2$.

12. $a^2 > a^2 - (b - c)^2$ $\because$ $(b - c)^2$ is positive $b^2 > b^2 - (a - c)^2$ $\because$ $(a - c)^2$ is positive, $c^2 > c^2 - (a - b)^2$ $\because$ $(a - b)^2$ is positive $\therefore$ $a^2b^2c^2 > \{a^2 - (b - a)^2\}\{b^2 - (a - c)^2\}\{c^2 - (a - b)^2\} > (a + b - c)^2 (a + c - b)^2(b + c - a)^2$ $\therefore$ $abc > (a + b - c)(a + c - b)(b + c - a)$.

13. $4x - 7 < 2x + 3$ $\therefore$ $2x < 10$ or $x < 5$ again $3x + 1 > 13 - x$ $\therefore$ $4x > 12$ or $x > 3$ $\therefore$ $x = 4$.

14. $x^2y^2 - (ac + bd)^2 = (a^2 + b^2)(c^2 + d^2) - (ac + bd)^2$ $= a^2d^2 + b^2c^2 - 2abcd = (ad - bc)^2$

$$\therefore \; xy - (ac + bd) = \frac{(ad - bc)^2}{xy + (ac + bd)} = \text{a positive quantity unless } ad=bc$$

$\therefore$ $xy > ac + bd$ unless $ad = bc$.

15. $\therefore$ the square of any quantity either positive or negative is positive $\therefore$ $(n - 1)^2$ is positive $\therefore$ $(n - 1)^2 > 0$ or $n^2 - 2n + 1 > 0$ or $n^2 - n + 1 > n$ or $(n + 1)(n^2 - n + 1) > n(n + 1)$ or $n^3 + 1 > n^2 + n$.

16. Multiply by 12 the L. C. M. of the denominators, then $3x + 6 + 4x < 6x - 24 + 36$, and $> 6x + 6 + 4$ or $7x + 6 < 6x + 12$ and $> 6x + 10$ or $x < 6$, and > 4 ; Hence $x = 5$.

17. From Ex. 1 $x^2 + y^2 > 2xy \quad \therefore \; z(x^2 + y^2) > 2xyz$

Similarly $y(x^2 + z^2) > 2xyz$

and $x(y^2 + z^2) > 2xyz$

$\therefore \; z(x^2 + y^2) + y(x^2 + z^2) + x(y^2 + z^2) > 6xyz$

or $zx^2 + zy^2 + yx^2 + yz^2 + xz^2 + xy^2 > 6xyz$

or $xz(x + z) + yz(y + z) + xy(x + y) > 6xyz$

18. $3(x^2 - x + 1) - (x^2 + x + 1) = 2x^2 - 4x + 2 = 2(x - 1)^2 =$ a positive quantity $\therefore \; 3(x^2 - x + 1) > x^2 + x + 1 \; \therefore \dfrac{x^2 - x + 1}{x^2 + x + 1} > \frac{1}{3}$.
again $3(x^2 + x + 1) - (x^2 - x + 1) = 2x^2 + 4x + 2 = 2(x + 1)^2 =$ positive quantity $\therefore \; 3(x^2 + x + 1) > x^2 - x + 1 \; \therefore \; 3 > \dfrac{x^2 - x + 1}{x^2 + x + 1} \; \therefore \dfrac{x^2 - x + 1}{x^2 + x + 1}$
< 3, and $> \frac{1}{3}$.

19. $\dfrac{x^n - y^n}{x - y} = x^{n-1} + x^{n-2}y + x^{n-3}y^2 \ldots\ldots y^{n-1}$

now since $x > y \;\therefore\; x^n > y^n \;\therefore\; x^{n-1} + x^{n-2}y + x^{n-3}y^2 \ldots\ldots y^{n-1}$

$> y^{n-1} + y^{n-1} + {}^{n-1} \ldots\ldots y^{n-1}$ or $> ny^{n-1}$ Hence $\dfrac{x^n - y^n}{x - y}$

$> ny^{n-1}$ *i. e.* $x^n - y^n > ny^{n-1}(x - y)$

Again $x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots\ldots + y^{n-1} < x^{n-1} + x^{n-1} + x^{n-1}$

$\ldots\ldots\ldots + x^{n-1}$ or $< nx^{n-1}$

$\therefore \dfrac{x^n - y^n}{x - y} < n, x^{n-1} \;\therefore\; x^n - y^n < nx^{n-1}(x - y)$

20. $(x^4 + y^4 + 1)(z^4 + w^4 + 1) - (x^2z^2 + y^2w^2 + 1)^2 = x^4z^4 + x^4w^4 + x^4 + y^4z^4 + y^4w^4 + y^4 + z^4 + w^4 + 1 - (x^4z^4 + y^4w^4 + 2x^2z^2 + 2y^2w^2 + 2x^2y^2z^2 w^2 + 1) = x^2w^2 + y^2z^2 + x^4 + y^4 + z^4 + w^4 - 2x^2z^2 - 2y^2w^2 - 2x^2y^2z^2w^2,$
$= (x^2w^2 - y^2z^2)^2 + (x^2 - z^2)^2 + (y^2 - w^2)^2$ which is always positive.

21. $x^6 + x^4y^2 + x^2y^4 + y^6 >$ or $< (x^2 + y^3)^2$ if the former $>$ or $< x^6 + 2x^3y^3 + y^6$ or $x^2y^2(x^2 + y^2) >$ or $< 2x^3y^3$ or $x^2 + y^2 >$ or $< 2xy$
but by Ex. 1, $x^2 + y^2 > 2xy \quad \therefore$ the first $>$ the second.

22. $\{(\frac{1}{2})^{\frac{1}{2}}\}^6 > =$ or $< \{(\frac{2}{3})^{\frac{2}{3}}\}^6$ according as $(\frac{1}{2})^3 > =$ or $< (\frac{2}{3})^4$ according as $\frac{1}{8} > =$ or $< \frac{16}{81}$ but $\frac{16}{81} > \frac{1}{8}$ $\therefore$ last quality > the first.

23. $(a-b)-(\sqrt{a}-\sqrt{b})^2 = 2\sqrt{(ab)}-2b = 2\sqrt{b}(\sqrt{a}-\sqrt{b}) =$ positive quantity $\because a > b$ $\therefore a-b$ is greater.

24. Let $x = 3+r$, if $x^{\frac{1}{2}} = (x+1)^{\frac{1}{3}}$ then $(3+r)^{\frac{1}{2}} = (4+r)^{\frac{1}{3}}$ or $(3+r)^3 = (4+r)^2$ or $27+9r+3r^2+r^3 = 16+8r+r^2$ or $r^3+2r^2+r+11=0$ which is absurd for any positive value of r; it is neither less for then $r^3+3r^2+9r+27 < 16+8r+r^2$ or $r^3+2r^2+r+11 < 0$ which is absurd for any positive value of r $\therefore$ 1st > 2nd.

25. Let $x =$ no.; then $2x+7 =$ or < 19 and $3x-5 =$ or > 13 $\therefore$ $2x =$ or < 12 and $3x =$ or > 18 $\therefore$ $x =$ or > 6 and $x =$ or > 6 $\therefore$ $x = 6$.

26. $2(x^4+x^2+1)-3x(1+x^2) = 2(1-2x+x^2)+x-3x^3+2x^4$
$= 2(1-x)^2+x(1-3x^2+2x^3) = 2(1-x)^2+(1-x)^2(x+2x^2)$
$= (1-x)^2(2+x+2x^2)$ which is positive $\because$ x is not $=1$ and
$\therefore$ the former is greater.

27. $\left(\frac{x+y}{2}\right) = xy + \left(\frac{x-y}{2}\right)^2$ $\therefore \left(\frac{x+y}{2}\right)^2 > xy$ Let $x = \frac{a+b}{2}$ and $y = \frac{c+d}{2}$ then $\left\{\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2}\right\}^2 > \frac{a+b}{2} \cdot \frac{c+d}{2}$. or $\{\frac{1}{4}(a+b+c+d)\}^2 > \frac{a+b}{2} \cdot \frac{c+d}{2}$, but again $\frac{a+b}{2} > \sqrt{(ab)}$ and $\frac{c+d}{2} > \sqrt{(cd)}$ $\therefore \frac{a+b}{2} \cdot \frac{c+d}{2} > \sqrt{(abcd)}$ $\therefore$ $\{\frac{1}{4}(a+b+c+d)\}^2 > \sqrt{(abcd)}$ or $\frac{1}{4}(a+b+c+d) > \sqrt[4]{(abcd)}$.

Ex. 31. ELIMINATIONS.

1. Squaring the 1 eqn we have $x^2+y^2+2xy=m^2$, substitute for x^2+y^2 then $2xy=m^2-n$ ∴ $xy=\frac{m^2-n}{2}$. Divide the 3rd eqn by the 1st, then $\frac{x^3+y^3}{x+y}=\frac{p}{m}$ or $x^2-xy+y^2=\frac{p}{m}$ or $(x^2+y^2)-xy=\frac{p}{m}$ ∴ $n-\frac{m^2-n}{2}=\frac{p}{m}$ $2p=3mn-m^3$

2 From the 1st eqn $m+n=x-a$, from the 3rd $mn=\frac{z}{a}$, Substitute these in the 2nd eqn then $mn+a(m+n)=y$ or $\frac{z}{a}+a(x-a)=y$ ∴ $z+a^2(x-a)=ay$ ∴ $a^3-a^2x+ay=z$.

3 $a^2x^2+2abxy+b^2y^2=c^2x^2+d^2y^2$
$a^2y^2-2abxy+b^2x^2=c^2y^2+d^2x^2$

add, then $(a^2+b^2)(x^2+y^2)=(c^2+d^2)(x^2+y^2)$ or $a^2+b^2=c^2+d^2$

4 Squaring the 1st equation we have $x^2+y^2-2xy=a^2$ ∴ $x^2+y^2=a^2+2xy$, divide the 2nd by the 1st then $x^2+xy+y^2=\frac{b}{a}$ or $(x^2+y^2)+xy=\frac{b}{a}$ ∴ $a^2+2xy+xy=\frac{b}{a}$ ∴ $a^2+3xy=\frac{b}{a}$ ∴ $a^2+a^2=\frac{b}{a}$ ∴ $b=2a^3$

5. From the 2nd eqn. $x^3+y^3+3xy(x+y)=m$ ∴ $\frac{m}{2}+n\sqrt[3]{m}=m$ ∴ $n\sqrt[3]{m}=\frac{m}{2}$ ∴ $n^3m=\frac{m^3}{8}$ or $m^2=8n^3$

6. From the 1st eqn. $y^2-x^2=ay-bx$ multiply this by y and the 2nd eqn. by x then add ∴ $y^3+3x^2y=a(x^2+y^2)=a$. Again multiply the 1st by x and the 2nd by y, then subtract the 1st result from the 2nd and we have $x^3+3xy^2=b(x^2+y^2)=b$ ∴ $a+b=(x+y)^3$

$\therefore\ (a+b)^{\frac{1}{3}}=x+y \quad \therefore\ (a+b)^{\frac{2}{3}}=(x+y)^2$ also $a-b=(x-y)^3$

$\therefore\ (a-b)^{\frac{1}{3}}=x-y \quad \therefore\ (a-b)^{\frac{2}{3}}=(x-y)^2 \quad \therefore\ (a+b)^{\frac{2}{3}}+(a-b)^{\frac{2}{3}}$
$=(x+y)^2+(x-y)^2=2(x^2+y^2)=2.$

7. From 1st and 3rd eqns we get $\left(m-\frac{z}{x}\right)\left(m-\frac{x}{y}\right)\left(m-\frac{y}{z}\right)=p$

$\therefore\ p=m^3-m^2\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)+m\left(\frac{z}{x}\cdot\frac{x}{y}+\frac{z}{x}\cdot\frac{y}{z}+\frac{x}{y}\cdot\frac{y}{z}\right)-\frac{z}{x}\cdot\frac{x}{y}\cdot\frac{y}{z}$
$=m^3-m^2\cdot m+mn-1 \quad \therefore\ mn=1+p.$

8 Square the 2nd eqn then $x^2+2xy+y^2=b^2$ and substituting the 1st eqn we have $xy=b^2-a$ or $a^2=b^2-a \quad \therefore\ a^2+a=b^2$

9 Square the 1st eqn. then $a^2+b^2+c^2+2(ab+ac+bc)=m^2$

Substitute then $n+2p=m^2$.

10 From the 2nd eqn $x^3-y^3=a^3-3a^2b+3ab^2-b^3$

$\therefore\ x^3-a^3-(y^3-b^3)=3ab(b-a)\ldots(1),\quad 1-\frac{y^3-b^3}{x^3-a^3}=\frac{3ab(b-a)}{x^3-a^3}$,

$1-\frac{3a+2b}{2a+3b}=\frac{3ab(b-a)}{x^3-a^3},\quad \frac{1}{2a+3b}=\frac{3ab}{x^3-a^3} \qquad x^3-a^3=6a^2b+9ab^2$

$\therefore\ x^3=a^3+6ab^2+9a^2b \qquad x^{\frac{3}{2}}=a^{\frac{3}{2}}+3a^{\frac{1}{2}}b$, in the same manner dividing (1) by y^3-b^3, $\quad y^{\frac{3}{2}}=b^{\frac{3}{2}}+3ab^{\frac{1}{2}} \quad \therefore\ x^{\frac{3}{2}}+y^{\frac{3}{2}}$
$=a^{\frac{3}{2}}+3ab^{\frac{1}{2}}+3a^{\frac{1}{2}}b+b^{\frac{3}{2}}=(a^{\frac{1}{2}}+b^{\frac{1}{2}})^3=c^{\frac{3}{2}} \quad \therefore\ a^{\frac{1}{2}}+b^{\frac{1}{2}}=c^{\frac{1}{2}}$

11 From the 1st eqn $\frac{a^3}{xyz}=\left(1-\frac{y}{x}\right)\left(1-\frac{z}{y}\right)\left(1-\frac{x}{z}\right)$

$1-\left(\frac{y}{x}+\frac{z}{y}+\frac{x}{z}\right)+\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)$, from the 2nd eqn. $\frac{b^3}{xyz}$

$$=\left(1+\frac{y}{x}\right)\left(1+\frac{z}{y}\right)\left(1+\frac{x}{z}\right)=2+\left(\frac{y}{x}+\frac{z}{y}+\frac{x}{z}\right)+\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)\ldots(a)$$

$\therefore\ \frac{a^3+b^3}{2xyz}=1+\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$ from the 1st, 2nd and 3rd eqns,

$(x^2-y^2)(y^2-z^2)(z^2-x^2)=a^3b^3$ and $(x^2+y^2)(y^2+x^2)(z^2+x^2)=\frac{a^6}{2}$

and these two equations are like the 1st and 2nd given equations and

$\therefore\ \frac{a^3b^3+\frac{a^6}{2}}{2x^2y^2z^2}=1+\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}\quad\therefore\ \frac{(a^3+b^3)^2}{4x^2y^2z^2}-\frac{a^3b^3+\frac{a^6}{2}}{2x^2y^2z^2}$

$=2\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)+2\left(\frac{y}{x}+\frac{z}{y}+\frac{x}{z}\right)=\frac{2b^3}{xyz}-4$ by (a) or $\frac{b^6}{4x^2y^2z^2}=\frac{2b^3-4xyz}{xyz}$

$\therefore\ \frac{b^6}{4xyz}=2b^3-4xyz\quad\therefore\ 4xyz=b^3$ Again from the 1st, 2nd, 3rd and 4th $(x^2-y^2)(y^2-z^2)(z^2-x^2)=a^3b^3$, $(x^2+y^2)(y^2+z^2)(z^2+x^2)$ $=\frac{a^6}{2}$ and $(x^4+y^4)(y^4+z^4)(z^4+x^4)=\frac{a^6b^6}{2}$ which are the 1st, 2nd and 3rd given equations, we have $4x^2y^2z^2=\frac{a^6}{2}\quad\therefore\ 2a^6=b^6$

12. $\frac{x}{m}-\frac{p}{m}+\frac{y}{n}+\frac{q}{n}=0\quad\therefore\ \frac{x}{m}+\frac{y}{n}=\frac{p}{m}-\frac{q}{n}$ but $\frac{pm}{a^2}=-\frac{qn}{b^2}$,

$\frac{m^2}{a^2}=\frac{n^2}{b^2}\quad\therefore\ \frac{p}{m}=-\frac{q}{n}$ but $\frac{m}{a}=\frac{n}{b}\quad\therefore\ \frac{x}{a}+\frac{y}{b}=\frac{2p}{a}=-\frac{2a}{b}$

and $\frac{p^2}{a^2}+\frac{q^2}{b^2}=1=\frac{2p^2}{a^2}=\frac{1}{2}\left(\frac{x}{a}+\frac{y}{b}\right)^2\quad\therefore\ \frac{x}{a}+\frac{y}{b}=\sqrt{2}.$

13. From (1) $b+c=-(a+p)\quad\therefore\ a(b+c)=-a(a+p)$ but from (2) $a(b+c)=q-bc\quad\therefore\ -a(a+p)=q-bc\quad\therefore\ -a^2-ap=q-bc$ or $-a^3-a^2p=qa-abc=qa+r$ from (3); or $a^3+a^2p+qa+r=0.$

14. $\frac{x^a}{m^{a+b}} = \frac{y^a}{n^{a+b}} = \frac{z^a}{p^{a+b}} \quad \therefore \left(\frac{y}{n}\right)^a = \left(\frac{n}{m}\right)^b \cdot \left(\frac{x}{m}\right)^a$, and $\left(\frac{z}{p}\right)^a$ $= \left(\frac{p}{m}\right)^b \left(\frac{x}{m}\right)^a \quad \therefore \left(\frac{x}{m}\right)^a + \left(\frac{n}{m}\right)^b \left(\frac{n}{m}\right)^a + \left(\frac{p}{m}\right)^b \left(\frac{x}{m}\right)^a = 1$

$\therefore \left(\frac{x}{m}\right)^a = \frac{m^b}{m^b + n^b + p^b} = \left(\frac{m}{q}\right)^b \quad \therefore \frac{m^{a+b}}{q^b} = x^a$ or $\left(\frac{m}{q}\right)^{a+b} = \left(\frac{x}{q}\right)^a$

$\therefore \left(\frac{m}{q}\right)^b = \left(\frac{x}{q}\right)^{\frac{ab}{a+b}}$; similarly $\left(\frac{n}{q}\right)^b = \left(\frac{y}{q}\right)^{\frac{ab}{a+b}}$ and $\left(\frac{p}{q}\right)^b = \left(\frac{z}{q}\right)^{\frac{ab}{a+b}}$

$\therefore \left(\frac{x}{q}\right)^{\frac{ab}{a+b}} + \left(\frac{y}{q}\right)^{\frac{ab}{a+c}} + \left(\frac{z}{q}\right)^{\frac{ab}{a+b}} = 1$ or $x^{\frac{ab}{a+b}} + y^{\frac{ab}{a+b}} + z^{\frac{ab}{a+c}} = q^{\frac{ab}{a+b}}$

APPLICATION OF ALGEBRA TO GEOMETRY.

Ex. 32. 1 Let x = other side then $8^2 + x^2 = 10^2 \quad \therefore x^2 = 10^2 - 8^2 = (10+8)(10-8) = 18 \times 2 = 6^2$.

2, Let x = other side then $x + 1$ = hypotenuse $\therefore 5^2 + x^2 = (x+1)^2$ or $5^2 + x^2 = x^2 + 2x + 1 \quad \therefore 2x + 1 = 25 \quad \therefore 2x = 24 \quad \therefore x = 12$.

3 Let x = other side then $50 - x$ = hypotenuse $\therefore 40^2 + x^2 = (50 - x)^2$ or $40^2 + x^2 = 2500 - 100x + x^2$ $\therefore 100x = 2500 - 1600 - 900 \quad \therefore x = 9$.

4. Let x = one side then $7 - x$ = other side $\therefore x^2 + (7 - x)^2 = 5^2$ or $x^2 + 49 - 14x + x^2 = 25 \quad \therefore 2x^2 - 14x = 25 - 49 = -24 \quad \therefore x^2 - 7x = -12 \quad \therefore x^2 - 7x + (\tfrac{7}{2})^2 = \tfrac{49}{4} - 12 = \tfrac{1}{4} \quad \therefore x - \tfrac{7}{2} = \tfrac{1}{2} \quad \therefore x = \tfrac{7}{2} + \tfrac{1}{2} = 4$

5. Let x = one side then $x + 8$ = hypotenuse and the other side $= (x + 8) - 4 = x + 4 \quad \therefore x^2 + (x+4)^2 = (x+8)^2 \quad \therefore x = 12$

5. (*a*) Let x = one side then $\sqrt{10^2 - x^2}$ = other side, then $\tfrac{1}{2}x\sqrt{10^2 - x^2} = 24 \quad \therefore x\sqrt{10^2 - x^2} = 48 \quad \therefore x^2(10^2 - x^2) = 48^2$ $\therefore x^4 - 100x^2 + 2500 = 14^2 \quad \therefore x^2 - 50 = 14, \quad x^2 = 64 \quad \therefore x = 8$,

6. Let x = segment adjacent to the side 50, then $x=\sqrt{50^2-40^2}=30$ similarly the other segment $=9$ $\therefore$ base $=39$.

7. Let x = side then altitude $=\frac{\sqrt{3}}{2}x$ $\therefore$ $\frac{1}{2}x\times\frac{\sqrt{3}}{2}x=16\sqrt{3}$

$\therefore$ $x^2=\frac{16\sqrt{3}\times4}{\sqrt{3}}=64$ $\therefore$ $x=8$ $\therefore$ perimeter $=24$.

8. Let ABC be the $\triangle$ (see fig. 1) and AD the altitude, then $AD^2=x^2-5^2$ also $AD^2=(x+2)^2-9^2$ $\therefore$ $x^2-5^2=(x+2)^2-9^2$ $\therefore$ $x=13$.

9. Let ABC be a $\triangle$ (see fig. 2) and let ABDC the described $\odot$. Draw AD the diameter, draw $AE\perp BC$; produce CA to F and make $AF=AB$; produce DA to G and make $AG=AE$; join FG. Because $\angle ABE=\angle ADC$ and $\angle AEB=\angle ACD=$ one right angle $\therefore$ $\angle BAE=\angle CAD=\angle FAG$; also $\because$ GA, $AF=AE$, AB and $\angle FAG=\angle BAE$ $\therefore$ the 2 $\triangle$S are equal $\therefore$ $\angle FGA=\angle AEB=$ one rt $\angle$ $\therefore$ if a circle be described on FD as diameter it will pass through G and C $\therefore$ $DA\cdot AG=CA\cdot AF$ (Euc. III. 35 or $DA\cdot AE=CA\cdot AB$ or $DA\cdot BC\cdot AE=CA\cdot AB\cdot BC$ or $DA=\frac{CA\cdot AB\cdot BC}{BC\cdot AE}=\frac{\text{Product of the 3 sides}}{\text{2 area of the }\triangle}$. In the present case therefore diameter $DA=\frac{15\times15\times18}{2\times108}=18\frac{3}{4}$. See Todhunter's Mens. p. 76.

10. Let AB, fig 3, be the surface of a lake and CD the stem of the lotus, ED the portion of the stem visible on the surface of the water. Let CE be the depth of the water $=x$ $\therefore$ $CD=x+1$ $\because$ $CF=x+1$, now $CF^2=CE^2+EF^2$ or $(x+1)^2=x^2+5^2$ or $x^2+2x+1=x^2+25$ $\therefore$ $2x+1=25$ $\therefore$ $x=12$.

11. Let x = length of the string $\therefore$ $x=\sqrt{(32^2+24^2+9^2)}=41$.

12. Let ABC be a triangle (fig 17); bisect angle ABC, ACB by BO, CO then O is the centre and perpendiculars OD, OE, OF to the

sides are equal ∴ they are radii of the circle ; now ABC = AOB $+ \text{AOC} + \text{BOC} = \frac{1}{2}cr + \frac{1}{2}br + \frac{1}{2}ar = \frac{1}{2}(a+b+c)r \therefore r = \frac{\text{ABC}}{\frac{1}{2}(a+b+c)}$ In the present case $= \frac{24}{12} = 2$.

13. BD is the side of the decagon (Fig of the 10th, proposition of the 4th book of Euclid)

Let $\text{BD} = x = \text{AC}$ then $\text{BC} = 10 - x \therefore \text{AB}.\text{BC} = \text{AC}^2$

$\therefore 10(10-x) = x^2$ or $x^2 + 10x = 100$ or $x^2 + 10x + 25 = 125$

$\therefore x + 5 = \sqrt{(125)} = 11.3 \,..\, \therefore x = 11.3 - 5 = 6{\cdot}3$.

The length of the perpendiculr from the centre on this side can be calculated and the fig is divided into 10 equal triangles having 6·3 for the base and this perpendicular for the altitude.

14. Let AC be the ladder (fig 4) in the position at fist put and CE the position in the 2nd time then it can be proved by Euclid 1.26 that the 2 ◁ s ABC, CDE are equal in all respects ∴ BC = 52 and CD = 39 ∴ BD = BC + CD = 52 + 39 = 91.

15. $x = \sqrt{(10^2 + 10^2)} = \sqrt{(2 \times 10^2)} = \sqrt{2}{\cdot}10$.

16. Let ABC be a ◁ and CD a perpendicular on AB (fig 5) ; first les $\text{BD} = x$ then $\text{AC}^2 = \text{BC}^2 + \text{BA}^2 - 2\,\text{AB}.\text{BD}$ (Euc. II. 13) or $b^2 = a^2 + c^2 - 2cx \therefore 2cx = a^2 + c^2 - b^2 \therefore x = \frac{a^3 + c^2 - b^2}{2c}$ ∴ from the right angled ◁ BCD, $\text{CD} = \sqrt{a^2 - \left(\frac{a^2+c^2-b^2}{2c}\right)^2} = \frac{1}{2c}\sqrt{4a^2c^2 - (a^2+c^2-b^2)}$

$$= \frac{1}{2c}\sqrt{(2ac + a^2 + c^2 - b^3)(2ac - a^2 - c^2 + b^2)}$$

$$= \frac{1}{2c}\sqrt{\{(a+c)^2 - b^2\}\{b^2 - (a - \text{[illegible]})^2\}}$$

$$= \frac{1}{2c}\sqrt{(a+c+b)(a+c-b)(b+a-c)(b-a+c)}$$

$$=\frac{1}{2c}\sqrt{2s\,2(s-b)\cdot 2(s-c)\cdot 2(s-a)}=\frac{2}{c}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\therefore \text{ area}=\tfrac{1}{2}\text{AB}\cdot\text{CD}=\frac{c}{2}\times\frac{2}{c}\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{s(s-a)(s-b)(s-a)}$$

18. Let AB (fig. 6) be the steeple and let AC be 150 feet; bisect BC in D then DB = DA = DC and ACD is equilateral and BC = 2AC = 2 × 150 = 300 ∴ AB = $\sqrt{300^2-150^2}$ = 259,8 ..

19. Let AB be the tower (fig 7), C and D are two mile posts then evidently CAD = 30° ∴ AC = CD = 1 mile, make BE = BC and join AE then ACE is an equilateral triangle each side of which is one mile and the steeple AB is the altitude of the equilateral ◁.

20. Let AB (fig. 8) be the light house and C the ship; then proceeding as in Ex. 18 we find BC = 346 feet.

21. Let ABC, ADC be two circles, (fig. 9) DA = 30, PA = 25 then AR the perpendicular = 24 ∴ chord AC = 2AR = 48.

22. Lat ABCD be the rectangle, (fig. 11) BD the diagonal $=d$, draw AG ⊥ BD and from G draw GH, GK perpendiculars to DC, CB, let GH $=p$, GK $=q$; for the sake of convenience let us assume AD $=a$ then AB $=\sqrt{d^2-a^2}$ then $\frac{1}{2}\cdot$AG$\cdot d$ = area of the ◁ ABD $=\frac{1}{2}\cdot$AD $\cdot$AB $=\frac{1}{2}a\sqrt{a^2-a^2}$ ∴ AG $=\frac{a\sqrt{d^2-a^2}}{d}$ and from the right-angled ◁ ADG where AD $=a$ and AG $=\frac{a\sqrt{d^2-a^2}}{d}$ we calculate DG $=\frac{a^2}{d}$ and from the ◁ ABG we calculate BG $=\frac{d^2-a^2}{d}$ Again $\frac{1}{2}\cdot a\cdot$GE = area of ◁ ADG $=\frac{1}{2}$AG$\cdot$DG $=\frac{1}{2}\cdot\frac{a\sqrt{d^2-a^2}}{d}\cdot\frac{a^2}{d}$ ∴ GE $=\frac{\sqrt{d^2-a^2}\cdot a^2}{d^2}$, also $\frac{1}{2}\sqrt{a^2-a^2}\cdot$GF = area of ◁ ABG $=\frac{1}{2}$AG$\cdot$BG $=\frac{1}{2}\cdot\frac{a\sqrt{d^2-a^2}}{d}\cdot\frac{(d^2-a^2)}{d}$

$\therefore$ GF $= \frac{a(d^2-a^2)}{d^2}$ Now GE + GK = EK = AB or $\frac{\sqrt{(d^2-a^2)}\cdot a^2}{d^2} + q$

$= \sqrt{(d^2-a^2)}$ $\therefore$ $(d^2-a^2)\sqrt{(d^2-a^2)} = qd^2$ or $(d^2-a^2)^{\frac{3}{2}} = qd^2$

$\therefore q = \frac{(d^2-a^2)^{\frac{3}{2}}}{d^2}$ $\therefore$ $q^{\frac{2}{3}} = \frac{d^2-a^2}{d^{\frac{4}{3}}}$ (1) Also GF + GH = FH = AD or

$\frac{a(d^2-a^2)}{d^2} + p = a$ or $a^3 = pd^2$ $\therefore$ $p = \frac{a^3}{d^2}$ $\therefore$ $p^{\frac{2}{3}} = \frac{a^2}{d^{\frac{4}{3}}}$......(2) ; adding (1)

and (2) we have $q^{\frac{2}{3}} + p^{\frac{2}{3}} = \frac{d^2-a^2+a^2}{d^{\frac{4}{3}}} = d^{\frac{2}{3}}$.

23 Let A be the kite (fig. 13) and AB = kite string; bisect AB in D then BD = DA = DC $\therefore$ AC = AD = 90.

24. Let x = distance from the root, then the part broken is $50 - x$. $\therefore$ $x^2 + 10^2 = (50-x)^2 = 2500 - 100x + x^2$ $\therefore$ $100x = 2\ 00$ $\therefore$ $x = 24$.

25. Let AB be the pillar (fig. 14), B the snakes hole, BD = 3AB = 27, let x = BC, C being the place where the snake was pounced upon then AC = CD = $27 - x$. Now AB² + BC² = AC² or $9^2 + x^2 = (27-x)^2 = 729 - 54x + x^2$ $\therefore$ $54x = 648$ $\therefore$ $x = 12$.

26. Let AB (fig 15) be the tree, AC = x = distance vaulted by one of the monkeys then $\because$ AB + BD = 100 + 200 = 300 = AC + CD $\therefore$ CD $300 - x$ Now BC² + BD² = CD² or $(100+x)^2 + 200^2 = (300-x)^2$ $\therefore$ $x = 5$.

27. If DC be produced to meet the circumference at E (figure of Euc. IV. 10 or figure 12 of this book) and EB be joined then EB = p = side of the inscribed pentagon $\because$ $\angle$ EAB being at the centre = 2 $\angle$ EDB = 2 $\angle$ BAD = $\frac{1}{5}$ of 4 right angles also BD = CD = AC = d = side of the inscribed decagon $\because$ $\angle$ BAD = $\frac{1}{10}$ of 4 right angles and AB = AE = EC = h = side of the inscribed hexagon $\because$ $\angle$ ACE = $\angle$ BCD = 2 $\angle$ CDB $\therefore$ EF a perpendicular to AC will bisect it Now BE² = EF²

$+ FB^2$ and $EC^2 = EF^2 + FC^2$ $\therefore$ $BE^2 - EC^2 = FB^2 - FC^2 = (FB + FC)(FB - FC) = AB \cdot BC = AC^2 = BD^2$ or $p^2 - h^2 = d^2$ or $p^2 = h^2 + d^2$.

28. Let ABD be the circle (fig. 10) AB = AD = radius then AB is the side of the inscribed hexagon and AC of the dodecagon when C is the point of bisection of AB, AE a perpendicular on DB and produced to D then AD is the side of an equilateral triangle inscribed; area of the dodecagon = 6 times the trapezium $AOBC = \frac{1}{2} \times 6 \times OC \times AB = 3r^2$ also $AE = \frac{\sqrt{3}r}{2}$ $\therefore$ $AD = \sqrt{3}r$ $\therefore$ $AD^2 = 3r^2$.

29. Let $AD = x$ then $AB^2 + AC^2 = 2\ BD^2 + 2AD^2$ (Euc II. A)
or $16^2 + 20^2 = 2 \times 9^2 + 2x^2$ $\therefore$ $x = 15 \cdot 7$ nearly.

30. Let O be the point where the two strings cross each other (fig 18) and let OE be drawn perpendicular to AC; let $OE = x$ and $AC = d$; then OEC is similar to BAC $\therefore 12 : d :: x : EC$ $\therefore$ $EC = \frac{dx}{12}$

$\therefore AE + d - EC = \frac{12d - dx}{12}$ now $\therefore$ OEA is similar to DCA $6 : d :: x : AE$

$\therefore AE = \frac{dx}{6} \therefore \frac{dx}{6} = \frac{12d - dx}{12}$ or $x = \frac{12 - x}{2}$ $\therefore$ $x = 4$.

31. Let ABCD (fig 19) be the quadrilateral, $AC = 40$, $BD = 50$ the diagonals and $\angle AOB = 45^\circ$; draw $BE \perp AC$, draw $BF \parallel AC$ and draw $DF \perp BF$; now $\because BF \parallel AC$ $\therefore$ $\angle OBF = 45$ $\therefore$ $\angle BDF = 45^\circ$ then $DF = BF$, let each of them $= x$ then $x^2 + x^2 = 50^2$ $\therefore$ $x = 25\sqrt{2}$ $\therefore$ area of the quadrilateral $= \frac{1}{2}AC \times (BE + DG) = 20 \times DF = 20 \times 25\sqrt{2} = 500\sqrt{2}$.

32. Let AB (fig 20) be the bar C the middle point of AB so that $AC = 7$, $AD = 8$, $DB = DE = 6$ then $AE = 10$; $2AF^2 + 2CF^2 = AC^2 + CE^2$ (II. A.) or $2 \cdot 5^2 = 2CF^2 = 7^2 + CD^2 + DE^2 = 7^2 + 1^2 + 6^2 = 86$ $\therefore$ $CF^2 = 18 = 9 \times 2$ $\therefore$ $CF = 3\sqrt{2}$.

33. Let ABCD (fig 21) be the rhombus and let $AE \perp DC$, $\angle ADE = 30^\circ$ $\therefore$ $\angle DAE = 60^\circ$; bisect AD in F then $FE = FA$ $\therefore$ $\triangle AFE$ is

equilateral $\therefore$ $AE = AF = \frac{1}{2}AD = \frac{1}{2} \times 20 = 10$ $\therefore$ area $= DC \times AE = 20 \times 10 = 200$

34 Let AH (fig 16) be the height of the hill $= \frac{1}{2}$ mile and $AB = 63$ then HB is approximately equal to $AB = 63$. Let $x =$ radius of the earth then $OH^2 = BH^2 + OB^2$ or $(x + \frac{1}{2})^2 = 63^2 + x^2$ or $x^2 + x + \frac{1}{4} = 63^2 + x^2$ $\therefore$ $x = 63^2 - \frac{1}{4} = 3969$ nearly.

35. Proceed as in Ex 13 to find the side of the inscribed decagon thus let $x =$ side of the inscribed decagon then $16 - x =$ other part of the radius then $16(16 - x) = x^2$ or $256 - 16x = x^2$ or $x^2 + 16x = 256$ $\therefore$ $x^2 + 16x + 64 = 256 + 64 = 320$ $\therefore$ $x + 8 = 17.8$ nearly $\therefore$ $x = 9{\cdot}8$ nearly, again from equ $p^2 = h^2 + d^2$ in Ex. 27 we have $p^2 = 16^2 + (9{\cdot}8)^2 = 256 + 96{\cdot}04 = 352{\cdot}04$ $\therefore$ $p = 18{\cdot}7$.

36. Draw $BD \perp AC$ (fig. 22) then $BD = AD$ $\therefore$ $AD^2 + BD^2 = AB^2$ or $2AD^2 = 40^2 \times 2$ $\therefore$ $AD = 40$ $\therefore$ $DC = 70 - 40 = 30$ and $BD = AD = 40$ $\therefore$ $BC = 50$.

38. Let AB (fig 23) be the light house, CD the observer; from the right angled triangle AOE, $OE = 4000$ miles $OA = 4000 + \frac{120}{5280}$ $\therefore$ AE is known, again from the right angled triangle EOD, $OE = 4000$, $OD = 4000 + \frac{6}{5280}$ $\therefore$ ED is known $\therefore$ AD or approximately the arc BC is known.

40. 1st method. Let $BD = x$ (fig. 24) then $15^2 = 13^2 + 14^2 - 2 \times 14 \times x$ (II. 13) whence $x = 5$ $\therefore$ $AD = 12$.

2nd method. $AC^2 = AD^2 + CD^2$, and $AB^2 = AD^2 + BD^2$ $\therefore$ $AC^2 - AB^2 = CD^2 - BD^2$ $\therefore$ $(15 + 13)(15 - 13) = (CD + BD)(CD - BD) = 14(CD - BD)$ $\therefore$ $CD - BD = 4$ and $CD + BD = 14$ $\therefore$ $CD = 9$ $\therefore$ from the right angled triangle ACD, $AD = 12$.

3rd method. Let $AD = x$ then $BD = \sqrt{(13^2 - x^2)}$ and $CD = \sqrt{(15^2 - x^2)}$ $\therefore$ $BD + CD = \sqrt{(13^2 - x^2)} + \sqrt{(15^2 - x^2)}$ or $14 = \sqrt{(13^2 - x^2)} + \sqrt{(15^2 - x^2)}$, whence $x = 12$.

41 Produce B A to E (fig. 25) make AE = AB join EC then AD CE and the triangle ABD is similar to the triangle EBC

$$\therefore \frac{BE}{AB}=\frac{BC}{BD} \therefore \frac{BE-AB}{AB}=\frac{BC-BD}{BD} \text{ (Dividendo)}$$

or $\frac{AE}{AB}=\frac{CD}{BD}$ or $\frac{AC}{AB}=\frac{CD}{BD}$ or $\frac{6}{4}=\frac{3}{2}=\frac{CD}{BD}$ *i e.* the line BC is to be divided in the ratio of 2 : 3.

42. From ex 12, $r=\frac{\text{area}}{\frac{1}{2}(a+b+c)}$ and from ex. 9. $R=\frac{abc}{4\ \text{area}}$

$$\therefore\ Rr=\frac{abc}{2(a+b+c)} \quad \therefore\ 2Rr=\frac{abc}{a+b+c}.$$

43. Let ABCD be a quadrilateral inscribed in a circle (fig 26) Let AB $=a$, BC $=b$, CD $=c$, AD $=d$, AC $=x$ BD $=y$ Draw CE $\perp$ AB and AF $\perp$ CD; ABC + ADC = 2 right angles ADF + ADC = 2 right angles, the angle ADF = ABC and consequently the triangles ADF, BCE are similar and BE : DF :: BC : AD :: b : d $\therefore$ $d\times$ BE $=b\times$ DF ....(a) but $x^2=a^2+b^2-2a\times$ BE and $x^2=c^2+d^2+2c\times$ DF

$\therefore$ BE $=\frac{a^2+b^2-x^2}{2a}$, DF $=\frac{x^2-c^2-d^2}{2c}$. Hence substituting these valus in eqn (a), $cd(a^2+b^2-x^2)=ab(x^2-c^2-d^2)$ $\therefore$ $(ab+cd)x^2$ $=a^2cd+b^2cd+abc^2+abd^2=(ac+bd)(ad+bc)$

$\therefore\ x=\sqrt{\left[\frac{(ac+bd)(ad+bc)}{ab+cd}\right]}$ In like manner, if we subtitute b, c d and a for a, b, c and d we shall have the value of the diagonal BD.

44. From Ex. 43, $xy=ac+bd$.

45. From Ex 43 we have $a^2+b^2-2a\times$ BE $=x^2$

$=\frac{a^2cd+b^2cd+abc^2+abd^2}{ab+cd}$ multiplying by $ab+cd$, and subtracting a^2cd+b^2cd from both sides of the eqn, $(a^2+b^2)ab - 2a\times BE \times(ab+cd) = abc^2 + abd$ $\therefore$ $BE = b\frac{a^2+b^2-c^2-d^2}{2ab+2cd}$. Now $CE^2 = BC^2 - BE^2 = (BC+BE)\,(BC-BE)$; and $BC+BE = b + b\frac{a^2+b^2-c^2-d^2}{2ab+2cd}$

$$= b\frac{2ab+2cd+a^2+b^2-c^2-d^2}{2ab+2cd} = b\frac{(a+b)^2-(c-d)^2}{2ab+2cd}$$

$= b\frac{(a+b+c-d)(a+b-c+d)}{2ab+2cd}$; let $a+b+c+d=s$, $a+b+c-d = 2s - 2d$, &c $=$ &c $\therefore$ $BC+BE = b\frac{(2s-2d)(2s-2c)}{2ab+2cd} = 2b\frac{(s-c)(s-d)}{ab+cd}$.

In the same manner it may be shewn that $BC - BE = 2b\frac{(s-a)(s-b)}{ab+cd}$ $\therefore$ $CE^2 = 4b^2\frac{(s-a)(s-b)(s-c)(s-d)}{(ab+cd)^2}$

Hence the area of the triangle $ABC = \frac{1}{2}a\times CE$ $= ab\frac{\sqrt{(s-a)(s-b)(s-c)(s-d)}}{ab+cd}$ In like manner the area of the triangle $ACD = cd\frac{\sqrt{(s-a)(s-b)(s-c)(s-d)}}{ab+cd}$

46. Suppose the inscribed circle to touch the sides in the points a, b, c, d (fig. 27) then $Ad = Aa$, $Dd = Dc$, $Cb = Cc$, $Bb = Ba$ $\therefore$ adding these equations we get $AD+BC = AB+DC$ or $a+c=b+d$ $\therefore$ $\frac{1}{2}$ perimeter $s=a+c$ or $=b+d$ $\therefore$ area $=\sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{(abcd)}$.

47. Let A and B (fig. 28) be the two points then AB is a tangent to the earths surface at its middle point D; $AD = DE$ nearly $= 4$ miles and $AE = 10$ ft. $= \frac{10}{3\times1760}$ miles. Let C be the earths centre and $CD = r$, then $AE\cdot(2r+AE) = AD^2$ or $2r\cdot\frac{10}{3\times1760} = 16$ nearly (AE^2 being too small may be neglected) $\therefore$ $r = 4224$ miles.

48 Let O (fig. 29) be the centre of the escribed circle and $r=$ its radius then $\triangleleft$ AOB = trapm. OACB $-\triangleleft$ ABC $=\triangleleft$ OBC $+\triangleleft$ OAC $-\triangleleft$ ABC $=\frac{1}{2}ar+\frac{1}{2}br-\triangleleft$ ABC but $\triangleleft$ AOB $=\frac{1}{2}cr$ $\therefore \frac{1}{2}cr$ $=\frac{1}{2}ar+\frac{1}{2}br-\triangleleft$ ABC $\therefore r=\frac{\triangleleft ABC}{\frac{1}{2}(a+b-c)}=\frac{\text{area of the triangle}}{s-c}$

ADFECTED QUADRATIC EQUATIONS.

Ex. 33. Add the square of half the coefficient of the 2nd term to both sides of the eqn. then $x^2-3x+(\frac{3}{2})^2=(\frac{3}{2})^2-2=\frac{1}{4}$ $\therefore x-\frac{3}{2}=\pm\frac{1}{2}$ $\therefore x=\frac{3}{2}\pm\frac{1}{2}=2$ or 1. Otherwise, $x^2-3x+2=0$ $\therefore (x-1)(x-2)=0$ $\therefore x-1=0$ and $x-2=0$ $\therefore x=1$ and $x=2$.

6. Divide the eqn. by 12 the coefficient of the 1st term then $x^2-\frac{7}{12}x=-\frac{1}{12}$ then proceed as in ex. 1.

10. $x^2+2x+1+12\sqrt{(x^3+2x+1)}=48$ $\therefore (x^2+2x+1)$ $+2\sqrt{(x^2+2x+1)}+1=49$ $\therefore \sqrt{(x^2+3x+1)}+1=7$ $\therefore \sqrt{(x^2+2x+1)}$

12. $x^4+1=0$, divide by x^2 then $x^2+\frac{1}{x^2}=0$ $\therefore x^2+2+\frac{1}{x^2}=2$ $\therefore x+\frac{1}{x}=\pm\sqrt{2}$ $\therefore x^2\mp\sqrt{2}x=-1$ $\therefore x^2\mp\sqrt{2}x+\frac{1}{2}=\frac{1}{2}-1=\frac{-1}{2}$ $\therefore x\mp\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{(-1)}}{\sqrt{2}}$ $\therefore x=\frac{\pm 1\pm\sqrt{(-1)}}{\sqrt{2}}$.

13. $a^3-1=(x-1)(x^2+x+1)=0$ $\therefore x-1=0$ $\therefore x=1$ again $x^2+x+1=0$ $\therefore x^2+x--1$ $\therefore x^2+x+\frac{1}{4}=\frac{1}{4}-1=\frac{-3}{4}$ $\therefore x+\frac{1}{2}=\pm\frac{\sqrt{(-3)}}{2}$ $\therefore x=-\frac{1}{2}\pm\frac{\sqrt{(-3)}}{2}$.

14. $x^3+1=2x$ ∴ $x^3-x=x-1$ or $x(x^2-1)=(x-1)$ ∴ $x(x+1)=1$ or $x^2+x=1$ then proceed as in ex. 13.

15. $x=4$, $x=5$ ∴ $x-4=0$, $x-5=0$ ∴ $(x-4)(x-5)=0$ or $x^2-9x+20=0$ ∴ $x^2-9x=-20$.

MAXIMA AND MINIMA.

Ex. 34. 1. Let x = one part then $8-x$ = other ∴ $x(8-x)=8x-x^2$ is maximum which let $=m$, then $x^2-8x=-m$ ∴ $x^2-8x+16=16-m$ ∴ $x=4\pm\sqrt{(16-m)}$ ∴ when m is maximum, $x=4$ ∴ the other part $=4$.

2. When $\frac{x}{(8+x)(2+x)}$ is maximum we must have $\frac{(8+x)(2+x)}{x}$ a minimum or $\frac{x^2+10x+16}{x}=\frac{x^2+16}{x}+10$ a minimum and as 10 is a constant given quantity we must have $\frac{x^2+16}{x}$ a minimum which let $=m$ ∴ $x^2+16=mx$. Solving this quadratic, $x=\frac{m}{2}\pm\sqrt{(\frac{m^2}{4}-16)}$, and m cannot be taken so small as to make $\frac{m^2}{4}$ less than 16, ∴ when m is minimum we must have $\frac{m^2}{4}=16$ ∴ $m=8$ ∴ $x=\frac{m}{2}=4$.

The same solved without impossible roots;

In the eqn. $x^2-mx=-16$ let $x=y+\frac{m}{2}$ ∴ $x^2-mx=y^2+my+\frac{m^2}{4}-my-\frac{m^2}{2}=y^2-\frac{m^2}{4}=-16$ ∴ $m^2=4y^2+64$ which is evidently a min. when $y=0$ ∴ $m=8$ and $x=\frac{m}{2}=4$.

This is the solution of the dynamical problem to find the magnitude of the body which must be interposed between two others, so that the velocity communicated from the one to the other shall be a maximum.

3. Let $p-(x-6)^2=m$ ∴ $p-x^2+12x-36=m$ ∴ $x^2-12x=p-m-36$. Solving this quadratic we find $x=6\pm\sqrt{(p-m)}$ and here it is

evident that m cannot be greater than p $\therefore$ when m is maximum we must have $m=p$ $\therefore$ $x=6$.

4. $\dfrac{x}{1+x^2}$ is maximum $\therefore$ $\dfrac{1+x^2}{x}$ is minimum which let $=m$ $\therefore x^2-mx=-1$. Solving this quadratic eqn, we find $x=\dfrac{m}{2}\pm\sqrt{\left(\dfrac{m^2}{4}-1\right)}$ and here it is evident that m or $\dfrac{m^2}{4}$ cannot be taken so small as to be less than 1 $\therefore$ when m is minimum we must hava $\dfrac{m^2}{4}=1$ $\therefore$ $m=2$ and $x=\dfrac{m}{2}=1$.

5. Let $x=$ one of the factors $\therefore$ $\frac{16}{x}=$ other factor $\therefore$ $x^2+\dfrac{256}{x^2}$ is minimum $=m$ $\therefore$ $x^4+256=mx^2$. Solving this quadratic we find $x^2=\dfrac{m}{2}\pm\sqrt{\left(\dfrac{m^2}{4}-256\right)}$. It is manifest that when m is minimum we must have $\dfrac{m^2}{4}=256$ or $m=32$ $\therefore$ $x=\sqrt{\dfrac{m}{2}}=4$.

6. Let x be the fraction required then let $x-x^2=m$, its maximum value $\therefore$ $x^2-x=-m$ whence $x=\frac{1}{2}\pm\sqrt{(\frac{1}{4}-m)}$ whence if m be maximum it cannot be greater than $\frac{1}{4}$ $\therefore$ $m=\frac{1}{4}$ $\therefore$ $x=\frac{1}{2}$.

7. $\because$ m is a constant given quantity, the expression is minimum when $\sqrt[3]{(m^3-2m^2x+mx^2)}$ is minimum or its cube $m^3-2m^2x+mx^2$ is minimum or $m^2-2mx+x^2$ is minimum which let $=p$ $\therefore$ $x^2-2mx=p-m^2$ $\therefore$ $x=m+\sqrt{p}$ and here it is manifest that when p is minimum it must be $=0$ $\therefore$ $x=m$.

8. Let ABC be a triangle (fig. 30) the base $BC=a$ and AD the altitude $=p$ let the altitude of the inscribed rect. $=DE=x$ $\therefore$ $AE=p$

$-x$, Now from similar triangles BC : BD :: FG : AE or $a : p$ :: FG : $p-x$. ∴ FG $=\frac{a(p-x)}{p}$ ∴ area of the rect $=\frac{a(p-x)x}{p}=\frac{a}{p}$. $(p-x)x$ and this is maximum when $(p-x)x$ is maximum, let this $=m$ then $x^2-px=-m$, whence $x=\frac{p}{2}\pm\sqrt{\left(\frac{p^2}{4}-m\right)}$ whence $m=\frac{p^2}{4}$ ∴ $x=\frac{p}{2}$.

9. Let a = area of the triangle and x one of the sides, then the other side would be $\frac{2a}{x}$ ∴ the sum $=x+\frac{2a}{x}$ is minimum, let this $=m$ ∴ $m^2-mx=-2a$ ∴ $x=\frac{m}{2}\pm\sqrt{\left(\frac{m^2}{4}-2a\right)}$ ∴ $\frac{m^2}{4}=2a$ ∴ $m^2=8a$ ∴ $m=2\sqrt{(2a)}$ ∴ $x=\sqrt{(2a)}$ ∴ the other side is also $=\sqrt{(2a)}$.

10. Let x and y be the variable sides of the triangle then $18=8+x+y$ or $y=10-x$ and the area of the triangle $=\sqrt{9(9-8)(9-x)(9-y)}$ $=3\sqrt{(9-x)(9-y)}$ is maximum and ∵ 3 is a constant factor, $\sqrt{(9-x)(9-y)}$ is maximum or its square $(9-x)(9-y)$ is maximum or $(9-x)(9-10+x)$ is maximum or $(9-x)(x-1)$ is maximum which let $=m$ ∴ $x^2-10x=-m-9$ whence $x=5\pm\sqrt{(16-m)}$ ∴ when m is maximum, it is $=16$ and ∴ $x=5$ and ∴ $y=5$.

11. Let CB = CD = CA = 2 (fig. 31) where CD is at right angles to AB, let CG $=x$ then FG $=\sqrt{(4-x^2)}$ ∴ area of the inscribed rect $=2x\sqrt{(4-x^2)}$ is maximum but as 2 is a constant quantity, $x\sqrt{(4-x^2)}$ is maximum ∴ its square $x^2(4-x^2)$ is maximum ∴ $4x^2-x^4$ is maximum which let $=m$ ∴ $x^4-4x^2=-m$ ∴ $x^2=2\pm\sqrt{(4-m)}$ ∴ when m is maximum it is $=4$ ∴ $x^2=2$ ∴ $x=\sqrt{2}$.

13. Let ABCD be the given square (fig. 32) and EFGH the inscribed square; let DH $=x$ then AH $= 8 - x$ and it may be proved that AE $=$ HD $= x$ $\therefore$ area of the square HF $= \mathrm{HE}^2 = \mathrm{AE}^2 + \mathrm{AH}^2 = x^2 + (8 - x)^2 = 2x^2 - 16x + 64 = 2(x^2 - 8x + 32)$ is minimum $\therefore$ $x^2 - 8x + 32$ is min. which let $= m$, then $x^2 - 8x = m - 32$ $\therefore$ $x^2 - 8x + 16 = m - 16$ $\therefore x = 4 \pm \sqrt{(m - 16)}$ $\therefore$ when m is min. it is $= 16$ $\therefore x=4$, *i.e.* the angular points of the inscribed square will rest at the middle points of the given square.

13. Let ABC be a quadrant (fig. 33) B being the centre, let BD $= y$, BE $= x$, DE being the tangent to the quadrant at T $\because$ $\triangleleft$EBT is similar to FBT $\therefore$ EB : BT :: BT : BF $\therefore$ $\mathrm{BF} = \frac{\mathrm{BT}^2}{\mathrm{EB}} = \frac{r^2}{x}$ where $r =$ radius) and $\mathrm{TF} = \sqrt{\left(r^2 - \frac{r^4}{x^2}\right)} = \frac{r}{x}\sqrt{(x^2 - r^2)}$. Also BD : BT :: BT : (BG $=$ TF) or $y : r :: r : \frac{r}{x}\sqrt{(x^2 - y^2)}$ $\therefore$ $y = \frac{rx}{\sqrt{(x^2 - r^2)}}$ $\therefore$ area of the $\triangleleft$ BDE $= \frac{1}{2}xy = \frac{1}{2}r \cdot \frac{x^2}{\sqrt{(x^2 - r^2)}}$ is minimum.

Now $\because$ $\frac{1}{2}r$ is a constant quantity $\therefore$ $\frac{x^2}{\sqrt{(x^2 - r^2)}}$ or its square $\frac{x^4}{x^2 - r^2}$ is minimum which let $= m$ $\therefore$ $x^2 = \frac{m}{2} \pm \sqrt{\frac{\{m(m - 4r^2)\}}{4}}$ and here it is manifest that m cannot be less than $4x^2$ $\therefore$ it is $= 4r^2$ when it's min. $\therefore$ $x^2 = \frac{m}{2} = 2r^2$ and $x = \sqrt{2}$ and $y - \frac{rx}{\sqrt{(x^2 - r^2)}} = a\sqrt{2}$ or the triangle described must be isosceles.

14. We can reject m^4 which is a constant quantity and see when $n^3x - p^2x^2$ becomes a maximum or $\frac{n^3}{p^2}x - x^2$ is maximum which let $= a$ then $x^2 - \frac{n^3}{p^2} = -a$ $\therefore$ $x = \frac{n^2}{2p^2} \pm \sqrt{\left(\frac{n^6}{4p^4} - a\right)}$ whence $x = \frac{n^2}{2p^2}$.

15. Let AC be the rectangle (fig. 34) and EF a diameter bisecting DC, let OE$=x$ then EF$=2x$, also DE$=\sqrt{(4^2-x^2)}$ $\therefore$ DC$=2\sqrt{(4^2-x^2)}$ $\therefore$ rect. AC$=2x\times 2\sqrt{(4^2-x^2)}=4x\sqrt{(1^2-x^2)}$, now this is maximum when $x\sqrt{(4^2-x^2)}$ is max. or when its square $x^2(1^2-x^2)=16x^2-x^4$ is max. which let$=m$ $\therefore x^4-16x^2=-m$ $\therefore x^2=8\pm\sqrt{(16-m)}$ and it is manifest that m cannot be greater than 16 $\therefore$ when m is max, it must be$=16$ $\therefore x^2=8$ $\therefore x=2\sqrt{2}$ $\therefore$ side of the square$=4\sqrt{2}$.

16. Let $x=$no required, then $\frac{1}{x}$ is its reciprocal, then $x+\frac{1}{x}\ \frac{x^2+1}{x}$ is min. which let$=m$ $\therefore x^2+1=mx$ $\therefore x^2-mx=-1$ $\therefore x=\frac{m}{2}\pm\sqrt{\frac{m^2}{4}-1)}$ $\therefore \frac{m^2}{4}=1$ $\therefore m=2$ $\therefore x=1$.

17. $\because m^2$ is a constant quantity we must see when $x(m-x)$ is a maximum, let this$=a$ then $mx-x^2=a$ $\therefore x^2-mx=-a$ whence $x=\frac{m}{2}\cdot\sqrt{\left(\frac{m^2}{4}-a\right)}$ It is manifest that a cannot be greater than $\frac{m^2}{4}$

$\therefore a=\frac{m^4}{4}$ [illegible]

19. Let AB be the horizontal surface, (fig. 35) C the point at which the candle is placed Inoptics it has been shewn that the intensity of the illumination varies inversely as the square of the distance,and with different degrees of obliquity, the distance being the same, as the sine of the angle which the rays make with the surface $\therefore$ intensity at A $=\frac{1}{AC^2}\sin a=\frac{BC}{AC^3}$, then the illuminating power on the surfac at A$=\frac{BC}{AC}\cdot\frac{AB^2}{AC^2}\times\frac{1}{AB^2}=\sin a,\frac{\cos^2 a}{4^2}$ is max $\therefore \sin a\cos^2 a=\sin a(1-\sin^2 a)$ $=\sin a-\sin^3 a$ is $max=m$ Now let $\sin a=y$ $\therefore y-y^3=m$ $\therefore y^3-y+m=0$ let one of the negative roots of this cubic eqn$=-n$ and it is evident

that $y+n$ must exactly divide $y^3-y+m=0$ and $\frac{y^3-y+m}{y+n}=y^2-ny+n^2-1=0$......(1) rem $=m-(n^3-n)$ $\therefore$ $m=n^3-n$ $\therefore$ $n^2-1=\frac{m}{n}$, by eqn (1) we find $y^2-ny+\frac{m}{n}=0$ whence $y=\frac{n}{2}\pm\sqrt{\left(\frac{n^3-4m}{4n}\right)}$ and here it is evident that the greatest value of m is when $n^3=4m=4n^3-4n$ $\therefore$ $n=\frac{2}{\sqrt{3}}$ and $y=\frac{n}{2}=\frac{1}{\sqrt{3}}$ $\therefore$ $BC=AB\times\frac{1}{\sqrt{2}}$.

21. Let $x=$ one part then $16-x$ the other $\therefore$ $x(16-x)-x^2$ is max. or $16x-2x^2=2(8x-x^2)$ is max. leaving aside 2 a constant factor we have $8x-x^2=$ max $=m$ say $\therefore$ $x^2-8x=-m$ $\therefore$ $x=4\pm\sqrt{(16-m)}$ $\therefore$ when m is max. $x=4$.

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Ex. 35.

7. $$\left\{\frac{x}{a}+\frac{2x^2}{a(b-x)}\right\}\left\{\frac{a}{x}-\frac{2ax}{x(b+x)}\right\}=\frac{abx-ax^2+2ar^2}{a^2(b-x)}$$
$$\times\frac{abx+ax^2-2ax^2}{x^2(b+x)}=\frac{abx+ax^2}{a^2(b-x)}\times\frac{abx-ax^2}{x^2(b+x)}=\frac{a^2b^2x^2-a^2x^4}{a^2x^2(b^2-x^2)}$$
$$=\frac{a^2x^2(b^2-x^2)}{a^2x^2(b^2-x^2)}=1.$$

10. $\frac{a}{b}=\frac{c}{d}$ or $\frac{ma}{mb}=\frac{nc}{nd}$ or $\frac{ma}{nc}=\frac{mb}{nd}$ or $\frac{ma+nc}{nc}=\frac{mb+nd}{nd}$ or $\frac{ma+nc}{mb+nd}=\frac{nc}{nd}=\frac{c}{d}$; $\frac{a}{b}=\frac{c}{d}$, squaring, $\frac{a^2}{b^2}=\frac{c^2}{d^2}$, or $\frac{a^2}{c^2}=\frac{b^2}{d^2}$ or $\frac{a^3+c^2}{c^2}=\frac{b^2+d^2}{d^2}$ or $\frac{a^2+c^2}{b^2+d^4}=\frac{c^2}{d^2}$ extracting the square root $\frac{(a^2+c^2)^{\frac{1}{2}}}{(b^2+d^2)^{\frac{1}{2}}}=\frac{c}{d}$ $\therefore$ $\frac{ma+nc}{mb+nd}=\frac{(a^2+c^2)^{\frac{1}{2}}}{(b^2+d^2)^{\frac{1}{2}}}$ $\therefore$ $ma+nc : mb+nd :: (a^2+c^2)^{\frac{1}{2}} : (b^2+d^2)^{\frac{1}{2}}$

12. Let x be the rate of the stream per hour in miles and y that of the boat; in going upstream the boat has to go against the stream therefore its motion is $y-x$ miles per hour; and in going down stream its rate is $x+y$ miles ∴ by the question,

$$\frac{30}{y-x}+\frac{44}{x+y}=10\ldots\ldots(1)$$

$$\frac{40}{y-x}+\frac{55}{x+y}=13\ldots\ldots(2)$$

multiply (1) by 4 & (2) by 3 then

$$\left.\begin{array}{l}\dfrac{120}{y-x}+\dfrac{176}{x+y}=40\\[2ex]\dfrac{120}{y-x}+\dfrac{165}{x+y}=39\end{array}\right\}\ \therefore\ \frac{11}{x+y}=1\ \therefore\ x+y=11\ldots\ldots(3)$$ subtracting (1) from (2), $\frac{10}{y-x}+\frac{11}{x+y}=3$, substituting the value of $x+y$, $\frac{10}{y-x}+1=3$ or $\frac{10}{y-x}=2$ $\therefore y-x=5\ldots\ldots(1)$, adding (4) & (3), $2y=16$ $\therefore$ $y=8$ $\therefore$ $x=3$.

1881.

2. $$\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}+\frac{1}{b(b-a)(x-b)}=\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}$$

$$-\frac{1}{b(a-b)(x-b)}=\frac{(a-b)(x-a\ (x-b)+bx(x-b)-ax(x-a)}{abx\ (a-b)(x-a)(x-b)}$$

$$=\frac{ax^2-a^2x-abx+a^2b-bx^2+abx+b^2x-ab^2+bx^2-b^2x-ax^2+a^2x}{abx(a-b)(x-a)(x-b)}$$

$$=\frac{a^2b-ab^2}{abx(a-b)(x-a)(x-b)}=\frac{ab\ (a-b)}{abx(a-b)(x-a)(x-b)}=\frac{1}{x(x-a)(x-b)}.$$

$x^2-5ax-66a^2=x^2-11ax+6ax-66a^2=x(x-11a)+6a(x-11a)$
$=(x+6a)(x-11a)$.

3. If we can from that $\frac{10x}{y}+\frac{10y}{x}-20$ is positive then what is required is done or if $\frac{x}{y}+\frac{y}{x}-2$ is positive or if x^2+y^2-2xy is posi-

tive. But we know that $x^2 + y^2$ is always greater than $2xy$ when x and y are unequal see Ex. 1. page 128. $\therefore \frac{10x}{y} + \frac{10y}{x} - 2$ is positive.

(1) $\sqrt{x^2+11x+20} - \sqrt{x^2+5x-1} = 3$ or $\sqrt{x^2+11x+20} = 3 + \sqrt{x^2+5x-1}$ Squaring $x^2+11x+20 = 9 + x^2 + 5x - 1 + 6\sqrt{x^2+5x-1}$ or $6x+12 = 6\sqrt{x^2+5x-1}$ or $x+2 = \sqrt{x^2+5x-1}$. Squaring, $x^2+4x+4 = x^2+5x-1$ $\therefore x = 5$.

(2) $\frac{4\cdot05}{9x} - \frac{\cdot3}{\cdot8-2x} = \frac{1\cdot8}{x} - \frac{3\cdot6}{2\cdot4-6x}$ or $\frac{3\cdot6}{2\cdot4-6x} - \frac{\cdot3}{\cdot8-2x} = \frac{1\cdot8}{x} - \frac{4\cdot05}{9x}$ or $\frac{3\cdot6-\cdot9}{2\cdot4-6x} = \frac{16\cdot2-4\cdot05}{9x}$ or $\frac{2\cdot7}{2\cdot4-6x} = \frac{12\cdot15}{9x}$ or $\frac{1}{2\cdot4-6x} = \frac{4\cdot5}{9x} = \frac{\cdot5}{x}$ or $x = 1\cdot2 - 3x$ or $4x = 1\cdot2$ $\therefore x = \cdot3$.

(3) $ax + by = c$ .. (1), $a^2x + b^2y = c^2$ ... (2), multiply (1) by c and subtract it from (2).

$$\left.\begin{aligned} a^2x + b^2y &= c^2 \\ abx + b^2y &= bc \end{aligned}\right\} \quad \therefore \text{ by subtraction.}$$

$(a^2 - ab)x = c^2 - bc$. $\therefore x = \frac{c(c-b)}{a(a-b)}$; again multiply (1) by a, and subtract it from (2).

$$\begin{aligned} a^2x + b^2y &= c^2 \\ \underline{a^2x + aby} &\underline{= ac} \end{aligned} \quad (b^2 - ab)y = c^2 - ac \quad \therefore y = \frac{c(c-a)}{b(b-a)} = \frac{c(a-c)}{b(a-b)}$$

6. Let x = no of yds. A rides per second and y that B rides

$$\left.\begin{aligned} \frac{1040}{x} &= \frac{920}{y} + 5 \\ \frac{1080}{x} &= \frac{1040}{y} - 5 \end{aligned}\right\} \quad \text{by the question.}$$

Adding up, $\frac{2120}{x} = \frac{1960}{y}$ $\therefore \frac{x}{y} = \frac{2120}{1960} = \frac{53}{49}$

$\therefore y = \frac{49x}{58}$ Substituting the value of y in the first of the equation.

$\frac{1040}{x} = \frac{48760}{49x} + 5$ or $\frac{50960 - 48760}{49x} = 5$ or $\frac{2200}{49x} = 5$

$\therefore x = \frac{440}{49}$ $\therefore y = \frac{440}{53}$. A will take $\frac{1040 \times 49}{440} = \frac{1274}{11} = 115\frac{9}{11}$ seconds to ride and B will take $\frac{1040 \times 53}{440} = \frac{1378}{11} = 125\frac{3}{11}$ sec.

MISCELLANEOUS EXAMPLES.

Ex. 36. 2. Multiply the whole eqn. by $2abc$ the L. C M. of the denominators then $a(b^2 + c^2 - a^2) + b(c^2 + a^2 - b^2) + c(a^2 + b^2 - c^2)$ $= 2abc$ or $ab^2 + ac^2 - a^3 + bc^2 + a^2b - b^3 + a^2c + b^2c - c^3 - 2abc = 0$ or $(a^2b - b^3 + 2b^2c - bc^2) + (a^2c - b^2c + 2bc^2 - c^3) - (a^3 - ab^2 + 2abc - ac^2)$ $= 0$ or $b(a^2 - b^2 + 2bc - c^2) + c(a^2 - b^2 + 2bc - c^2) - a(a^2 - b^2 + 2bc - c^2)$ $= 0$ or $(b + c - a)(a^2 - b^2 + 2bc - c^2) = 0$ or $(b + c - a)\{a^2 - (b - c)^2\} = 0$ or $(b + c - a)(a + b - c)(a - b + c) = 0$ $\therefore$ one of the 3 factors $= 0$ let $a + b - c = 0$ then $a + b = c$ $\therefore$ $\frac{b^2 + c^2 - a^2}{2bc}$ the 1st fraction $= \frac{b^2 + (a + b)^2 - a^2}{2b(a + b)}$ $= \frac{b^2 - a^2}{2b(a + b)} + \frac{(a + b)^2}{2b(a + b)} = \frac{b - a}{2b} + \frac{a + b}{2b} = 1.$

Similarly the 2nd fraction $\frac{c^2 + a^2 - b^2}{2ac} = \frac{(a + b)^2 + a^2 - b^2}{2a(a + b)} = \frac{a + b}{2a} + \frac{a - b}{2a}$ $= 1$ and then the 3rd fraction $\frac{a^2 + b^2 - c^2}{2ab} = \frac{a^2 + b^2 - (a + b)^2}{2ab} = \frac{-2ab}{2ab} = -1$

3. $\because$ n is any positive integer $2n$ is an even no $\therefore$ $2n + 1$ is an odd no. $\therefore$ $13^{2n+1} + 1$ is divisible by $13 + 1$ *i. e.* by 14.

4. $\frac{\text{Sum of all the antecedents}}{\text{Sum of all the consequents}} =$ each ratio is $\therefore$ $\frac{10(x + y - z)}{10(x + 1)}$ $= \frac{x + y - z}{6}$ *i. e.* $\frac{1}{x + 1} = \frac{1}{6}$ $\therefore$ $x = 5$. Substitute this value in the 3 eqns and we will get $y = 4$, $z = 3$.

5. Let each of the given expressions $=p$ then $bx+ay-cz=p(a^2+b^2), cy+bz-ax=p(b^2+c^2), az+cx-by=p(a^2+c^2)$ $\therefore$ $x=p(b+c)$ $y=p(a+c)$, $z=p(a+b)$ $\therefore$ $p=\dfrac{x}{b+c}=\dfrac{y}{a+c}=\dfrac{z}{a+b}$ $\therefore$ $\dfrac{x+y+z}{2(a+b+c)}$

$$=\frac{ax}{a(b+c)}=\frac{by}{b(a+c)}=\frac{cz}{c(a+b)} \text{ or } \frac{x+y+z}{2(a+b+c)}=\frac{ax+by+cz}{2(ab+bc+ac)}$$

$$\therefore \frac{x+y+z}{a+b+c}=\frac{ax+by+cz}{ab+bc+ac}.$$

9. $a=\dfrac{1-x^2}{x}=\dfrac{y^2}{x}$ $\therefore$ $a^{\frac{2}{3}}=\dfrac{y^{\frac{4}{3}}}{x^{\frac{2}{3}}}$; $b=\dfrac{1-y^2}{y}=\dfrac{x^2}{y}$ $\therefore$ $b^{\frac{2}{3}}=\dfrac{x^{\frac{4}{3}}}{x^{\frac{2}{3}}}$

$$\therefore a^{\frac{2}{3}}+b^{\frac{2}{3}}=\frac{y^{\frac{4}{3}}}{x^{\frac{2}{3}}}+\frac{x^{\frac{4}{3}}}{y^{\frac{2}{3}}}=\frac{x^2+y^2}{(xy)^{\frac{2}{3}}}=\frac{1}{(xy)^{\frac{2}{3}}};\ a^{\frac{2}{3}}b^{\frac{2}{3}}=\frac{y^{\frac{4}{3}}}{x^{\frac{2}{3}}}\cdot\frac{x^{\frac{4}{3}}}{y^{\frac{2}{3}}}=\frac{(xy)^{\frac{4}{3}}}{(xy)^{\frac{2}{3}}}=(xy)^{\frac{2}{3}}$$

$$\therefore a^{\frac{2}{3}}b^{\frac{2}{3}}(a^{\frac{2}{3}}+b^{\frac{2}{3}})=\frac{1}{(xy)^{\frac{2}{3}}}\cdot\frac{(xy)^{\frac{2}{3}}}{1}=1.$$

10. $x^2+y^3+z^2-xy-yz-xz=0$ $\therefore$ $(x+y+z)(x^2+y^2+z^2-xy-yz-xz)=0$ $\therefore$ $x^3+y^3+z^3-3xyz=0$ $\therefore$ $x^3+y^3+z^3=3xyz$.

11. $a^2d^2-b^2c^2=a^2(1-c^2)-(1-a^2)c^2=a^2-a^2c^3-c^2+a^2c^2=a^2-c^2$

12. Cubing both sides of the eqn. we have $x^3=a+\sqrt{(a^2+b^3)}+a-\sqrt{(a^2+b^3)}+3\sqrt[3]{(a^2-a^2-b^3)}\times[\sqrt[3]{\{a+\sqrt{(a^2-b^3)}\}}+\sqrt[3]{\{a-\sqrt{(a^2+b^3)}\}}]$ $=2a+3\cdot-b\cdot x=2a-3bx$ $\therefore$ $x^3+3bx=$[illegible].

13. The left side expression of the eqn. to be proved $=4a^2b^2-(a^2+b^3-c^2)^2$; here $a^2=(x^4+1-2x^2)$, $b^2=4x^2$ $\therefore$ $a^2+b^2=x^4+1+2x^2=(x^2+1)^2$ but $c^2=(x^2+1)^2$ $\therefore$ $a^2+b^2-c^2=(x^2+1)^2-(x^2+1)^2=0$.

14. $a+b+c=0$ $\therefore$ $a+b=-c$ $\therefore$ $a^2+2ab+b^2=c^2$ or $(a^2+b^2-c^2)^2=4a^2b^2$ or $a^4+b^4+c^4=2a^2b^2+2b^2c^2+2a^2c^2$.

15. $x = \frac{a-b}{2\sqrt{(ab)}}$ $\therefore x^2 = \frac{(a-b)^2}{4ab}$ $\therefore \sqrt{(1-x^2)} = \sqrt{\left(1+\frac{(a-b)^2}{4ab}\right)}$

$= \frac{a+b}{2\sqrt{(ab)}}$ $\therefore$ expression $= \frac{\frac{2a(a+b)}{2\sqrt{(ab)}}}{\frac{2a}{2\sqrt{(ab)}}} = a+b.$

16. By Ex 14 in page 120 we have $(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)$ $-3(x^2-yz)(y^2-xz)(z^2-xy) = (x^3+y^3+z^3-3xyz)^2$ or $(-8)^3+1^3+10^3$ $-3\times-8\times10 = (x^3+y^3+z^3-3xyz)^2$ or $(x^3+y^3+z^3-3xyz)^2 = 27^2$ $\therefore x^3$ $+y^3+z^3-3xyz = 27 \ldots (1)$. But $(x^2-yz)^2-(y^2-xz)(z^3-xy) = x(x^3+y^3$ $+z^3-3xyz) = (-8)^2 - 1\times10 = 54 \ldots (2)$ $\therefore \frac{(1)}{(2)} = x = 2$; similarly y and z can be found.

17. Let $x =$ digit in the units place and $y =$ digit in the tens place then the no $= 10y+x$ then $\frac{10y+x}{x+y} = y+1$ and $\frac{10y+x}{x+y} = 4x$ $\therefore y+1$ $= 4x$. $\therefore y = 4x-1$. Substitute this in the 1st eqn. then $\frac{10(4x-1)+x}{x+4x-1}$ $= 4x-1+1 = 4x$ or $\frac{41x-10}{5x-1} = 4x$ $\therefore 41x-10 = 20x^2-4x$ $\therefore 20x^2$ $-45x = -10$ $\therefore x^2-\frac{9}{4}x = -\frac{1}{2}$ $\therefore x^2-\frac{9}{4}x+\frac{81}{64} = \frac{81}{64}-\frac{1}{2} = \frac{49}{64}$ $\therefore x-\frac{9}{8} = \frac{7}{8}$ $\therefore x = 2$ $\therefore y = 4x-1 = 8-1 = 7$ $\therefore$ no $= 72$.

19. Let AB (fig. 36) be the wall, AC the flagstaff, BE the horizontal plane, then if a circle be described so that it passes through A and C and touch BE at D then D is the point where the flagstaff will subtend the maximum angle, for take any other point E in the plane then $<$ ADC $=$ $<$ HFC but $<$ AFC is greater than AEC $\therefore$ $<$ ADC is greater than the $<$ AEC. Now $BD^2 = BC.BA$ (III. 36) $= (3+5\frac{1}{3})\times3$ $= 8\frac{1}{3}\times3 = \frac{25}{3}\times3 = 25$ $\therefore$ BD $= 5$.

20. The left hand expression $= \frac{2x+2\sqrt{(x^2-25)}}{2x-2\sqrt{(x^2-25)}}$

$$=\frac{\{\sqrt{(x+5)}+\sqrt{(x-5)}\}^2}{\{\sqrt{(x+5)}-\sqrt{(x-5)}\}^2}\quad\therefore\quad\frac{\{\sqrt{(x+5)}+\sqrt{(x-5)}\}^2}{\{\sqrt{(x+5)}-\sqrt{(x-5)}\}^2}$$

$$=125\frac{\sqrt{(x+5)}-\sqrt{(x-5)}}{\sqrt{(x+5)}+\sqrt{(x-5)}}\quad\therefore\quad\frac{\{\sqrt{(x+5)}+\sqrt{(x-5)}\}^3}{\{\sqrt{(x+5)}-\sqrt{(x-5)}\}^3}$$

$$=125=5^3\quad\therefore\quad\frac{\sqrt{(x+5)}+\sqrt{(x-5)}}{\sqrt{(x+5)}-\sqrt{(x-5)}}=\frac{5}{1}\quad\therefore\frac{\sqrt{(x+5)}}{\sqrt{(x-5)}}=\frac{5+1}{5-1}=\frac{6}{4}=\frac{3}{2}$$

$$\therefore\frac{x+5}{x-5}=\frac{9}{4}\quad\therefore\quad\frac{x}{5}=\frac{9+4}{9-4}=\frac{13}{5}\quad\therefore\ x=13.$$

21. $\frac{x^2-x^2}{x^2-x^2}=\frac{0}{0}$ now zero in the denominators may be contained in zero of the denominator once or twice or any no. of times $\therefore\frac{x^2-x^2}{x^2-x^2}$ = indefinite quantity and to make it $=1$ is wrong.

22. Multiply the 1st eqn. by 3 then $3^{x+1}+3\cdot2^{4}=51$; Subtract this from 2nd then $2^{y+2}-3.2^{y}=8$ or $2^{y}(2^2-3)=8$ or $2^{y}=8=2^3$ $\therefore$ $y=3$ and from the 1st $x=2$.

23. A B C is a right angled triangle (fig. 37) right angled at A; let D the middle point of BC be the place of their 1st. meeting and E of the 2nd; let 11 m and 13 m be the valocities then

$$\frac{AC+CD}{13m}=\frac{AB+BD}{11m}\text{ or }\frac{AC+CD}{AB+BD}=\frac{13}{11}\text{ or }\frac{(AC+CD)-(AB+BD)}{AC+CD+(AB+BD)}$$

$$=\frac{13-11}{13+11}=\frac{2}{24}=\frac{1}{12}\text{.......(1) also }\frac{BD+AB+30}{13m}=\frac{DC+AC-30}{11m}\ \therefore\text{ or}$$

$\frac{BD+AB+30}{DC+AC-30}=\frac{13}{11}$ or $\frac{AB+AC+BC}{AB-AC+60}=\frac{24}{2}=12$ .. (2); multiplying (1) and (2) we have $\frac{AC-AB}{AB-AC+60}=1$ or $AC-AB=AB-AC+60$ or $AC-AB=30$ subtituting this in (1), $\frac{30}{AB+BC+CA}=\frac{1}{12}$ $\therefore$ $AB+BC+AC=360$

24. $\left(\frac{x+a}{b}\right)^{\frac{2}{3}} - 2\left\{\frac{(x+a)(x+c)}{bd}\right\}^{\frac{1}{3}} + \left(\frac{x+c}{d}\right)^{\frac{2}{3}} = 0$, find the sq. root

$\therefore \left(\frac{x+a}{b}\right)^{\frac{1}{3}} - \left(\frac{x+c}{d}\right)^{\frac{1}{3}} = 0 \quad \therefore \left(\frac{x+a}{b}\right)^{\frac{1}{3}} = \left(\frac{x+c}{d}\right)^{\frac{1}{3}} \quad \therefore \frac{x+a}{b} = \frac{x+c}{a}$

$\therefore dx + ad = bx + bc \quad \therefore (d-b)x = bc - ad \quad \therefore x = \frac{bc-ad}{d-b}$

25. Let x = distance in miles then $\frac{x}{7\frac{1}{2}}$ = no of hours required by the 1st person and $\frac{x}{30}$ = no of hours required by the 2nd person then

$\frac{x}{7\frac{1}{2}} = \frac{x}{30} + \frac{1}{2} \quad \therefore x = 5.$

28 $365^2 : 221^2 :: 95{,}000{,}000^3 : x^3 \quad \therefore x = 68{,}610{,}000$

29. Divide the expression by $x - m$ and the remainder after 3 terms in the quotient $= m^2(2m - 6n) - 6m^2n = 0 \quad \therefore 2m - 6n = 6n \quad \therefore n = \frac{1}{6}m.$

30. $a + b + c = 0 \quad \therefore a + b = -c \quad \therefore a^2 + b^2 - c^2 = -2ab \quad \therefore a^2 + b^2 + c^2 = 2(c^2 - ab)$, $a^4 + b^4 + c^4 = 4a^2b^2 - 2(a^2b^2 - a^2c^2 - b^2c^2) = 2(a^2b^2 + + a^2c^2 + b^2c^2) = 2\{a^2b^2 + c^2(a^2 + b^2)\} = 2(a^2b^2 + c^4 - 2abc^2) = 2(c^2 - ab)^2$; again $\therefore a + b = -c \quad \therefore a^5 + b^5 + c^5 = -5ab\,(a^3 + 2a^2b + 2ab^2 + b^3) = -5ab(a+b)\,(a^2 + ab + b^2) = -5ab(a+b)\,\{c^2 - ab\}$; also $a^3 + b^3 + c^3 = -3ab(a+b)$.

$$\therefore \frac{a^5 + b^5 + c^5}{5(a^3 + b^3 + c^3)} = \frac{-5ab(a+b)(c^2 - ab)}{-15ab(a+b)} = \frac{c^2 - ab}{3} = \frac{2(c^2 - ab)^2}{6(c^2 - ab)}$$

$$= \frac{a^4 + b^4 + c^4}{3(a^2 + b^2 + c^2)}$$

31. Let $\frac{m(x-y)}{m(y-z)} = \frac{n(3x-y)}{n(y-3z)} = \frac{p(4x-z)}{p(z-4y)} \quad \therefore m(x-y) + n(3x-y) + p(4x-z) = 19x - 3y + 3z \quad \therefore (m + 3n + 4p)x = 19x \quad \therefore m + 3n + 4p = 19$ ..(1), $(-m-n)y = -3y \quad \therefore m + n = 3$...(2) ; $-p = -3 \quad \therefore p = 3$...(3) and from (1), (2), (3), $m = 1$, $n = 2$, $p = 3$; in the same manner the 2nd portion of the equality may be treated.

32. Height $=16.1\times 30^2=14490$ ft.

33. It has been proved in Euclid 11.5 that if a square and a rectangle have the same perimeter the area of the square is greater than the area of the rectangle ∴ 8 ft. is the side of the square.

34. $4a^4-37a^2b^2+49b^4=(4a^4-28a^2b^2+49b^4)-9a^2b^2=(2a^2-7b^2)^2-(3ab)^2=(2a^2-7b^2+3ab)(2a^2-7b^2-3ab)$

35. ∵ the perimeter of a rectangle is greater than that of a square of the same area, the cost of the walls will be least if the form of the room be a square.

38. $x^3+1=2x$ ∴ $x^3-x=x-1$ or $x(x^2-1)=x-1$ or $x(x+1)(x-1)=x-1$ ∴ $x(x+1)=1$.

39. Dividing the eqn. by 8 we have $x^3-3x=\frac{65}{8}$; let $x=p+p^1$ then $\left(p+\frac{1}{p}\right)^3-3\left(p+\frac{1}{p}\right)=\frac{65}{8}$ or $p^3+\frac{1}{p^3}=\frac{65}{8}$ ∴ $p^6-\frac{65}{8}p^3=-1$ ∴ $p^6-\frac{65}{8}p^3+\left(\frac{65}{16}\right)^2=\left(\frac{65}{16}\right)^2-1=\frac{3969}{16^2}$ ∴ $p^3-\frac{65}{16}=\frac{63}{16}$ ∴ $p^3=8$ ∴ $p=2$ ∴ $x=p+\frac{1}{p}=2\frac{1}{2}$.

40. $\frac{(2+\sqrt{x})^{\frac{1}{2}}}{x^{\frac{1}{6}}}+\frac{(2-\sqrt{x})^{\frac{1}{2}}}{x^{\frac{1}{3}}}=x^{\frac{1}{6}}$ ∴ $(2+\sqrt{x})^{\frac{1}{2}}+(2-\sqrt{x})^{\frac{1}{2}}=x^{\frac{1}{3}}$.

41. $\frac{1}{24}$ of the circumference is travelled by the 1st person and $\frac{1}{648}$ of the circumference by the 2nd person in an hour ∴ gain in each hour by the 1st person $=\frac{1}{24}-\frac{1}{618}=\frac{625}{648\times 24}$ ∴ $\frac{625}{648\times 24}:1::1:x$ ∴ $x=24$ hrs. 55 min.

43. First method. Let x^2 and y^2 be two square numbers and let $x^2+y^2=(nx-y)^2=n^2x^2-2nxy+y^2$ ∴ $x^2=n^2x^2-2nxy$ ∴ $x=n^2x-2ny$ ∴ $x=\frac{2ny}{n^2-1}$: if the numbers be integers then $y=n^2-1$, then $x=2n$. Thus if $n=2$ then $y=3$ and $x=4$ (Diophantine analysis).

Second method. If n represent any number and m any other number then n^2+m^2 will represent the hypotenuse of a right angled triangle of which the other two sides are n^2-m^2 and $2mn$ ie. $(n^2-m^2)^2+4m^2n^2=(n^2+m^2)^2$.

46. When $x=1$, then $m+n+p=9$. (1), when $x=2$, $4m+2n+p=18$...(2), when $x=3$, $9m+3n+p=31$. (3). from (1), (2) and (3) determine m, n, p and then with the value of x, find the value of the expression.

47. $x^3+y^3+z^3=3xyz$ by ex. 1 p 121 $\therefore\ xyz=(a-b)(b-c)(c-a)$, but by the 1st eqn. $z=-(x+y)$ $\therefore\ -xy(x+y)=(a-b)(b-c)(c-a)$...(1) and the 2nd eqn. becomes $x(b+c)+y(a+c)+(-x-y)(a+b)=0$ $\therefore\ x(c-a)-y(b-c)=0$ $\therefore\ y=x\frac{c-a}{b-c}$. (2),

thus (1) becomes $z\cdot x\frac{c-a}{b-c}\left\{x+z\frac{c-a}{b-c}\right\}=(b-c)(a-b)(c-a)$

$\therefore\ -x^3\frac{c-a}{b-c}\left(1+\frac{c-a}{b-c}\right)=(a-b)(b-c)(c-a)$ $\therefore\ x^3=(b-c)^3$

$\therefore\ x=b-c$ and by (2) $y=c-a$ and hence $z=a-b$.

48. If $ax+by+cz=0$. and $a'x+b'y+c'z=0$ then $\frac{x}{bc'-b'c}=\frac{y}{ca'-c'a}=\frac{z}{ab'-a'b}$ See Todhunter's Algebra, new edition Art 385 Here $x(b-c)+y(c-a)+z(a-b)=0$ and $a(b-c)+b(c-a)+c(a-b)=0$

$$\therefore\ \frac{b-c}{cy-bz}=\frac{c-a}{az-cx}=\frac{a-b}{bx-ay} \text{ or } \frac{bz-cy}{b-c}=\frac{cx-az}{c-a}=\frac{ay-bx}{a-b}$$

49. $\frac{x_0}{x_r}=\left(\frac{x_0}{x_1}\right)^r$ $\therefore\ \frac{x_0}{x_1}=\left(\frac{x_0}{x_r}\right)^{\frac{1}{r}}$, $\frac{x_1}{x_r}=\left(\frac{x_0}{x_1}\right)^{r-1}$ $\therefore\ \frac{x_0}{x_1}=\left(\frac{x_1}{x_r}\right)^{\frac{1}{r-1}}$,

Similarly $\frac{x_0}{x_1}=\left(\frac{x_2}{x_r}\right)^{\frac{1}{r-2}}$ and $\therefore\ \frac{x_0}{x_1}=\left(\frac{x_{r-2}}{x_r}\right)^{\frac{1}{r-(r-2)}}=\frac{x^{\frac{1}{2}}_{r-2}}{x^{\frac{1}{2}}}$ also $\frac{x_0}{x_1}$

$$=\frac{x_{r-1}}{x_r}\quad\therefore\ \frac{x_0^{\frac{1}{r}}}{x_r^{\frac{1}{r}}}=\frac{x_1^{\frac{1}{r-1}}}{x_r^{\frac{1}{r-1}}}=\frac{x_2^{\frac{1}{r-2}}}{x_r^{\frac{1}{r-2}}}=\&c.=\frac{x_{r-1}^{\frac{1}{1}}}{x_r^{\frac{1}{1}}}=\frac{x_{r-1}}{x_r}$$

∴ each of the expressions $=\dfrac{\text{Sum of all the numerators}}{\text{Sum of all the denominators}}$

51. Let x = side of the equilateral triangle then the altitude $=\frac{\sqrt{3}}{2}x$ and ∴ the area of a triangle $=\frac{1}{2}\times$ base $\times$ altitude $=\frac{1}{2}x\times\frac{\sqrt{3}}{2}x$ = area $=16\sqrt{3}$ or $\frac{x^2}{4}=16$ ∴ $x^2=64$ ∴ $x=8$

52. $mn+1=0$ ∴ $mn=-1$ ∴ $m=-\frac{1}{n}$ ∴ $y=-\frac{x}{n}+\sqrt{\left(\frac{a^2}{n^2}+b^2\right)}$

or $y+\frac{x}{n}=\sqrt{\left(\frac{a^2+n^2b^2}{n^2}\right)}$ or $\frac{x+yn}{n}=\sqrt{\frac{a^2+n^2b^2}{n^2}}$

∴ $\frac{x^2+2nxy+n^2y^2}{n^2}=\frac{a^2+n^2b^2}{n^2}$ ∴ $x^2+2nxy+n^2y^2=a^2+n^2b^2$

Again from the 2nd eqn $(y-nx)^2=a^2n^2+b^2$ or $y^2+n^2x^2-2nxy$ $=a^2n^2+b^2$, add $x^2+y^2+n^2x^2+n^2y^2=a^2+b^2+a^2n^2+b^2n^2$ ∴ $(x^2+y^2)+n^2(x^2+y^2)=(a^2+b^2)+n^2(a^2+b^2)$ or $(1+n^2)(x^2+y^2)$ $=(a^2+b^2)(1+n^2)$ ∴ $x^2+y^2=a^2+b^2$

53. $x=a^2-bc$, $y=b^2-ac$, $z=c^2-ab$ ∴ $x+y+z=a^2+b^2+c^2$ $-ab-ac-bc$ ∴ $(x+y+z)(a+b+c)=(a^2+b^2+c^2-ab-ac-bc)$ $(a+b+c)=a^3+b^3+c^3-3abc$

$\frac{x^2-yz}{a}=\frac{(a^2-bc)^2-(b^2-ac)(c^2-ab)}{a}=\frac{a(a^3+b^3+c^3-3abc)}{a}$ $=a^3+b^3+c^3-3abc$ in the same manner the other expressions are equal

55. Put the values of a and b in the first expression then

$$(a+b)^{\frac{2}{3}}-(a-b)^{\frac{2}{3}}=\left(\frac{x^2}{y}+\frac{3y}{1}+\frac{y^2}{x}+\frac{3x}{1}\right)^{\frac{2}{3}}-\left(\frac{x^2}{y}+\frac{3y}{1}-\frac{y^2}{x}-\frac{3x}{1}\right)^{\frac{2}{3}}$$

$$=\frac{\{(x+y)^3\}^{\frac{2}{3}}}{(xy)^{\frac{1}{3}}}-\frac{\{(x-y)^3\}^{\frac{2}{3}}}{(xy)^{\frac{1}{3}}}=\frac{x^2+2xy+y^2-x^2+2xy-y^2}{(xy)^{\frac{1}{3}}}=\frac{4xy}{(xy)^{\frac{1}{3}}}$$

$=4(xy)^{\frac{2}{3}}=4c^{\frac{2}{3}}$

58 $xy+yz+xz+x^2=30$ or $(y+z)x+x(x+z)=30$ ∴ $(x+y)(x+z)=30$, $xy+yz+xz+y^2=35$ ∴ $x(y+z)+y(y+z)=35$ ∴ $(x+y)(y+z)=35$, $xy+yz+xz+z^2=42$ ∴ $y(x+z)+z(x+z)=42$ or $(y+z)(x+z)=42$ ∴ $(x+y)^2(x+z)^2(y+z)^2=30\times35\times42$ then proceed as in ex 23 p 98

59 $2^x+2^{2-x}=4$ or $2^x+\frac{4}{2^x}=4$ or $(2^x)^2-4\cdot2^x+4=0$ or $(2^x-2)^2=0$ or $2^x=2$ ∴ $x=1$.

61 $3^x-1=\frac{2}{\sqrt{3}+1}=\frac{3-1}{\sqrt{3}+1}=\sqrt{3}-1$ ∴ $3^x=3^{\frac{1}{2}}$ ∴ $x=\frac{1}{2}$.

64 If 845 be the G C M then let $845x$ and $845y$ be the two numbers then $845x+845y=4225$ ∴ $845(x+y)=4225$ ∴ $x+y=5$ when $x=1$ $y=4$ and when $x=2$, $y=3$ so that 845 and 3380 make a pair and 1690 and 2535 make another pair

65. Let x=no of eggs she bought at 2 a penny ∴ x=no she bought at 3 a penny ∴ whole sum spent $=\frac{x}{2}+\frac{x}{3}=\frac{5x}{6}$, and whole sum realised from selling $=\frac{2x}{5}\times2=\frac{4x}{5}$ ∴ $\frac{5x}{6}-\frac{4x}{5}=4$ ∴ $x=120$ ∴ sum laid out $=\frac{5x}{6}=100d=8s\ 4d$.

66 $$\frac{\frac{4ab}{a+b}+2a}{\frac{4ab}{a+b}-2a}+\frac{\frac{4ab}{a+b}+2b}{\frac{4ab}{a+b}-2b}=\frac{\frac{4ab+2a^2+2ab}{a+b}}{\frac{4ab-2a^2-2ab}{a+b}}+\frac{\frac{4ab+2ab+2b^2}{a+b}}{\frac{4ab-2ab-2b^2}{a+b}}$$

$$=\frac{6ab+2a^2}{2ab-2a^2}+\frac{6ab+2b^2}{2ab-2b^2}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}=\frac{3b+a}{b-a}-\frac{3a+b}{b-a}=$$

$$=\frac{3b+a-3a-b}{b-a}=\frac{2b-2a}{a-b}=2.$$

67. Square both sides then the rational parts in one side of the eqn. = rational parts in the other side and the irrational parts in one side = irrational parts in the other, here making the rational parts equal we have $2x^3-10ax+7a^2=2x^3-16ax-9a^2$ $\therefore$ $6ax=-16a^2$ $\therefore$ $x=-\frac{8}{3}a$.

Otherwise $(x^3-3ax+2a^2)-(x^3-7ax+5a^2)=(x^3-6ax-6a^2)-(x^3-10ax-3a^2)$; divide this by the given eqn. then $\sqrt{(x^3-3ax+2a^2)}-\sqrt{(x^3-7ax+5a^2)}=(x^3-6ax-6a^2)-\sqrt{(x^3-10ax-3a^2)}$ add this to the given eqn and square the result then x can be easily found.

68. $100'=1\frac{2}{3}$ hrs. $=\frac{5}{3}$hrs., let x = rate of the waterman in miles per hour then $\frac{3\frac{1}{2}}{2+x}$ = no of hrs. required to go down stream and $\frac{3\frac{1}{2}}{x-2}$ = no of hrs. required to go up stream by the question $\frac{3\frac{1}{2}}{2+x}+\frac{3\frac{1}{2}}{x-2}=\frac{5}{3}$. whence $x=5$.

69. Let x = no of £ he began with then $\frac{2}{3}x$ = remainder after the 1st game then $\frac{2}{3}x+\frac{1}{2}=\frac{4x+3}{6}$ = sum he had after the 2nd game, and $\frac{2}{3}\,\frac{4x+3}{6}+1=x$ $\therefore$ $x=2\frac{2}{5}$.

72. Cubing both sides we have $\frac{2x+3}{2x-3}+\frac{2x-3}{2x+3}+3.\ \frac{8}{13}\cdot\frac{4x^2+9}{4x^2-9}=\frac{8^3}{13^3}\cdot\left(\frac{4x^2+9}{4x^2-9}\right)^3$ or $\frac{8x^2+18}{4x^2-9}+\frac{24}{13}\,\frac{4x^2+9}{4x^2-9}=\frac{8^3}{13^3}\cdot\left(\frac{4x^2+9}{4x^2-9}\right)^3$ or $2+\frac{24}{13}=\frac{50}{13}=\frac{8^3}{13^3}\cdot\left(\frac{4x^2+9}{4x^2-9}\right)^3$ $\therefore$ $\left(\frac{4x^2+9}{4x^2-9}\right)^3=\frac{50}{13}\times\frac{13^3}{8^3}=\frac{50\times13^2}{8^3}$ $\therefore$ $x=\frac{27}{14}$

74. Dividend $=x^{(2^3)^{\frac{3x+1}{3}}} - y^{(2^3)^{\frac{3x+1}{3}}} = x^{2^{3x+1}} - y^{2^{3x+1}}$ and divisor $=x^{(2^3)^x} + y^{(2^3)^x} = x^{2^{3x}} + y^{2^{3x}}$ ∴

$$\begin{array}{l}
x^{2^{3x}}+y^{2^{3x}}\Big)\, x^{2^{3x+1}} - y^{2^{3x+1}} \Big(\, x^{2^{3x}} - y^{2^{3x}} \\
\qquad\qquad\quad \underline{x^{2^{3x+1}} + x^{2^{3x}}y^{2^{3x}}} \\
\qquad\qquad\qquad\qquad - x^{2^{3x}}y^{2^{3x}} + y^{2^{3x+1}} \\
\qquad\qquad\qquad\qquad \underline{- x^{2^{3x}}y^{2^{3x}} + y^{2^{3x+1}}} \\
\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}$$

Here $2^{3x+1} - 2^{3x}$
$=2\cdot2^{3x} - 2^{3x}$
$=2^{3x}(2-1) = 2^{3x}$

75. Let $x+m$ be their common measure then by ex. 85 $m=\dfrac{b-b'}{a-a'}$ now divide x^2+ax+b by $x+m$ it will be found that the remainder is $b-m+m^2$ and this must be $=0$ for the quantity x^2+ax+b is divisible by $x+m$ ∴ $b-am+m^2=0$ or $b-a\cdot\dfrac{b-b'}{a-a'}+\left(\dfrac{b-b'}{a-a'}\right)^2=0$ or $b(a-a')^2 - a(b-b')(a-a')+(b-b')^2=0$ or $(a-a')\{b(a-a')-a(b-b')\} + (b-b')^2=0$ or $(a-a')(ab'-a'b)+(b-b')^2=0$.

77. $\dfrac{81\sqrt{3}(\sqrt{3}+4\sqrt{x})}{(4\sqrt{x}+\sqrt{3})(4\sqrt{x}-\sqrt{3})}=(4\sqrt{x}-\sqrt{3})^2$ ∴ $(4\sqrt{x}-\sqrt{3})^3=81\sqrt{3}$ $=3^4\cdot3^{\frac{1}{2}}=3^{\frac{9}{2}}$ ∴ $4\sqrt{x}-\sqrt{3}=3^{\frac{3}{2}}$ ∴ $4\sqrt{x}=3^{\frac{1}{2}}+3^{\frac{3}{2}}=3^{\frac{1}{2}}+3^{\frac{1}{2}}\cdot3=3^{\frac{1}{2}}(1+3)=3^{\frac{1}{2}}\cdot4$ ∴ $x^{\frac{1}{2}}=3^{\frac{1}{2}}$ ∴ $x=3$.

78. 1st quantity $=x^3+x^2y+x^2z-xy^2-xz^2+2xyz-y^3+2y^2z-z^2y-y^2z+2yz^2-z^3=(x^3+x^2y+x^2z)-(xy^2+y^3+y^2z)-(xz^2+yz^2+z^3)+(2xyz+2y^2z+2yz^2)=x^2(x+y+z)-y^2(x+y+z)-z^2(x+y+z)+2yz(x+y+z)=(x+y+z)(x^2-y^2-z^2+2yz)$, and the 2nd quantity $=(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$ ∴ $x+y+z$ is the G. C. M.

79. The given eqn. may be put in this form $\frac{\sqrt{(x+1)}+\{-\sqrt{(x-1)}\}}{\sqrt{(x+1)}-\{-\sqrt{(x-1)}\}}$ $=\frac{x+\sqrt{(x^2-1)}}{x-\sqrt{(x^2-1)}}$ or $\frac{\sqrt{(x+1)}}{-\sqrt{(x-1)}}=\frac{x}{\sqrt{(x^2-1)}}$ or $\frac{\sqrt{(x+1)}}{-1}=\frac{x}{\sqrt{(x+1)}}$ or $x+1=-x$ ∴ $x=-\frac{1}{2}$.

80. $\frac{x}{b}-\frac{a}{b}+\frac{x}{a}-\frac{b}{a}+\frac{x}{c}-\frac{a}{c}-\frac{b}{c}=\frac{2x}{a+b}$ or $x\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{2x}{a+b}$ $=\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}$ or $\frac{x\left\{(a+b)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-2\right\}}{a+b}=\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}$

∴ $\frac{x}{a+b}\left(\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}\right)=\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}$ ∴ $\frac{x}{a+b}=1$.

81. Transposing, $5+2\sqrt{6}=x+y+2\sqrt{(xy)}$ ∴ $x+y=5$ and $2\sqrt{(xy)}=2\sqrt{6}$. Squaring the 1st eqn., $x^2+2xy+y^2=25$. Squaring the 2nd $4xy=24$ Subtract the 2nd from the 1st then $x^2-2xy+y^2=1$ ∴ $x-y=1$ and $x+y=5$ ∴ $2x=6$ ∴ $x=3$, $y=2$.

82. $x^3+3cx^2+2c^2x+5c^3=x^3+3cx^2+3c^2x+c^3$ ∴ $4c^3=c^2x$ ∴ $x=4c$.

83. Let x = no. of men in front of the solid square then by the question $x+3$ = no. of men in front of the hollow square then the no. of men in the hollow square $=(x+3)^2-(x+3-24)^2$ and in the solid square is x^2 ∴ $(x+3)^2-(x-21)^2=x^2$ whence $x=36$ ∴ $x^2=1296$.

85. Each of the quantities x^2+ax+b and x^2+cx+d is divisible by $x+m$ without remainder ∴ it is the G. C. M. ∴ dividing thus

$$\begin{array}{l}
x+m\,)\,x^2+ax+b\,(\,x+(a-m)\\
\qquad\quad \underline{x^2+mx}\\
\qquad\quad (a-m)x+b\\
\qquad\quad \underline{(a-m)x+(a-m)m}
\end{array}$$

Remainder $=b-(a-m)m=0$

Simly. $d-(c-m)m=0$

$\therefore b-(a-m)m=d-(c-m)m$ or $b-d=am-cm=(a-c)m$

$$\therefore m=\frac{b-d}{a-c}.$$

87. Let AH (fig. 16) be the light house, then approximately arc AB = HB which is the tangent to the earth from the top H of the light house $\therefore$ CH·HA = HB² or (CA + HA) HA = HB² or CA HA + HA² = HB² neglecting HA² which is too small when compared with the radius of the earth we have CA·HA = HB² but HA = 132 ft. $=\frac{1}{40}$ mile and CA = diameter = 8000 miles $\therefore$ HB² $=8000\times\frac{1}{40}=200=2\times100$ $\therefore$ HB $=10\sqrt{2}$.

88. $$\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(x-y)(z-y)}+\frac{z^2}{(z-x)(z-y)}=1,$$ (Ex. 67 p 43)

$$\frac{x}{(x-y)(x-z)}+\frac{y}{(x-y)(z-y)}+\frac{z}{(z-x)(z-y)}=0,\quad \frac{1}{(x-y)(x-z)}$$

$$\frac{1}{(x-y)(z-y)}+\frac{1}{(z-x)(z-y)}=0$$ (Ex 56p 42) $\therefore$ the given expression = 1.

90 From the 1st eqn. $mx+ny=\sqrt{(m^2+n^2)}$ $\therefore mx=\sqrt{(m^2+n^2)}-ny$...(A) from the 2nd eqn $y^2=1-x^2$, $x^2=1-y^2$ $\therefore y=\sqrt{(1-x^2)}$, $x=\sqrt{(1-y^2)}$ Substitute these in (A) $\therefore m\sqrt{(1-y^2)}=\sqrt{(m^2+n^2)}-ny$ squaring we have $m^2(1-y^2)=m^2+n^2+n^2y^2-2ny\sqrt{(m^2+n^2)}$ $\therefore (m^2+n^2)y^2-2ny\sqrt{(m^2+n^2)}+n^2=0$, $\therefore \sqrt{(m^2+n^2)}\,y-n=0$

$$\therefore y=\frac{n}{\sqrt{(m^2+n^2)}}\quad\therefore x=\sqrt{(1-y^2)}=\frac{m}{\sqrt{(m^2+n^2)}}$$ Substituting these in the 3rd eqn. we have $$\frac{m^2}{p^2(m^2+n^2)}+\frac{n^2}{q^2(m^2+n^2)}=\frac{1}{m^2+n^2}\text{ or }\frac{m^2}{p^2}+\frac{n^2}{q^2}=1.$$

91. If x^2+mx+n^2 being divided by $x+p$ leaves no remainder then it is a multiple of $x+p$ but we see that after the operation of division is gone through, there is a remainder $n^2-(m-p)p$ $\therefore$ the condition that x^2+mx+n^2 may be a multiple of $x+p$ is that $n^2-(m-p)p=0$, or $n^2+p^2=mp$.

92 $$\frac{x^2}{3}+\frac{11x}{6}\sqrt{x+1}-x-1\Big)\,x^2-\frac{x}{4}-\frac{1}{4}\Big(3$$

$$x^2+\frac{11x}{2}\sqrt{x+1}-3x-3$$

$$-\frac{11x}{2}\sqrt{x+1}+\frac{11x}{4}+\frac{11}{4}$$

$$=-\frac{11}{2}\sqrt{x+1}\Big[x-\tfrac{1}{2}\sqrt{x+1}\Big]$$

$$\frac{x^2}{3}+\frac{11x}{6}\sqrt{x+1}-x-1$$

$$\underline{\hspace{6em}}\quad\underline{6}$$

$$x-\tfrac{1}{2}\sqrt{(x+1)}\Big)\,2x^2+11x\sqrt{(x+1)}-6x-6\Big(2x+12\sqrt{(x+1)}$$

$$\underline{2x^2-\quad x\sqrt{(x+1)}}$$

$$12x\sqrt{(x+1)}-6x-6$$

$$\underline{12x\sqrt{(x+1)}-6x-6}$$

93. $2b(ab-ac-b^2+bc)=(c-a)(a^2-ab+bc-c^2)$ or $2ab^2-2abc-2b^3+2b^2c=a^2c-abc+bc^2-c^3-a^3+a^2b-abc+ac^2$ or $a^3+c^3-2b^3=a^2c+a^2b+bc^2+ac^2-2ab^2-2b^2c$ or $2(a^3+c^3-2b^3)=a^2c+a^2b+a^3+bc^2+ac^2+c^3-2ab^2-2b^3-2b^2c=a^2(a+b+c)+c^2(a+b+c)-2b^2(a+b+c)=(a^2+c^2-2b^2)(a+b+c)$.

94. The road coils round the conical hill; if a string were put in this position round the hill, it would, when stretched, form the hypotenuse of a right angled triangle of which the height of the hill will be the perpendicular and the horizontal line from the foot of the hill would be the base and this imaginary triangle is similar to the right angled triangle of which the perpendicular $=1$ and hypotenuse $=8$; by a gradient of 1 in 8, it is meant that in a length of 8 in the direction of the slope there is a rise of 1 from the level. $\therefore 1:8::4500:x \quad \therefore x=36000$.

96. $xy+x+y=2 \quad \therefore x(y+1)=2-y \quad \therefore x=\frac{2-y}{y+1},\ \frac{xy(4-xy)}{xy-1}$

$$\frac{\frac{y(2-y)}{y+1}\left\{4-\frac{y(2-y)}{y+1}\right\}}{\frac{y(2-y)}{y+1}-1}=\frac{y(2-y)\{4(y+1)-4(2-y)\}}{(y-y^2-1)(y+1)}$$

$$=\frac{y(2-y)(4+2y+y^2)}{-1-y^3}=\frac{y(8-y^3)}{-(1+y^3)}=\frac{y(y^3-8}{1+y^3}=\frac{y^4-8y}{1+y^3}$$ Similarly y

$$=\frac{2-x}{x+1},\quad\therefore\ \frac{xy(4-xy)}{xy-1}=\frac{\frac{x(2-x)}{x+1}\left\{4-\frac{x(2-x)}{x+1}\right\}}{\frac{x(2-x)}{x+1}-1}$$

$$=\frac{x(2-x)\{4(x+1)-x(2-x)\}}{(x-x^2-1)(x+1)}=\frac{x(2-x)(4+2x+x^2)}{-1-x^3}=\frac{x(8-x^3)}{-(1-y^3)}$$

$$=\frac{x(x^3-8)}{1+x^3}=\frac{x^4-8x}{1+x^3}\quad\therefore\ \frac{x^4-8x}{1+x^3}=\frac{y^4-8y}{1+y^3}=\frac{xy(4-xy)}{xy-1}.$$

97. $yz+xz+xy=\frac{1}{xy}+\frac{1}{yz}+\frac{1}{xz}=3\quad\therefore\ x+y+z=3xyz$;

$\therefore\ yz+zx+xy=3\quad\therefore\ (1+xy)(1+yz)(1+xz)=1+xy+yz+zx+xyz$
$(x+y+z)+x^2y^2z^2=1+3((xy+yz+xz)-2(xy+yz+xz)+3xyz$
$(x+y+z)-2xyz(x+y+z)+x^2y^2z^2=1+(xy+yz+xz)^2-2(xy+yz+xz)$
$+(x+y+z)^2-2xyz(x+y+z)+x^2y^2z^2=1+(x+y+z)^2-2$
$(xy+yz+xz)+(xy+yz+xz)^2-2xyz(x+y+z)+x^2y^2z^2=1+x^2+y^2+z^2$
$+x^2y^2+y^2z^2+x^2z^2+x^2y^2z^2=(1+x^2)(1+y^2)(1+z^2)$.

98. $x+y+z+w=0\quad\therefore\ x+y=-(z+w)\quad\therefore\ x^2+y^2=(z+w)^2-2xy$
$\therefore\ x^2+y^2+z^2+w^2=2\{(z+w)^2-xy-zw\}$; $x^3+y^3+z^3+w^3=-3xy$
$(x+y)-3zw(z+w)=-3xy(x+y)+3zw(x+y)=-3(x+y)(xy-zw)$;
$x^5+y^5+z^5+w^5=-5xy(x^3+2x^2y+2xy^2+y^3)-5zw(z^2+2z^2w$
$+2zw^2+w^3=-5\{xy(x+y)^3-x^2y^2(x+y)+zw(z+w)^3-z^2w^2(z+w)\}$
$=-5\{xy(x+y)^3-x^2y^2(x+y)-zw(x+y)^3+z^2w^2(x+y)=-5(x+y)$
$\{xy(x+y)^2-zw(x+y)^2-x^2y^2+z^2w^2\}=-5(x+y)(xy-zw)\{(x+y)^2$

$-xy-zw\}$ ∴ $\frac{6(x^5+y^5+z^5+w^5)}{5(x^3+y^2+z^2+w^3)}=\frac{-30(x+y)(xy-zw)\{(x+y)^2-xy-zw\}}{-15(x+y)(xy-zw)}$

$=2\{(x+y)^2-xy-zu\}=2\{(z+w)^2-xy-zu\}=x^2+y^2+z^2+w^2$.

99. The 1st quantity $=a^nb^n-b^nc^n+c^nd^n-a^nd^n=b^n(a^n-c^n)-d^n(a^n-c^n)=(b^n-d^n)(a^n-c^n)$ but b^n-d^n is divisible by $b-d$ and a^n-c^n is divisiele by $a-c$ ∴ the 1st quantity is divisible by $(b-d)(a-c)$ or by $ab-bc+cd-ad$.

100. Let $x-a=m$, $x-b=n$, $x-c=p$ $m+n+p=3x-(a+b+c)=0$ ∴ $m+n=-p$ squaring $m^2+n^2+2mn=p^2$ ∴ $m^2+n^2-p^2=-2mn$, squaring $m^4+n^4+p^4+2m^2n^2-2m^2p^2-2n^2p^2=4m^2n^2$ ∴ $m^4+n^4+p^4=2(m^2n^2+m^2p^2+n^2p^2)$ substitute the values of m, n, p.

101. The expression $=\frac{2^{x+4}-2^{x+1}}{2^{x+2}\times 2^2}=\frac{2^{x+1}(2^3-1)}{2^{x+1}\cdot 2\cdot 2^2}=\frac{7}{8}$

102. $\frac{a+b}{x+b}-1+\frac{a+c}{x+c}-1=\frac{2(a+b+c)}{x+b+c}-2$ or $\frac{a-x}{x+b}+\frac{a-x}{x+c}$ $=\frac{2(a-x)}{x+b+c}$ or $\frac{1}{x+b}+\frac{1}{x+c}=\frac{2}{x+b+c}$ ∴ $\frac{1}{x+b}-\frac{1}{x+b+c}=\frac{1}{x+b+c}-\frac{1}{x+c}$ ∴ $\frac{c}{(x+b)(x+b+c)}=\frac{-b}{(x+b+c)(x+c)}$ ∴ $\frac{c}{x+b}=\frac{-b}{x+c}$

∴ $x=-\frac{b^2+c^2}{b+c}$.

103 Let $x=$ no of yds. travelled by A in 1 min.
y...B in ....

then $\frac{1716}{y}-\frac{1760}{x}=\frac{51}{60}$...(1) and $\frac{1672}{x}+1\frac{1}{4}=\frac{1760}{y}$. (2)

multiply (1) by 40 and (2) by 32 then $\frac{1716\times 40}{y}-\frac{70400}{x}=34$

and $\frac{1760\times 39}{y}-\frac{65208}{x}=\frac{195}{4}$, subtract the 1st from the 2nd then

$\frac{5192}{x}=\frac{195-136}{4}=\frac{59}{4}$ $\therefore x=352$ $\therefore$ no of min. $=\frac{1760}{352}=5$, also from

eqn. (1) $\frac{1716}{y}-5=\frac{51}{60}$ $\therefore y=\frac{34320}{117}$ $\therefore$ no of min to run a mile

$=\frac{1760\times 117}{34320}=6.$

106 Let AB $=220$ and Q, P, R be the positions when P is equidistant from Q and R,

A Q P R B

let $x=$ no of hours required by P, then AP $=25x$, AQ $=(x-\frac{5}{4})20$, BP $=220-25x$, BQ $=(x-\frac{7}{4})30$, now PQ $=$ PR, i.e., $25x$ $-(x-\frac{5}{4})20=(220-25x)-(x-\frac{7}{4})30$ $\therefore$ $x=5$.

109. $\frac{x}{a^2+x^2}=\frac{2y}{a^2+y^2}=\frac{4z}{a^2+z^2}$, then each fraction $=\frac{x-2y}{x^2-y^2}$

$=\frac{2y-4z}{y^2-z^2}=\frac{4z-x}{z^2-x^2}$ $\therefore$ $\frac{4z(x^2-y^2)}{4z(x-2y)}=\frac{x(2y^2-x^2)}{x(2y-4z)}=\frac{2y(z^2-x^2)}{2y(4z-x)}=k$ say

$\therefore$ $4z(x^2-y^2)+x(y^2-z^2)+2y(z^2-x^2)=k\{4z(x-2y)+x(2y-4z)$

$+2y(4z-x)\}=k\{4xz-8yz+2xy-4xz+8yz-2xy\}=0.$

110. Let $\frac{y-z}{x}=a$, $\frac{z-x}{y}=b$, $\frac{x-y}{z}=c$ then the left side expression $=(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=3+\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}$, now $\frac{b+c}{a}=\frac{1}{a}$

$(b+c)=\frac{x}{y-z}\left(\frac{z-x}{y}+\frac{x-y}{z}\right)=\frac{x}{y-z}\cdot\frac{xy-y^2+z^2-xz}{yz}=\frac{x}{y-z}.$

$\frac{(y-z)(x-y-z)}{yz}=\frac{x^2-xy-xz}{yz}=\frac{2x^2-x(x+y+z)}{yz}=\frac{2x^2}{yz}$. Similarly

$\frac{a+c}{b}=\frac{2y^2}{xz}$ and $\frac{a+b}{c}=\frac{2z^2}{xy}$ $\therefore \frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}=\frac{2x^2}{yz}+\frac{2y^2}{xz}+\frac{2z^2}{xy}$ $=\frac{2(x^3+y^3+z^3)}{xyz}=\frac{2\times 3xyz}{xyz}$ (by ex 1 p. 121) $=6$

111. $x+y+z=2 \therefore 1-x=y+z-1$, squaring $(1-x)^2=y^2+z^2+2yz+1-2(y+z)=2-x^2+2yz+1-2(2-x)=2yz-x^2+2x-1=2yz-(1-x)^2$ or $2(1-x)^2=2yz \therefore (1-x)^2=yz \therefore x(1-x)^2=xyz$; again $1-y=x+z-1$, squaring, $(1-y)^2=x^2+z^2+2xz+1-2(x+z)$ $=2-y^2+2xz+1-2(2-y)=2xz-y^2+2y-1=2xz-(1-y)^2 \therefore (1-y)^2$ $=xz \therefore y(1-y)^2=xyz$, similarly $(1-z)^2=xy \therefore z(1-z)^2=xyz$ $\therefore x(1-x)^2=y(1-y)^2=z(1-z)^2=xyz.$

112. The expression $=x^2(1-y^2)+y^2(1-x^2)+2xy\sqrt{(1-x^2)(1-y^2)}$ $+x^2y^2+(1-x^2)(1-y^2)-2xy\sqrt{\{(1-x^2)(1-y^2)\}}=x^2-x^2y^2+y^2-x^2y^2$ $+x^2y^2+1-x^2-y^2+x^2y^2=1.$

113. $\left(\frac{y}{z}+\frac{z}{y}\right)^2=m^2+4,\ \left(\frac{z}{x}+\frac{x}{z}\right)^2=n^2+4,\left(\frac{x}{y}+\frac{y}{x}\right)^2=p^2+4$

$\therefore \left(\frac{y}{z}+\frac{z}{y}\right)^2+\left(\frac{z}{x}+\frac{x}{z}\right)^2+\left(\frac{x}{y}+\frac{y}{x}\right)^2=m^2+4+n^2+4+p^2+4$ or

$4+\left(\frac{y}{z}+\frac{z}{y}\right)\left(\frac{z}{x}+\frac{x}{z}\right)\left(\frac{x}{y}+\frac{y}{x}\right)$ (Ex. 21 p. 120) $=m^2+n^2+p^2+12$ or

or $\sqrt{(4+m^2)(4+n^2)(4+p^2)}=m^2+n^2+p^2+8$, squaring both sides, we have $64+16(m^2+n^2+p^2)+4(m^2n^2+m^2p^2+n^2p^2)+m^2n^2p^2=m^4$ $+n^4+p^4+64+2m^2n^2+2m^2p^2+2n^2p^2+16(m^2+n^2+p^2)$ or $m^4+n^4+p^4$ $=2m^2n^2+2m^2p^2+2n^2p^2+m^2n^2p^2.$

115. $2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4=4a^2b^2-(a^2+b^2-c^2)^2$ $=3(xy+yz+xz)^2.$

118. The expression $=x^n-a^n-na^{n-1}x+na^n=x^n-a^n-na^{n-1}(x-a)$ $=(x-a)\{x^{n-1}+ax^{n-2}+a^2x^{n-3}+\ldots\ldots a^{n-1}\}-(x-a)na^{n-1}$: if this be

divided by $x-a$ the quotient $=x^{n-1}+ax^{n-1}+a^{n-2}+a^2x^{n-3}+\ldots\ldots a^{n-1}$ $-na^{n-1}$ but $na^{n-1}=a^{n-1}+a^{n-1}+a^{n-1}+\ldots\ldots$to n terms ∴ quotient $=(x^{n-1}-a^{n-1})+(ax^{n-2}-a.\,a^{n-2})+\ldots\ldots$to n terms $=(x^n-a^{n-1})+a(x^{n-2}-a^{n-2})+a^2(x^{n-3}-a^{n-3})+$ &c to n terms and this is again divisible by $x-a$ ∴ the expression is divisible by $(x-a)^2$

119. $1+\frac{3a}{x+a+m}+4-\frac{3a-6m}{x+a-m}=5$ or $\frac{3a}{x+a+m}-\frac{3(a-2m)}{x+a-m}=0$

or $\frac{a}{x+a+m}-\frac{a-2m}{x+a-m}=0$ ∴ $\frac{a}{a-2m}=\frac{x+a+m}{x+a-m}$

$$ax+a^2-am=ax+a^2+am-2mx-2am-2m^2$$

$$\therefore\ -2mx=2m^2\ \therefore\ x=-m.$$

120 Let x = height of the hill in miles, then $8x$ = length of the slope, now pace on level $=4$ ∴ pace up hill $=3$ and pace down hill $=5$ $\frac{8x}{3}+\frac{8x}{5}=5$ ∴ $x=\frac{75}{64}$ miles $=2062\frac{1}{2}$ yds

121. $\frac{\text{Sum of all the antecedents}}{\text{Sum of all the consequents}}=$ each ratio

$$\therefore\ \frac{x_1+x_2+x_3+\ldots x_n}{x_2+x_3+x_4+\ldots x_{n+1}}=\frac{x_1}{x_2}\ \therefore\ \left(\frac{x_1+x_2+x_3\ldots x_n}{x_2+x_3+x_4+\ldots x_{n+1}}\right)^n$$

$$=\left(\frac{x_1}{x_2}\right)^n=\frac{x_1}{x_2}\times\frac{x_2}{x_3}\times\frac{x_3}{x_4}\times\text{\&c}\ldots\frac{x_n}{x_{n+1}}=\frac{x_1}{x_{n+1}}.$$

124. The angle described by the planet in the unit of time is $\frac{360}{P}$, and that by the earth $\frac{360}{E}$, hence their separation in this time is $\frac{360}{P}-\frac{360}{E}$, but since they separate by 360 in the time T their

separation in the unit of time is also $\frac{360}{T}$; equating these quantities we have $\frac{1}{P}-\frac{1}{E}=\frac{1}{T}$ whence $P=\frac{TE}{T+E}$.

For the superior planets, the equation is $\frac{1}{E}-\frac{1}{P}=\frac{1}{T}$ whence $P=\frac{TE}{T-E}$.

125. $P=\frac{584\times365}{584+365}=224$ days.

126. $\left(2+\frac{1}{x+5}\right)-\left(3+\frac{3}{3x-4}\right)=\left(4+\frac{1}{x+3}\right)-\left(5+\frac{3}{3x-10}\right)$ or $\frac{1}{x+5}-\frac{3}{3x-4}=\frac{1}{x+3}-\frac{3}{3x-10}$ or $\frac{-19}{(x+5)(3x-4)}=\frac{-19}{(x+3)(3x-10)}$ ∴ $(x+5)(3x-4)=(x+3)(3x-10)$ or $3x^2+11x-20=3x^2-x-30$ ∴ $12x=-10$ ∴ $x=-\frac{5}{6}$.

127. $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}=m$ say; then $a=mx$; $b=my$; $c=mz$ ∴ $\frac{a^2}{m^2}=m^2\frac{x^2}{m^2}$; $\frac{b^2}{n^2}=m^2\frac{y^2}{n^2}$, $\frac{c^2}{p^2}=m^2\frac{z^2}{p^2}$ ∴ $\frac{a^2}{m^2}+\frac{b^2}{n^2}+\frac{c^2}{p^2}=m^2\left(\frac{x^2}{m^2}+\frac{y^2}{n^2}+\frac{z^2}{p^2}\right)$ $=m^2$, and $\frac{a^2}{x^2}=\frac{b^2}{y^2}=\frac{c^2}{z^2}=m^2=\frac{a^2+b^2+c^2}{x^2+y^2+z^2}$.

129. By transpositon $x^3-3x^2-10x+24=0$ or $(x-2)(x+3)(x-4)=0$ ∴ $x-2=0$, $x+3=0$, $x-4=0$ ∴ $x=2$, $x=-3$, $x=4$.

131. Let the given expression be the cube of $2x-p$; then $8x^3-4x^2(a+b+c)+2x(ab+bc+ac)-abc=(2x-p)^3=8x^3-12x^2p+6xp^2-p^3$; hence $-4(a+b+c)=-12p$ and $2(ab+bc+ac)=6p^2$

and $-abc=-p^3 \quad \therefore\ a+b+c=3p,\ ab+bc+ac=3p^2$ and $abc=p^3$

$$\therefore (ab+ac+bc)\left(\frac{1}{ab}+\frac{1}{ac}+\frac{1}{bc}\right)=(ab+ac+bc)\left(\frac{a+b+c}{abc}\right).$$

$$=3p^2\ \frac{3p}{p^3}=9=3^2.$$

133. $\frac{ax}{a+x}+\frac{by}{b+y}=\frac{c(a+b)}{a+b+c}=\frac{ac}{a+b+c}+\frac{bc}{a+b+c}\ \therefore \frac{ax}{a+x}$

$-\frac{ac}{a+b+c}=\frac{bc}{a+b+c}-\frac{by}{b+y}$, or $\frac{a^2x+abx+acx-a^2c-acx}{(a+x)(a+b+c)}$

$=\frac{b^2c+bcy-aby-b^2y-bcy}{(a+b+c)(b+y)}\ \therefore \frac{a(ax+bx-ac)}{a+x}=\frac{b(bc-ay-by)}{b+y}$

but $x+y=c \quad \therefore\ x=c-y$. Substituting the value of x in the left side, we have $\frac{a(ac-ay+bc-by-ac)}{a+x}$

$\frac{b(bc-ay-by)}{b+y} \quad \therefore\ bc-ay-by=0 \quad \therefore\ y=\frac{bc}{a+b} \quad \therefore\ x=\frac{ac}{a+b}.$

134. The man must first carry over the goat, and then return for the wolf; when he carries over the wolf, he must take back with him the goat, and leave it, in order to carry over the cabbage; he may then return and carry over the goat. By these means, the wolf will never be left with the goat, nor the goat with the cabbage, but when the boatman is present.

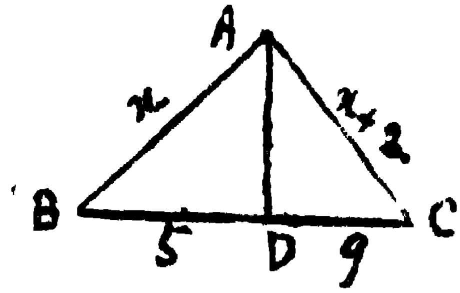
137. Two servants cross first, and one of them, rowing back the boat, carries over the third servant. One of the three servants then returns with the boat, and remaining, suffers the two gentlemen whose servants have crossed to go over in the boat. One of the gentlemen then carries back his servant, and leaving him on the bank, rows over the third gentleman. In the last place, the servant who had crossed enters the boat, and returning twice, carries over the other two servants.

ERRATA.

In page 4 ex. 17 for 2370 read 3432.
,, 5 ex. 67 for 48 read 61.
,, 11 ex. 16 for $+c$ read $-2c$.
,, 11 ex. 24 for $a-1$ read 1.
,, 12 ex. 14, $+2xy$ to be ommitted.
,, 13 ex 43 for 35 read 40.
,, 14 ex, 57 read $x^{\frac{1}{2}}+x^{\frac{1}{3}}y^{\frac{1}{6}}+x^{\frac{1}{6}}y^{\frac{1}{3}}+y^{\frac{1}{2}}$.
,, 17 ex, 27 read at the end "rem $2x^4$"
,, 22 ex. 6 after c^2 read $-ab$.
,, 27 ex. 93 for $5x^3$ read x^3.
,, 28 ex. 13 for $x-x$ read x^2-x.
,, 36 ex. 22 for x^3 read x^4.
,, 39 ex. 7 for $70b$ read $70b^2$.
,, 41 ex 41 for the denominator of the ans read a^4-x^4.
,, 41 ex. 45 for $x+8$ read $-7x$.
,, 41 ex. 47 for 25 read 34.
,, 62 ex. 16 for $2\sqrt{3}+2$ read $\sqrt{3}+1$.
,, 62 ex. 18 for 13 read 32.
,, 63 ex. 27 for $\frac{1}{\sqrt{3}}-\frac{1}{3}$ read $\frac{-1\pm\sqrt{(-3)}}{2}$.
,, 66 ex. 39 for $\frac{57}{241}$ read $\frac{57}{121}$.
66 ex. 46 for $\frac{1}{10}$ read $2\frac{1}{4}$.
,, 66 ex. 50, the eqn. being quadratic may be ommitted.
,, 66 ex. 52 for $x=2$ read $x=2\frac{1}{4}$.
,, 66 ex. 51 for the given eqn read $(x+2)(x+4)+5$
$=(x+3)^2+x$.
,, 67 ex. 58 for $x=3$ read $x=-\frac{11}{4}$.
,, 68 ex. 75 for $x=1$ read $x=8$
,, 81 ex. 291 for $(x^2+20x+51)^{\frac{1}{2}}$ read $\frac{3}{10}(x^2+20x+51)^{\frac{1}{2}}$
,, 102 ex. 11 for 19 read 10.

To be had at Sarodaic Poostakalay, Garanhatta, Calcutta.

Fig 1



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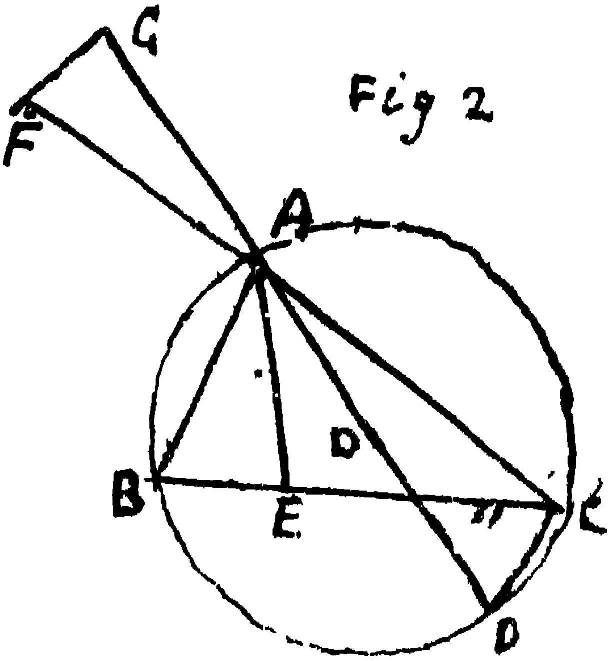

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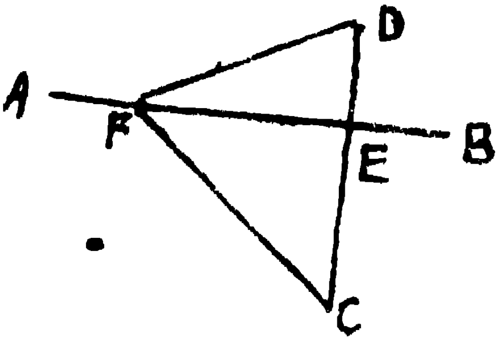

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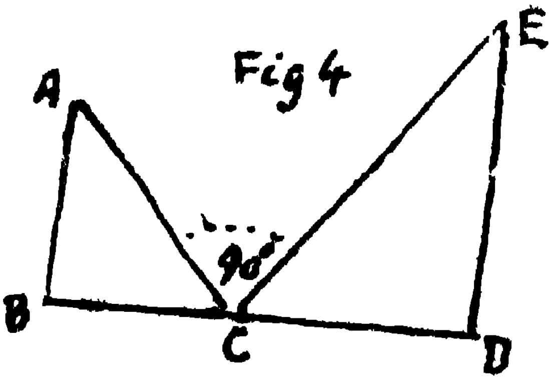

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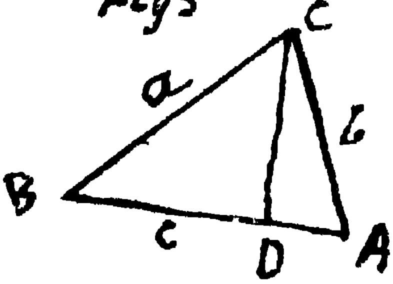

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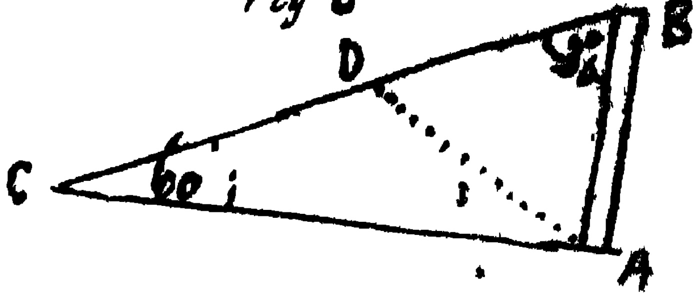

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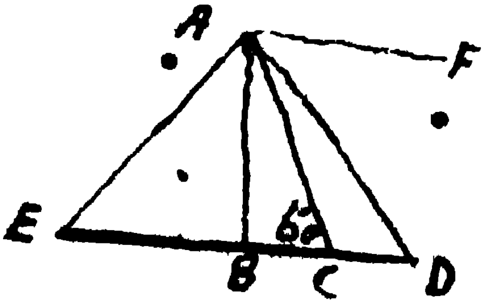

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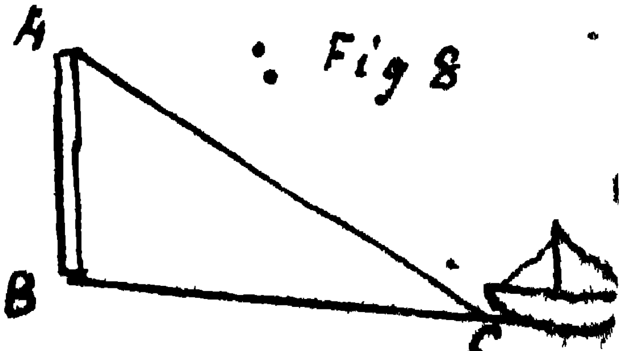

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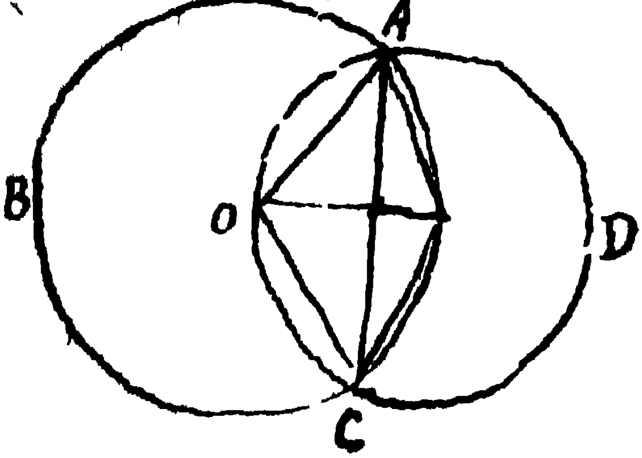

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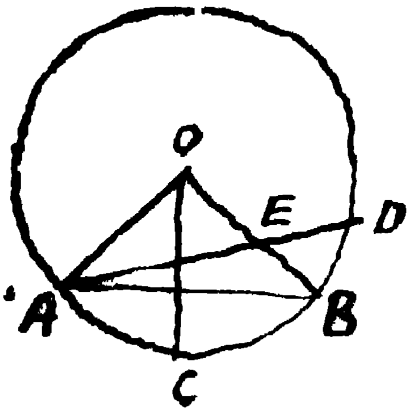

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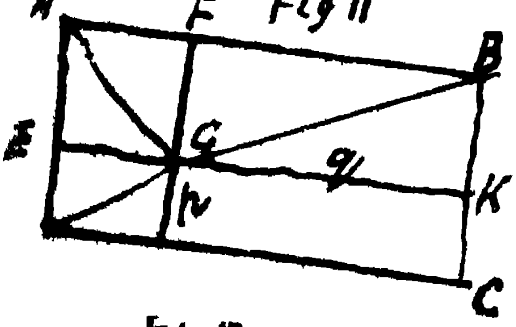

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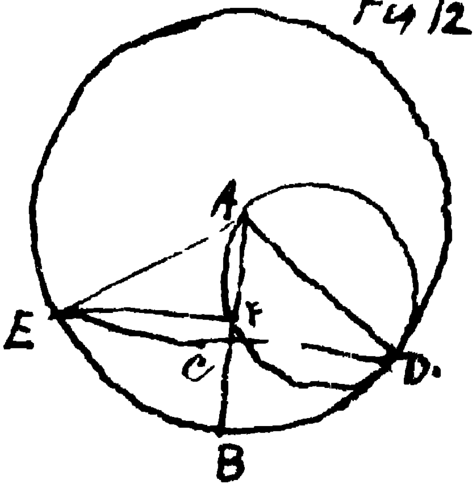

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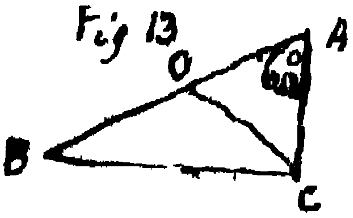

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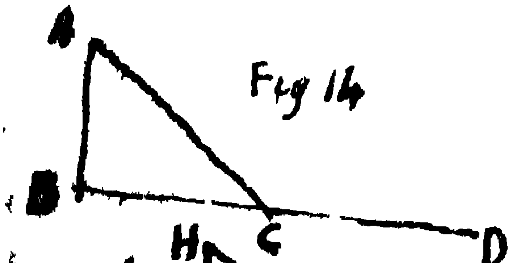

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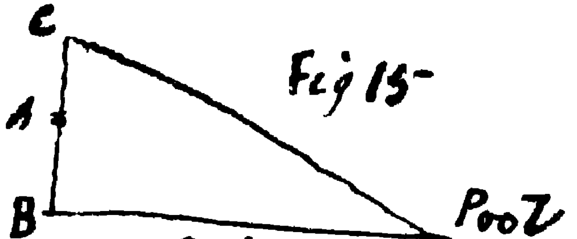

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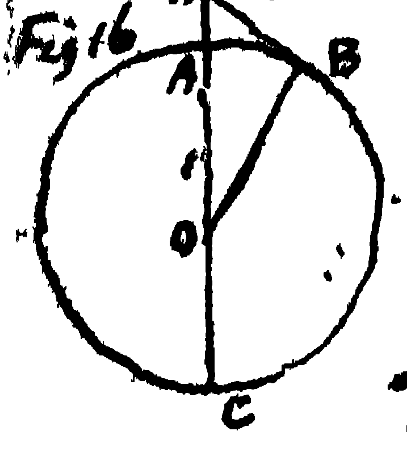

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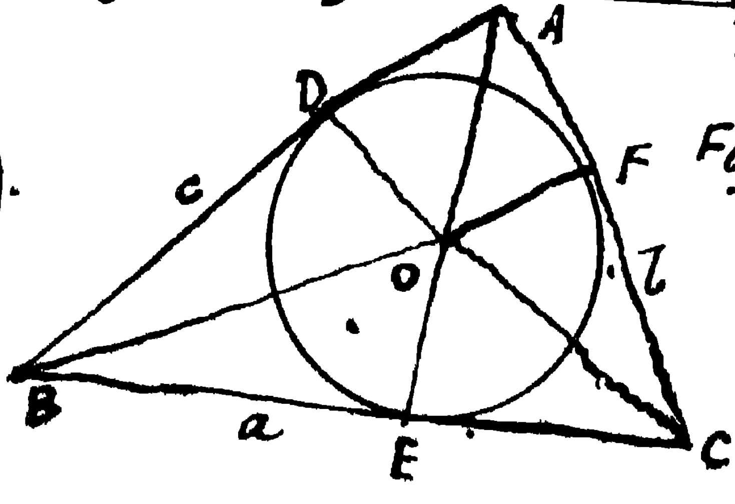

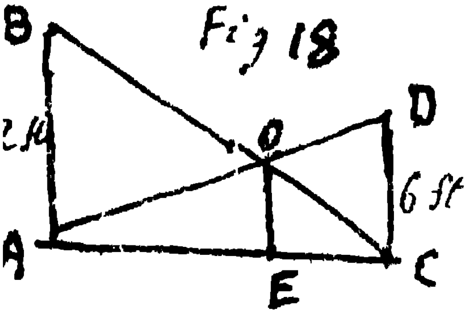
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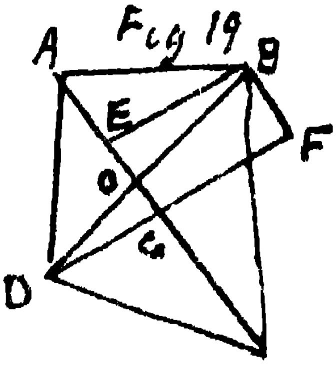
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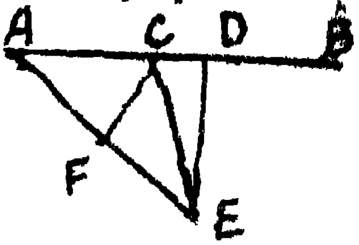
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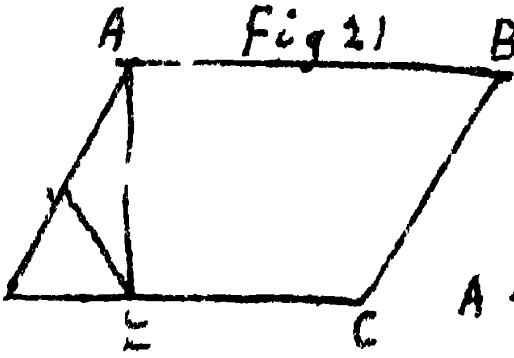
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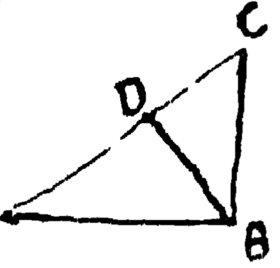
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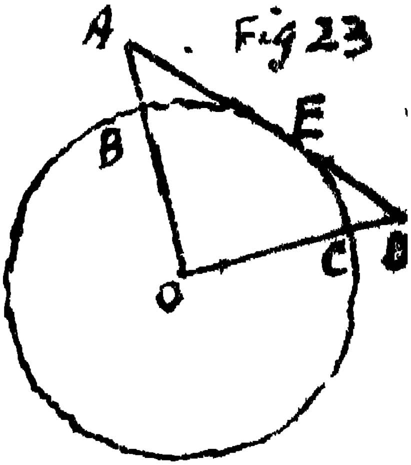
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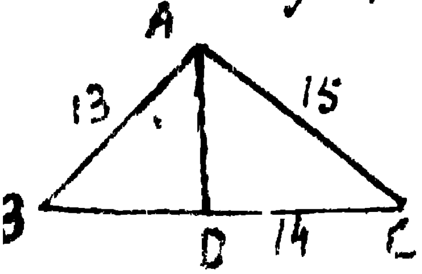
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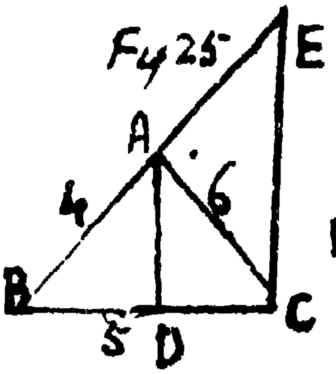
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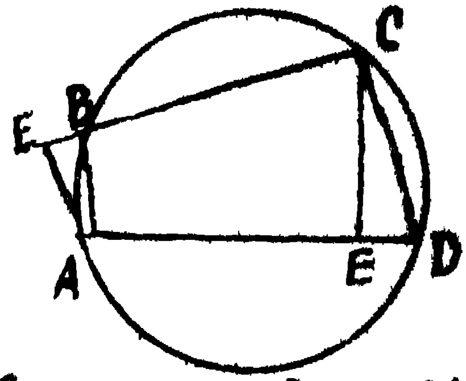
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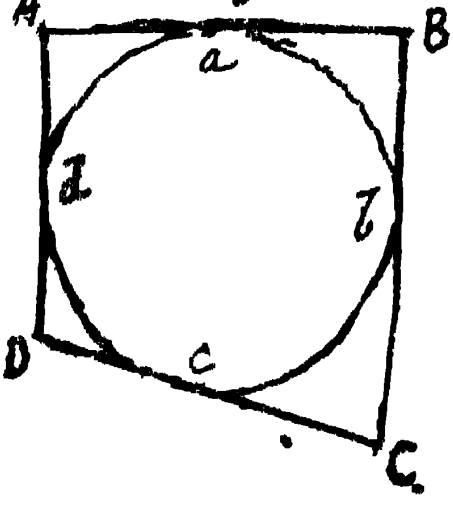
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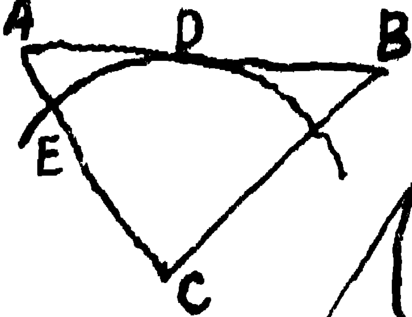
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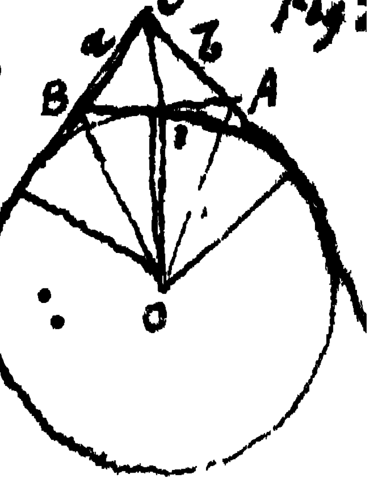
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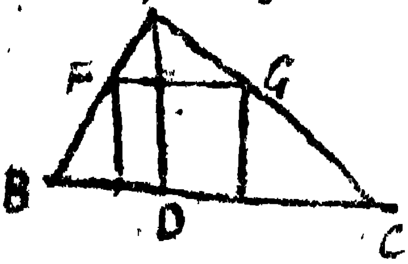
Fig 30



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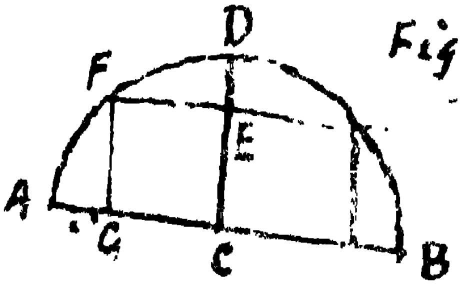

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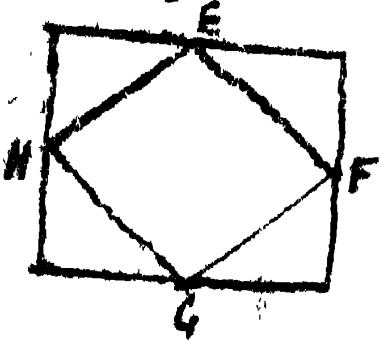

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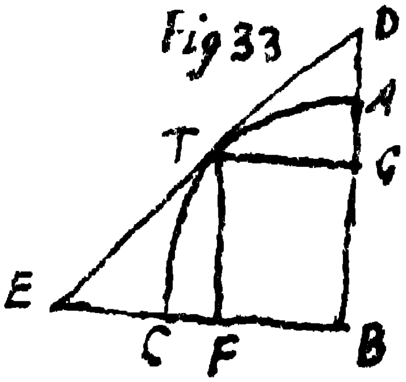

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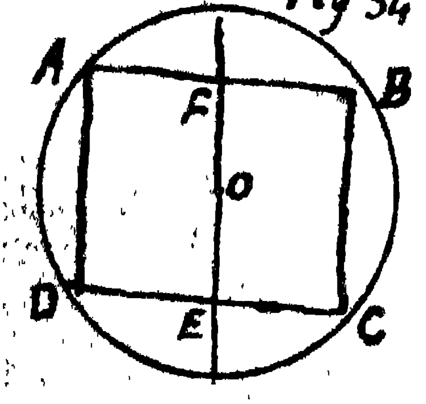

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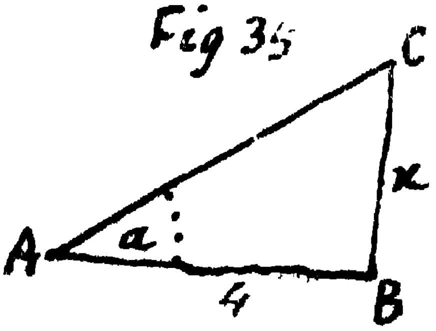

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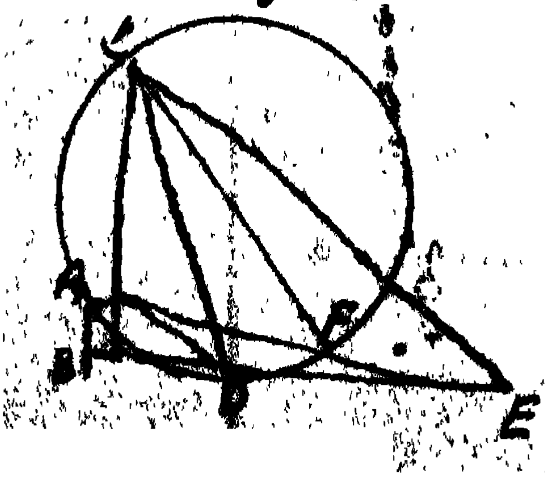

Fig 37